QUANTUM THEORETIC MACHINES

WHAT IS THOUGHT FROM THE POINT OF VIEW OF PHYSICS

AUGUST STERN

NORTH-HOLLAND
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AUGUST STERN
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Humanity will never solve its problems until we understand how we think.

Albert Einstein

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NOTATIONS

SYMBOLS

$\mathbf{Ç}$ - logical vacuum (phase space)
$\mathbf{\Phi}$- matrix statistics
L - **(2x2)** matrix logical operator
L - **(3x3)** logical (quark) operator
π- logical meson
U - quantum evolution operator
H - quantum Hamiltonian
H - Hilbert space
ψ - quantum-mechanical wavefunction
$\sqrt[i]{}$ - complex square root
σ_z - z-component of spin
$\hbar$ - action quantum
c - fundamental velocity
u /d - up/down quark
W(x,p) - Wigner distribution
W(L) - operator wave
w(L) - weight of a matrix operator
$e_n^{\mathbf{L}}$ - n-terms logical exponential
∞(k) - (topo)logical potential

T - topological energy

μ - integral logical charge

$\ddot{\imath}$ - iqon (half-quark) charge

β - bosonic topological charge

ϕ - fermionic topological charge

ϕ - golden ratio

F - Fredkin gate

$|q\rangle$ - normalized logical state

$|s_{\pm}\rangle$ - superposed state

λ - eigenvalue of an operator

λ - logical quantum number

n - principal quantum number

l - orbital quantum number

m - magnetic quantum number

B - Boolean energy constant

$\mathbf{L}^*$- dual logical operator

θ - unitary transform

A^+- conjugate transpose

$\tilde{\mathbf{L}}$ - transpose

γ_i, Γ^m - Dirac matrices

α - truth-value of a logical string

α' - fundamental string constant

D - dispersion function

p - classical probability

i - improbability

x - position operator

p - quantum momentum operator

q - logical position operator

$'$- differentiation operator

M - logical momentum operator

M - manifold

Tr - trace of an operator

Rt - retrace of an operator

$\sum(\mathbf{L})$ - global trace of **L**

$\multimap$ - right division

$\multimap$ - left division

[,] - Lie commutator
{ ,} - Wigner anticommutator
{ ,] - supercommutator
a^* - creation field operator
a - annihilation field operator
$\overline{A\,B}$ - contraction product
$\overline{L}$ - converse transpose
$\otimes$ - tensor product
$\oplus$ - addition **mod2**
$\emptyset$ - empty set
$\bullet$ - **AND**-product
t - scalar time
$\blacktriangle$ - time operator
Δ - Laplasian
H(·) - Hadamardian operator
$\exists$ - existential quantifier
$\forall$ - universal quantifier
ρ - quantum density matrix
Ω - logical density matrix
$\Box$ - necessarily
$\Diamond$ - possibly
Re/Im - real/imaginary component
{X} - coding error set
$\lrcorner$ - parity checks of matrix code
Q - quaternion number(matrix)
$\blacklozenge$ - Hadamard product
SU(n) - groups of unitary symmetry
F_n - Farey sequence of order n
f_n - Fibonacci number
| - stroke operator (identity)
0 - zero matrix(contradiction)
I - unit matrix
2 - two matrix
H - logical Lagrangian
1 - tautology (logical Hamiltonian)
GF(n) - Galois field with n elements
Ç(n, m) - code with n codewords of length m
Λ - empty symbol (puncturing error)
K(n) - knot with n crossings

OPERATORS

M	▼	▲	N	I	H	1	0
$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$
∧, n	∨	→	←	↓	↑	\, a*	/, a
$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$
⊥	⊤	⊣	⊢	$\vee^{-1}$	$\rightarrow^{-1}$	$\leftarrow^{-1}$	$\downarrow^{-1}$
$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}$
$\dot{H}$	$H\dot{}$	.H	H.	$-\dot{H}$	$-H\dot{}$	-.H	-H.
$\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$
$\circ\vee$	$\vee^{\circ}$	$^{\circ}\rightarrow$	$\rightarrow_{\circ}$	$^{\circ}\leftarrow$	$\leftarrow_{\circ}$	$\circ\downarrow$	$\downarrow^{\circ}$
$\begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix}$
W(⊥)	W(⊤)	ln(⊥)	ln(⊤)	W(⊣)	W(⊢)	ln(⊣)	ln(⊢)
$\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix}$	$\begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix}$	$\begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix}$
+	↕	$\rightarrow^2$	$\leftarrow^2$	σ, σ_z	XOR	∧	M
$\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$	$\begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}$	$\begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$	$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	•	$-\frac{d}{dq}$
Rt	θL	$\tilde{L}$	q	[q, M]	A	{∧, ∨]	Ω
Tr^{-1}	NLN	$\theta\tilde{L}$	q	1	$\frac{d^2}{dq^2}$	∧∨±∨∧	|q><q|

NAMES

OPERATOR	SYMBOL
YES	I
NOT	N
AND	$\wedge$
OR	$\vee$
IMPLY	$\rightarrow$
IF	$\leftarrow$
ON	\
OFF	/
NOR	$\uparrow$
NAND	$\downarrow$
TRUE	$\perp$
FALSE	$\top$
ONE	1
NONE	0
MOVE	M
ADD	+
COUNTER	$\rightarrow^n$
TIME	▲
SPIN	σ

QUANTUM NUMBERS

ATOMIC PHYSICS	MATRIX LOGIC
Principal quantum number n $n = 1, 2, ...$	Synergy logical number S $S = 0, 2$
Orbital quantum number l $l = 0, ..., n-1$	Boolean amplitude number A $A = 0, 1$
Magnetic quantum number m $-l \leq m \leq l$	Logical phase number P $P = -1, 0, 1$

NONSTANDARD OPERATIONS

RETRACE is the inverse of the trace tensor operation which recovers nonuniquely an unknown matrix from its trace,

$$\mathbf{Rt} \langle x|y\rangle = |x\rangle\langle y|.$$

Trace of a retrace is identity:

$$\mathbf{TrRt} = 1.$$

SUPERCOMMUTATION denoted by the bracket structure {A,B]. Represents an extended logical suprasymmetry when the commutator [A,B] = AB - BA and the anticommutator {A,B} = AB + BA are equal. Ideally, the operators A and B supercommute if

$$[A,B] = \{A,B\},$$

but typically a correction matrix $C \neq \mathbf{I}$ is required to enforce the equality

$$C[A,B] = C\{A,B\}.$$

ORTHOCOMMUTATION is denoted by the bracket structure [A, B). The operators A and B orthocommute if

$$[A, B) = A \bullet B - \theta B \bullet A = 0,$$

where θB is unitary logical transform of B:

$$\theta B = \mathbf{N} \bullet B \bullet \mathbf{N}$$

CONVERSE TRANSPOSITION is a matrix transposition against the secondary diagonal, as opposed to ordinary transposition which swaps matrix elements against the principle diagonal:

$$\overset{\sim}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \begin{pmatrix} a & c \\ b & d \end{pmatrix}$$

transposition

$$\overset{\urcorner\ulcorner}{\begin{pmatrix} a & b \\ c & d \end{pmatrix}} = \begin{pmatrix} d & b \\ c & a \end{pmatrix}$$

converse transposition

Transposition is used to identify a symmetric matrix, whose elements are equal with respect to the principle diagonal:

$$a_{ij} = a_{ji}$$

Converse transposition is used identify a persymmetric ($\mathbf{n} \times \mathbf{n}$) matrix, whose elements are equal with respect to the secondary diagonal:

$$a_{ij} = a_{n+1-j,\, n+1-i}$$

MATHEMATICAL ABSTRACT

$$\sqrt{0} = \mathbf{M}$$

$$\sqrt{\mathbf{I}} = +1, -1, \mathbf{N}$$

$$\mathbf{FALSE \cdot TRUE = N \cdot TRUE \cdot FALSE}$$

$$[q, M]\,|q\rangle = |q\rangle$$

$$|q\rangle = c|p\rangle + \bar{c}|r\rangle$$

$$\mathbf{IF}^{\mathbf{IF}} = e^{a^*}$$

$$e^{a} = \rightarrow, \qquad e^{a^*} = \leftarrow$$

$$\rightarrow^{\rightarrow} = \rightarrow = \cos a + \sin a, \qquad \leftarrow^{\leftarrow} = \leftarrow = \cos a^* + \sin a^*$$

$$\ln \rightarrow = a = \overleftarrow{\leftarrow}, \qquad \ln \leftarrow = a^* = \overrightarrow{\rightarrow}$$

$$\int_0^{\infty} \delta(x)dx \equiv \int_0^{1} \langle p|q\rangle dq \equiv \int_0^{1} \langle p|q\rangle dp$$

$$\langle q|M|q\rangle = \langle \psi|\sigma|\psi\rangle$$

$$(M - \frac{d}{dq})|q\rangle = 0$$

$$' = [\mathbf{FALSE, TRUE}] = \mathbf{FALSE - TRUE}$$

$$-\frac{d|q\rangle}{dq} = H\sigma|q\rangle, \quad \mathbf{H = - \ln NOT}$$

$$\frac{d^2q}{dt^2} = <8\sigma>$$

$$(H\blacktriangle + \frac{d}{dq})|q> = 0, \qquad (1 + \blacktriangledown\frac{d}{dq})|q> = 0$$

$$H(AND \cdot OR) = H(AND + OR)$$

$$H[a, a^*] = H[\vee, \wedge]$$

$$\frac{d<q|\rightarrow|q>}{dq} = <q|[\rightarrow, H]|q>$$

$$[NOT, AND] = [NOT, OR]$$

$$H \cdot H = H + H$$

$$H \wedge H \wedge H = H \vee H \vee H$$

$$\text{rot NOT} = <p|\,[\sigma_x, \sigma_z]\,|q>, \qquad \text{rot YES} = <p|\,[\sigma_z, \sigma_x]\,|q>$$

$$<q|-\frac{d}{dq}|q> = \bar{q}^2 - q^2 = \bar{q} - q$$

$$NOT = a^* + a, \quad AND = a^*a, \quad TIME = a - a^*$$

$$\mu(AND) = 1/3, \qquad \mu(OR) = 2/3$$

$$\mu(a^*) = \mu(a) = 1/6$$

$$\underline{\rightarrow}^4 = \underline{\rightarrow}, \qquad \underline{\rightarrow}^* = \underline{\rightarrow}^{-1}$$

$$[H(\cdot), TIME] = 0, \qquad TIME^5 = TIME$$

$$\sigma|s_{\pm}> = |s_{\mp}>$$

$$\oint_0^{4\pi}|K_{\pm}(n)>d\sigma = |s_{\pm}>$$

$$H = \hbar T$$

THE MATRIX PRINCIPLE

Future terrestrial historians may judge the late 20th century as a time when theorists were like children playing on the seashore, diverting themselves with the smoother pebbles or prettier shells of superstrings while the great ocean of matrix theory lay undiscovered before them.

M. Duff

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MINDWARE

MAKING SENSE OF INNER SENSE

'*Terra cognita*' is *terra incognita*. It is difficult to find someone not taken aback and fascinated by the incomprehensible but indisputable fact: **there are material systems which are aware of themselves.** Consciousness is self-cognizing code. During *homo sapiens*'s relentness and often frustrated search for self-understanding various theories of consciousness have been and continue to be proposed. However, it remains unclear whether and at what level the problems of consciousness and intelligent thought can be resolved. Science's greatest challenge is to answer the fundamental question: **what precisely does a cognitive state amount to in physical terms?**

Albert Einstein insisted that the fundamental ideas of science are essentially simple and can be expressed in a language comprehensible to everyone. When one thinks about the complexities which present themselves in modern physics and even more so in the physics of life, one may wonder whether Einstein really meant what he said. Are we to consider the fundamental problem of the mind, whose understanding seems to lie outside the limits of the mind, to be essentially simple too? Knowledge is neither automatic nor universally deductive. Great new ideas are typically counterintuitive and outrageous, and connecting them by simple logical steps to existing knowledge is often a hard undertaking. The notion of a tensor was needed to provide the general theory of relativity; the notion of entropy had to be developed before we could get full insight into the laws of thermodynamics; the notion of information bit is crucial for communication theory, just as the concept of a Turing machine is instrumental in the deep understanding of a computer. To understand something, consciousness must reach an adequate intellectual level, even more so in order to understand itself. Reality is full of

unending mysteries, the true explanation of which requires very technical knowledge, often involving notions not given directly to intuition. Even though the entire content and the results of this study are contained in the eight pages of the mathematical abstract, it would be unrealistic and impractical to suggest that anyone can gain full insight into the theory that presented here after just reading abstract.

In our quest for knowledge we are exploring the remotest areas of the macrocosm and probing the invisible particles of the microcosm, from tiny neutrinos and strange quarks to black holes and the Big Bang. But the greatest mystery is very close to home: the greatest mystery is human consciousness. The question before us is whether the logical brain has evolved to a conceptual level where it is able to understand itself.

The brain is the seat of the mind, *mindware*. **Mindware is a ware which is aware of itself.** What enables this in a way unremarkable and quite ugly object to produce the phenomenal effect of thought and consciousness is a tantalizing puzzle of cosmology. As opposed to the straightforward and clear definitions of software and hardware offered by computer science, mindware is the embodiment of the paradox. If the brain were so simple that we could understand it, we would be so simple that we couldn't. Are we able to think because our brain is sufficiently complex or is the brain complex because we think? Is there a point where the brain ends and the mind begins or the mind-brain is one undivided ware? These are fundamental questions. We live in a challenging and exciting time in the science of the brain. All the indications are that the evolution of self-consciousness is at a turning point. After a long and frustrating search, a scientific picture of the mind and its place in the natural order has begun to emerge. Many diverse fields, from mathematical logic and cognitive science to computer theory, fundamental physics and topology, are contributing to the new postbiological understanding that consciousness is wholly natural and physical.

Mental phenomena are a bundle of contradictions. The attempt to localize a thought in the brain strongly resembles the attempt to localize a quantum particle which happens to be everywhere and nowhere. Is the brain classical or more like a renormalized atom, an electron, a neutrino, a quark or a macroscopic nucleus? How many times have we thought that we have finally unlocked the puzzle and solved the great mystery? How many times have philosophers and scientific visionaries proclaimed: "consciousness is understood, consciousness is explained"? Yet with each new theoretical advance or technological leap forward, consciousness repeatedly vanishes like a dream, like a mirage in a desert, so desirable but unattainable and constantly shifting away from the thirsty traveller. Isn't it time we understood why? Isn't it time we realised that we are on a journey which may never end, that the incompleteness of consciousness is fundamental, not just the result of the deficiencies of the current level of science. Consciousness is knowledge, and gaining knowledge of consciousness inevitably changes consciousness itself,

perpetuating forever this unending loop of circular causation. Is this the fundamental limitation we must accept? Could there be only temporary understanding of consciousness? A theory of the mind can never be completed: it necessarily will give way to the *next* theory, and this change is in itself the only theory of consciousness one can ever possibly hope to achieve.

In the tension area between the finite and the infinite, between facts and illusions, of the two great extremes of modern philosophy: Descartes's *Cogito ergo sum* and Nietzsche's *God is dead*, we are still trying to understand consciousness but resolution is nowhere in sight. Again and again we find ourselves in a closed conceptual box, from which there is no escape. We can guess a great deal, learn some truths, but never the whole truth about consciousness. Unlike in the Einstein's great gedanken experiment, we cannot get out of the 'lift', even theoretically. Only help from outside the box may help us to solve the puzzle. For theology and religion it is God. For a physicist it is the power of mathematics, with its, in the candid words of Eugene Wigner, 'unreasonable effectiveness in the natural sciences'. A worst-case scenario would be if self-understanding is not part of the human equation and only contact with another nonterrestrial intelligence, if we were ever to establish such contact, could enable us truly to understand ourselves.

I've studied consciousness as long as I can remember myself. As a matter of fact, we all do, we are all researchers into consciousness, into SELF. Man's introspection begins as soon as the cognitive function is installed through the child's rather trivial but miraculous social development from no-mind to mind or, as some would prefer, from less to more mind. This is an endless quest. Those who nurture the illusion that one day we will reach a total understanding, the full truth of consciousness, and those who claim to have already done so, could both be profoundly at fault. Just as the quantum uncertainty principle sets up a fundamental limit on the observation process, consciousness is essentially and fundamentally incomplete. The great paradox and the beauty of SELF is that it can and does gain knowledge of itself, but it does so at the expense of increasing ignorance and uncertainty, leaving in tatters the gains that were made, and often returning us to square one.

In the context of the brain-mind problem we are confronted with very difficult scientific and philosophical questions: what is mindware? how does it work? what mechanism actually produces consciousness? The explanation of our inability to observe consciousness directly has been sought by suggesting that the mind is too fast in comparison to the time scale of the macroscopic world we exist in. Another proposal is that it is too small in comparison to the macrophysical space which our intuition is familiar with, or even that it resides in different dimensions than the dimensions of the Newtonian and relativistic mechanics. Many attempts to identify on what level the mechanism of consciousness actually operates have failed and the subject continues stubbornly to elude scientific demystification.

An even greater mystery surrounds the question: why does consciousness exist? Does the world need consciousness? and if so, what for?

Nobody seems able to give a totally satisfactory answer. Perhaps it is the answer that there is no answer, no complete explanation but a permanent question mark? If everything is in the world for a reason, wouldn't this be for consciousness the perfect *raison d'être*? From the two opposite poles of the scientific divide, the computer scientist tells us that the mind is nothing more than a computation, while the psychoanalyst insists that we cannot hope to conquer consciousness without understanding the mechanism of emotions, which are noncomputable. Illogical logic is a contradiction in terms. Not so for emotions. The dream of a man is to write a poem next to every mathematical equation. The *dream* of a robot is to reduce every poem to a mathematical formula. Is understanding of the mind a science or an art? Mathematics reveals the deep aspects of intelligence just as reading Dostoevsky or viewing Magritte conveys a great deal about the essence of consciousness, if only on an intuitive level. Indeed, what distinguishes research in neuroscience from other fields is that the study of consciousness is clearly at the border of science and art. However, unfortunately for some and fortunately for others, in conventional science we are forced by our peers to be artless. Should one dare to unleash the imagination and submit a poem to a physical journal, one would very quickly be put to the stake, or in a less tragic outcome be treated as excentric or insane.

I wonder why science and art are so incompatible. Why do numbers, machines and money make many people tick while rhythm and harmony put them fast to sleep? In another universe, less numerical then ours, could science and art evolve symbiotically? Have we been victimized by Pythagoras, the Greek mathematician and founder of a school which in the 6th century BC claimed to have found the principles of all things in numbers? Should there be only a quantal, either scientific or artistic, response to the question of consciousness? The French novelist Balzac once said that without numbers, the whole edifice of our civilization would fall to pieces. Although there is some deep and strong feeling in man's soul that persistently revolts against this idea when seeking an answer to the question, what is thought from the point of view of physics? We mustn't relay on intuition but on hard scientific facts. **No conclusions should be based on illusions.** Despite some surprising discoveries, we are vaguely glimpsing only small corners of the cognitive phenomenon and the SELF. The study of the soul is the exclusive province of art; it is completely outside science and, in all probability will remain art for another millenium, continuing to delight poets, antitechnologists and greens. Art is multiple, science is exact. The multiplicity of words is a great obstacle. We listen but do not grasp. We speak but do not communicate. A great variety of terms is used to describe mental phenomena. Mind, logic, consciousness, thinking, cognition, intelligence - all these and many other terms can be and often, depending on taste, culture or education, are interpreted freely, ovelapping and causing confusion. To avoid misreading and misconceptions we must from the very beginning fix the content of these terms to the greatest extent possible and refine them continuously.

THE INTELLIGENCE CODE

Eighteen months on the planet and already a psychological theorist.

D. Gustafson

Intelligence is an undeniable fact of reality. Various hypotheses exist which try to tie up the emergence of intelligence to the theory of natural evolution and growing complexity. However, the attempts to explain the mind in the framework of complexity theory and consciousness as a complex computing program fail. Complexity is a factor, but not a solution. We are confronted with critical questions: is the intelligence code fundamental or an artifact? Is it an accident of biology or a fulfilment of physical law? Seeking answers to these questions forces one to examine the phenomenon of intelligence in a broad conceptual framework, which takes us all the way back to the beginning of the universe, the Big Bang.

In a deterministic Laplacian systems one can deduce from the initial conditions what will happens to a system in future. A spider builds a highly ordered web relying on reflexes which are enacted by a complete set of data. If 'error', induced mechanically or by a chemical agent, occurs, a defective web is built. A computer program may crash if a circuit failure takes place. A child will step into deep water not understanding the consequences. Compared with these unreliabilities a high-level symbolic intelligence is robust in most incomplete and ill-defined situations. The decisions we make are often prone to numerous errors. Again and again we are capable of reaching the desired goals, and solve problems in *impossible* situations. The intelligence code is able to resist huge amounts of errors. It is this ability to remain operational and effective in the presence of errors and deficit of information that we consider here the central feature of high-level intelligence. A thought begins with fantasy and confabulation but in the end leads to an intelligent logical outcome.

In mathematical analysis we can typically solve a problem if all the necessary data and algorithms are available. One can then even delegate this job to a computer. But we are in a very different situation if the universe of data is incomplete, be it due to lack of knowledge or in virtue of the universe itself. Yet the human brain is a great theoretician, which remains an effective problem-solver even when the data are known only partially. In an uncertain, random or fuzzy environment some powerful mechanism of extrapolation takes charge. Moreover, not only does a problem get solved, the solution is often the optimal one, almost as though some natural law of optimization were

at work. When one rides a bicycle things seem natural and trivial: one is unaware of the complex computing machine which evaluates, controls and makes use of a complex set of differential equations of motions, decides in real time and tells hundreds of muscles in the body how to move and to keep its balance. This *trivial* job is a job for a powerful supercomputer. But the brain does it with ease and elegance, remaining prone to many errors which inevitably occur in its complex system.

The question which naturally presents itself is then, what language shall we use to describe the intelligent computations of this sort in fundamental terms? Is the code of the brain biological, physical or mathematical? Computer programmers, psychologists, neuroscientists, linguists and physicists have tried to address this highly controversial and still hotly debated issue. John von Neumann, a great cynic and one of the creators of the atomic bomb, questioned the validity of using the computer as an interpretive model for human brain, asserting that the language of the brain is not mathematical and that thought is noncomputable. What von Neumann, and some after him, did not realize that besides and beyond the **noncomputability** question we may have to face an even greater challenge if the brain is **unsolvable** at finite cost in the sense of complexity theory.

The objectives of cognitive science have been rapidly shifting but after much debate we are back to where we started. Nobody understands consciousness. Computable or not, all interpretive models of intelligent thought fall far short of explaining the actual mechanism. The language of mindware cannot be ordinary language, nor the language of the physical sciences, nor neurochemistry or electrophysiology. Thought cannot be reduced to a chemical reaction or equated with the spike electricity in the brain, even though these certainly play a part in the overall dynamics of the brain. An intricate complexity is built into the thought process which does not let us describe it either in pure physical terms alone or in terms of pure information or logic alone. Both elements are required for the description of the machinery with which the mindware does its job. The suitable funda*mental* language must involve both: logic and physics. However, things are not as easy as they may appear. The logics we know are inadequate for the description of thinking and a logical structure adequate to intelligent thought must be found. Likewise, a physics one may employ to describe mindware cannot involve ordinary physical units such as mass, position, momentum or energy, so that a new physics, which takes account of the information aspect of matter and fields, is needed. Just as the chemistry of DNA is more than merely a chemistry of life macromolecules, but the transmission of genetic information, the physics of the sentient brain cannot be reduced to the mechanics of its constituent elements, however complicated they may be. The physics of the brain is more than just the neurophysics of the action potentials of firing neurons, it is also essentially and fundamentally the encoding and decoding of cognitive information, the information which provides us with the power of knowledge, the sensations of touch, sound and colour and most importantly

with the perception of SELF. Self-awareness and modern physics have little, if anything, in common. Physics, as we know it at present, provides no room for phenomena associated with the mind and cognition. These terms alone may scare away a traditional physicist, who is used to working with more mundane and technical matters. However, physical theory is and will remain incomplete as long as it is unable to accommodate consciousness. In a strange twist of fate, the attempt to apply quantum-mechanical principles to the realm of consciousness leads to the conclusion that a modification of quantum theory itself is necessary to accommodate the puzzling problems of the brain and mindware.

An attempt to resolve the problem of mindware goes hand in hand with several fundamental issues, namely: *M(atrix) logic* and the *Quantum Computer*. At first glance it may seem that these are separate from and unrelated to the problem of consciousness. *M(atrix) logic* is a general field logic, seeking a grand unification of all particles and forces of nature. The *Quantum Computer* is an extension of Turing Machines into the domain of quantum mechanics. Over the last thirty years important progress and unexpected advances have been made in these fields, originating in the author's works in the 1970s. Surprisingly, the formulation of principles in an axiomatic way has revealed that the theory of matter, the theory of mindware (*thinking* matter) and the theory of quantum computing are intimately related.

Many difficult questions with which we are confronted in brain science are attributable to the fact that the laws of classical logic break down when one tries to analyse mental function. If the brain is classical, how then can it give rise to quantum theory which as a universal theory refutes the deterministic brain itself? If the brain is quantum-mechanical, then we have a quantum system attempting to describe and validate itself. But can quantum theory prove its own validity? The Gödel incompleteness theorems imply that this project is doomed to failure, since no formal system can obtain a proof of its own consistency by means of the system itself. When a theory refers to itself, fundamental limitations unavoidably emerge. The validation of the mind as a logical theory requires the enlargement of the theory which neither classical nor quantum mechanics can provide.

Quantum theory is considered to be a universal theory, and as such must be valid for the description of intelligent systems such as ourselves capable of creating theories such as quantum mechanics. Self-reference is inherent in fact that the brain that tests the validity of quantum theory must be itself be described by the same theory. According to the incompleteness theorems this inevitably should set fundamental limitations on the brain as it attempts to formulate a theory of it*self*. Then, to avoid paradoxes, one is forced to incorporate quantum-mechanical description into a larger system, just as in mathematics to resolve an inconsistency we extend a formal system with a new axiom. In this study I intend to show that m(atrix) logic is such a larger system, providing a proper theoretical framework for understanding the intelligence code. To understand why a matrix logic is more effective theory

of the brain then quantum mechanics, we will need many different tools ranging from topology and relativity theory to quantum codes and the new concept of improbability.

Matrix logic reveals that **potency laws** are a defining feature of a logic, determining the extent of the "logical resolution" of an intellect. A given potency law is the involutive identity

$$X^n = X$$

and can be viewed as a symmetry transformation which rotates a logical value to itself. Two-valued classical logic is 2-potent or idempotent logic, which is a far cry from the complexity we observe in developed thinking systems. The question naturally presents itself: which values can the power **n** take? The attempts to refine logical resolution that were made in multivalued logics is not fundamental because the introduction of extra truth-values, besides true and false, is arbitrary. The potency law enables us to define multivalued logics in a fundamental way. In this study we will identify the logical potency laws for **n = 2, 3, 4, 5, 7** which are the Galois field numbers. A Galois field **GF(n)** is a finite field containing **n** elements that must be a prime or a power of a prime, which excludes among others **n = 6**. The potency laws connect in fundamental way the theory of logic with the theory of prime numbers.

THE EVOLUTION OF THE MACHINE

The development of the theory of the intelligent brain follows in footsteps of technology, mimicking, often foolishly, the advances made in engineering. Struggling to obtain a theory of the brain with the tools available, each generation uses the machines of its time as a metaphor for the functioning of the brain. The Laplacian clock universe regarded the brain as a collection of gears, wheels and springs beating out the rhythm of thought. Later, with the emergence of the steam engine we saw the rise of the idea of the Analytical Engine. For the past half-century, and especially since we have entered the Internet age, the metaphor of choice has been the computer, which goes hand-in-hand with better understanding of complex systems and chaos. The secrets of the brain are presently under attack as never before, with computer science contributing pieces to the puzzle of the cognizing function. Can the cognitive brain be embodied in a computer? We must carefully evaluate the arguments for and against this claim. The conceptual foundation of the brain-as-computer metaphor is rooted in the work of Turing, who showed that every effectively computable process can be carried out on a Turing machine, roughly speaking, anything that can be computed by means of an algorithm can be computed using a Turing machine. But can one equate cognitive processes with algorithms? Given that inference is just a computation, a clear distinction must be made between computational inferences, which support the computer

metaphor, and inductive consciousness, where it breaks down. At the heart of the computer metaphor is the idea that thought processes can be formalized as algorithms. There is little doubt that we can simulate thought processes, but can we actually model them? Closely related to this claim is the question whether it is possible for an algorithm to understand itself. Can the brain understand itself, either as a computer or in any other machine terms? This leads to the question of the internal vs. external observation of a system. In other words, does self-measurement lead to a different interpretation of the system than measurement obtained by an external observer? The state of a system as seen from within is very different from the state measured by an outside observer.

The metaphor of the brain as a computer gave rise to the field of Artificial Intelligence. But as many attempts to simulate high-level intelligence failed computationally, and the understanding of the complexity of human intellect has grown, the idea of the brain as a computer has began to lose its appeal. The key difference between the brain and the computer is that in the computer the bits are the states of the machine which appear and disappear. In the brain the bits are physical particles whose number is conserved and on average remains unchanged during a computation. A computer is a superb number-crunching savant, but lacks creativity and free will. It makes up in speed for what it lacks in intelligence. Some higher-level and genuinely thinking agent, like ourselves, must input a theory into the computer before it can perform a calculation. **The computer cannot generate new theories by itself.**

A goal we set for ourselves is to introduce and to gain insight into the intelligent machines of new kind: quantum-theoretic machines. Thus far the notion of a machine has had an engineering, technological meaning. **The concept of a quantum-theoretic machine changes the notion of a machine into a notion of fundamental physics.** The evolution of the machine has come a long way from the ancient catapult and abacus to the space shuttle and the modern supercomputer. The initially purely mechanical concept of a machine was enriched by electro-mechanical and thermodynamical concepts. After the age of steam engines came the age of information machines, the historical era of computers in which one lives, or is lost, depending on one's situation. As information entered physics, physics itself began first to flirt superficially and then to make inroads into information theory and logic. At the fundamental level information is physical, and in the same vein one legitimately concludes that physics is digital, hence logical. As suggested by the author physics is a logic in fundamental sense, and since the foundation of physics is quantum-mechanical, one raises the stakes by suggesting that quantum physics is a fundamental logic. Just read off Hilbert space from logic. Most importantly, with introduction of logic into the machine, the concept of a quantum-theoretic machine began to take shape, and continues to grow under the umbrella of topological m(atrix) theory. Computing machines are necessarily implemented in the physical world. With

the convergence of physical and computational theories and with the bits becoming smaller and ultimately quantal, machines must inherit some of the constraints of the physical laws and explicitly reflect the fundamental principles of physics. Physics does not need to describe what might happen in any possible theory, just what does happen in the theory of the actual world. In consciousness, the distinction is more important and subtle because we design our own theoretical worlds.

Before we can meaningfully refer to the thinking brain as a quantum-theoretic machine, I have to explain in greater detail what is meant here by a quantum-theoretic machine and how it differs from other types of machines. Three aspects has to be tackled: Why it is quantum? Why it is theoretic? And why it is a machine?

The machines of the early industrial revolution required full supervision by human beings. Automation constituted a second industrial revolution, with the machine partly taking control over some of its functions, and man interfering only when a contingency arose. Climbing up the ladder of abstraction, the machines were empowered with feedback, a self-regulating mechanism which enables a machine to steer itself, adjusting to different loads or targets. It was thought in greatly exaggerated terms by the cyberpioneers that the fundamental principle of intelligence had been found. However, besides having **feedback**, the brain is also distinguished by having the **feedforward** mechanisms which enable us to probe and anticipate the future. Nonetheless, information technology have changed forever the concept of a machine in the scheme of things. Study of the biological machines, the brain and consciousness in particular, further expands this notion, liberating it from the constraints of pure data and information processing towards knowledge acquisition and abstract behaviour. The understanding of a machine as purely mechanical or electromechanical or thermomechanical is no longer tenable for the brain. The brain relies on and explores natural physics, arising from the fundamental level of physical interactions. Since this level is a level which is treated most adequately by quantum mechanics and quantum field theory, the term *quantum* comes naturally into the definition of a quantum-theoretic machine. The term *theoretic* requires a comparison with existing technologies. There are machines which process mass, like a chemical plant, or energy, like a combustion engine, or information, like a computer. **We are concerned with machines which process theories.** Systems which process theories are thinking systems. They differ from the systems that process data or information only (which could very well be the data of a theory) in that that they evolve and change as result of such processing, while an ordinary information machine (more like a coping machine) undergoes no such evolution or intrinsic knowledge acquisition.

A computer is indifferent to the content of the data it processes. It does not matter whether these data concern words or graphics, the weather forecast or the calculation of a nuclear explosion. The quantum-theoretic machine is fundamentally different: its states depend on its knowledge content.

A change of a theory changes and alters the organization of the machine, effectively creating a new machine over and over again. **The brain creates, destroys and maintains theories.** The fact that the brain can create and destroy theories is important, but even more significant and fundamental is its ability to **globally process** theories, which is very different from the local processing of data or information.

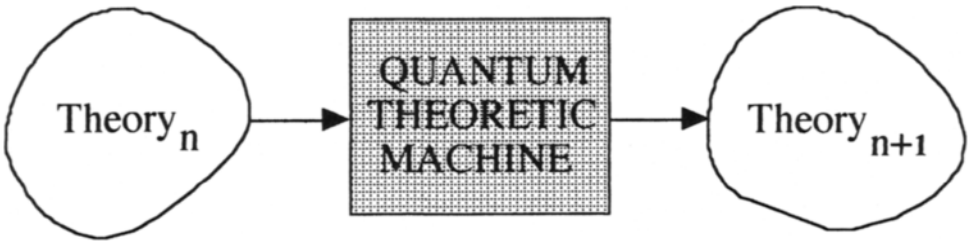


Fig. 1.

Quantum Theoretic Machine
takes in Theory_n and processes it into another Theory_{n+1}.

All machines, computers included, are effectively classical machines, even though they may rely on some underlying quantum processes in their operations. Although semiconductor chips do make use of quantum effects, still on a whole they are classical devices not only because their operations are directed towards precision and complete predictability, but also because they make no use of quantum information as such. Some thirty years ago the question began to arise whether a machine can make use of quantum effects directly [Ref 78]. Can a computer actually explore with benefit such typically quantum phenomena as coherent superposition, nonlocality and the uncertainty principle? For thousands of years men have built and explored deterministic or statistically deterministic machines. Can science and technology allow us to build useful quantum machines and ultimately, quantum-theoretic machine? A quantum machine with internal degrees of freedom is nondeterministic: its operations can be controlled only to the extent which is allowed by the uncertainty principle. What use can be made of such unruly systems? How can one benefit from a machine which in fact processes theories and unpredictable quantum information instead of well-defined ordinary data? Of course one would not wish to delegate to such a machine the control of a commercial

airliner, but one immediate application comes to mind right away: the problem- solving which requires intelligence and quantum parallelism.

The economy of the 21st digital century will be, if it is not already, driven by information and knowledge technologies, in the same manner as the 20th century was primarily the century of energy. The greater the benefits machines bring to our quality of life, the greater are the chances of harm when the machine fails to perform its functions or when it performs them incorrectly. The systematic design of fault-tolerant systems have seen successes and failures. Information machines are different from machines that process energy, and a reliable information machine can be built from unreliable components, which is not the case with energy machines like a nuclear reactor. As the product of hundred of thousands of years of evolution the brain is extraordinarily reliable system. But in technology high-confidence computing remains a costly privilege of a few critical, mostly military-oriented, applications. In large information systems, in complex computer technology a minor insignificant error can cause havoc. A standard way to design fault-tolerant systems is redundancy, which entails duplication of operational and storage coding elements. But if elements are quantum states fundamental problem arises because quantum states cannot be copied exactly. This also suggests that the brain's ability for error correction must be based on a different built-in principle, which differs fundamentally from classical error correction in communication and computers. The theme we develop in this study is that the intelligence code of the brain is a **self-correcting code** in which redundancy exists but is of secondary significance.

In spite of its many defects and deficiencies the computer metaphor of a thinking machine has been useful. Although the computer is not an apt model of the brain, it has helped to streamline and bring under one umbrella many different fields previously considered to be unrelated. Since the dawn of the computer age the idea of a thinking machine has had its ardent supporters and pronounced sceptics. In the 1960s the artificial intelligence movement was born and flourished until it got stuck several decades later, unable to either explain or simulate genuine intelligence or consciousness. Irrational exuberance has given way to a more reserved view on the artificial intelligence project. The purpose of this study is not to enter into this continuing dabate. What is important for us is that in the end of the 20th century the notion of a machine itself underwent a fundamental change, amounting to a paradigm shift. The important developments in mathematical logic have shaken our concept of a machine profoundly. One feature, however, has remained intact: solid state physics and rigid geometry dominate our concept of a machine and its design. This geometry-based approach has begun to crumble with the emergence of the concept of biological machines, in particular molecular biological machines. Everything in biology, from the embryo to the cognitive brain, is *soft*, floats, is dynamical and solvable. The DNA and protein supramolecules curl and uncurl, expand and contract, undergoing fascinating conformational changes. However, the information meaning of DNA, be it in

the state of the double helix or unrolled and divided, remains unchanged. It is *soft*ware in the direct sense of the word. In technical terms, geometry explicitly gives way to topology. It is this new topological concept of machines which we pursue in this study, from the elementary quantum computer to consciousness. **Nature *knew* all along how to explore the topological laws to create consciousness. We simply looked in the wrong direction and did not ask the right questions.**

Ideas along the lines of a quantum-theoretic machine and a quantum computer can be traced back to the late 1960s [Ref 73]. This early work focused on the development of computing concepts suitable for biomolecular systems in the brain and on paper quantum-mechanical designs. This effort was motivated by the desire to replicate the powerful information processing capabilities of the brain, such as pattern recognition, learning, effective parallelism, high adaptability, self-organization and self-programming. The fact that it is possible to simulate any process in nature with symbolic computations inspired a false hope that the intelligent brain could also lend itself to realization through classical logic. A deeper analysis, however, exposed the fundamental differences between the conventional computer and quantum-theoretic nature of the brain. This analysis reveals a wholly new and more more profound computing paradigm in which the distinction between hardware and software loses its clear contours, reviving in a sophisticated way the idea of analog continuum computing in which physical structure and the computing program are tied inseparably.

Machines which process theories climb up the ladder of abstraction, taking over more and more functions of self-control, and ultimately must control themselves entirely. Often the actions of such a machine are directed inward and not outward, and for a superficial observer the machine would seem to be doing nothing. There might be no noticeable external manifestations while in fact an active internal transformation of the machine is taking place as it seeks to reach a desired state. What is important is that the state in question must in physical terms be defined as an eigenstate. It is a principal new element of the description. A quantum theoretic machine is best described not as an automaton but as a dynamical system whose states are determined by the corresponding eigenequations. Symbolically we can write:

$$\text{MACHINE } |\text{state}\rangle = \text{NUMBER } |\text{state}\rangle$$

The machine reaches a desired solution when and if it is able *to change itself* in such a way that this equation is converted into the equation:

$$\text{OPERATOR } |\text{eigenstate}\rangle = \text{EIGENVALUE } |\text{eigenstate}\rangle$$

For example for the simple conjunction machine

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} |q> = \lambda |q>$$

one solves in steps the characteristic determinant

$$\left| \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} - I\lambda \right| = 0$$

$$\left| \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} \lambda & 0 \\ 0 & \lambda \end{pmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 0 \\ 0 & 1-\lambda \end{vmatrix} = 0$$

$$\lambda^2 - \lambda = 0$$

$$\lambda(1-\lambda) = 0$$

$$\lambda_1 = 0 \quad \text{and} \quad \lambda_2 = 1$$

Having calculating the eigenvalues, one then determines the corresponding orthogonal eigenstates of the machine:

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1 \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \end{pmatrix} = 0 \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

Much wisdom is built into the characteristic polynomial of the machine

$$\lambda^2 - \lambda = 0.$$

Apart from being satisfied by the consequents, the eigenvalues 1 and 0, it can also be boosted into the operator format which will be satisfied by the antecedent, which the machine's operator:

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}^2 - \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{0}.$$

As we indicated, the calculation of the eigenstates requires an intelligent agent such as ourselves. One either goes through the standard routine by setting the determinant to zero and computing the eigenvalues, and then determines the eigenstates by solving a corresponding system of equations, or one uses some special properties of the operator. In our example a shortcut directly to the characteristic polynomial

$$\lambda^2 - \lambda = 0$$

is possible by relying on the fact that we have the idempotent machine:

$$\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Although the eigenvalue problem possesses much self-control, some intelligent job still has to be done to determine the unknown eigenstates. Imagine, however, a system, whose operator has knowledge of itself and is able to compute an eigenstate directly while acting on arbitrary and unknown state of the machine:

$$\text{OPERATOR } |\text{state}> = |\text{eigenstate}>$$

This is an extraordinary situation, with which physics has never dealt before. No external intelligent agent is needed to solve the eigenstate problem. A solution comes directly. In self-cognizing fashion a problem *solves itself.* In

the course of this study we will determine that for quantum theoretic machines there exist a fundamental quadruplet of matrix logical operators which are the *knowledge operators*, solving themselves:

$$\{M, TRUE, FALSE, 1\}.$$

As we mentioned early, the fundamental difference between classical and quantum information theories finds its expression in the fact that quantum states cannot be copied perfectly. The thinking process is 'unconscious', occurring without any awareness of its underlying mechanisms in the brain. Conscious thought is very much like a stick of a blind man probing his way in total darkness, but it works amazingly well. Therefore the ability to obtain a required '**known**' logical state by acting on an **unknown** and moreover an **arbitrary** state becomes essential for effective consciousness. In this study we will establish that the knowledge operators enable the brain to do precisely that: with the aid of these operators thoughts 'dial' the logical eigenstates of the brain directly and without impediments. More generally, we will prove that cognitive brains are universal quantum-theoretic machines for translating classical information into quantum information and vice versa.

THE BRAIN'S UNKNOWN STATISTICS

Physics is a major source of inspiration for the theory of logic. First quanta, then fermions and quarks not only changed our concept of the world forever, they had also irreversibly broken the smooth fabric of our thinking and feeling which we erroneously view as continuous and differentiable. Hayden and Bach gave way to Shomberg and Stravinsky, Titian and El Greco to Picasso and Dali. Another key complication in the study of the logical brain is that neither physical equations alone nor logic alone fully describe it, both are needed. This important fact motivates a search for a unified theory which has power to simultaneously account for the physical and logical aspects of the brain. Frustrated by the repeated failures to connect the logical and physical brain, and in this way to identify the elusive *mindware*, one starts looking for a new physics underlying consciousness. Since neither classical nor quantum theories were able to solve the problem, an idea that an additional physics is needed to describe consciousness comes naturally. We lack, as the argument goes, a fundamentally important insight into physics, without which we will never be able to comprehend the mind.

In this study, seeking a new physics suitable for the thinking brain, we narrow this general problem to a specific problem of finding a new fundamental statistics. After all, at the core of any physical theory lies its

defining statistics. Modern physics views the world as being composed of quantum particles and fields which obey either Bose-Einstein statistics

$$\psi_b(a)\psi_b(b) = \psi_b(b)\psi_b(a)$$

or Fermi-Dirac statistics

$$\psi_f(a)\psi_f(b) = -\psi_f(b)\psi_f(a).$$

Equivalently, we say the bosonic fields commute

$$[\psi_b(a), \psi_b(b)] = 0$$

while the fermionic fields anticommute

$$\{\psi_f(a), \psi_f(b)\} = 0$$

Whenever a changeover of the fields or particles occurs the factor **+1** for bosons and **-1** for fermions appears

$$\psi(a)\psi(b) = (\pm 1)\, \psi(b)\psi(a),$$

determining the nature of the quantum statistics, and in this manner, the underlying physics of a process under investigation.

Does the physics of the mind comply with Bose-Einstein and Fermi-Dirac statistics? Theoretical physicists generally believe that these two statistics must describe all imaginable phenomena in the universe, consciousness including. However, if one accepts the proposal that the *mental* universe, the logical mind, obeys different physics and is controlled by a more general but unknown physical principle, this dogma is forced into retreat. The problem of finding a physics suitable for the cognitive brain then can be meaningfully formulated as the problem of finding new type of statistics

$$\psi(a)\psi(b) = \Phi\, \psi(b)\psi(a),$$

where the factor Φ, to be determined, is neither bosonic nor fermionic:

$$\Phi \neq \pm 1.$$

The goal of this study is to determine in the framework of m(atrix) theory the unknown factor Φ. We will show that its identification requires a conversion from numbers to matricial dimensions. Such conversion not only clarifies the essence of the new statistics, but in a surprising turn of events, matrix logic offers ultra unification where the factors determining Bose-Einstein, Fermi-Dirac and general Matrix Statistics are all obtained in a uniform fashion as the three different square roots of the matrix identity

$$\Phi = \sqrt{\mathbf{I}},$$
$$\Phi = +\mathbf{I},\ -\mathbf{I},\ \mathbf{N}.$$

M(atrix) logic thus reveals that there exists one universal scheme for obtaining different statistics. In a sense, a road map towards the Theory of Everything passes through a fundamental theory of the brain. Depending on the particular choice of the square rooting of the matrix identity, bosonic 'classical' physics, bosonic and/or fermionic quantum physics and matrix physics of the brain, emerge as particular, symmetric, antisymmetric and logical limits, of one general matrix theory.

THOUGHT AS MATRIX MULTIPLICATION

I link therefore I am.

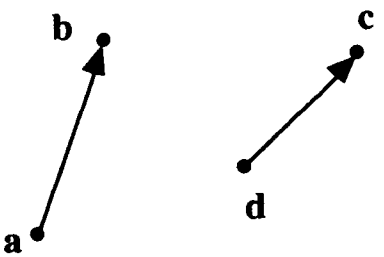
S. Singer

Matrix logic breaks new ground in the study of the fundamental laws of the brain. Logic develops naturally from matrix algebra. Logical inferences, no matter how sophisticated and abstract, can be directly related to fundamental matrix forms. The purpose of this foundational section is to show that by exploring matrix theory it is possible to take the complexities of human thought and reduce them to intelligible and workable simplicities. We begin with the formulation of the key theorem which provided the main motivation for the development of matrix logic.

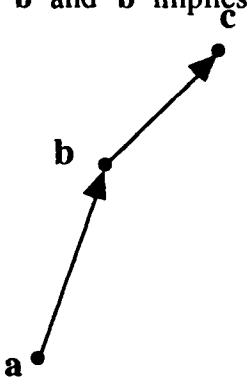
Matrix theorem

THE IMPLICATION INFERENCE IS THE MATRIX PRODUCT

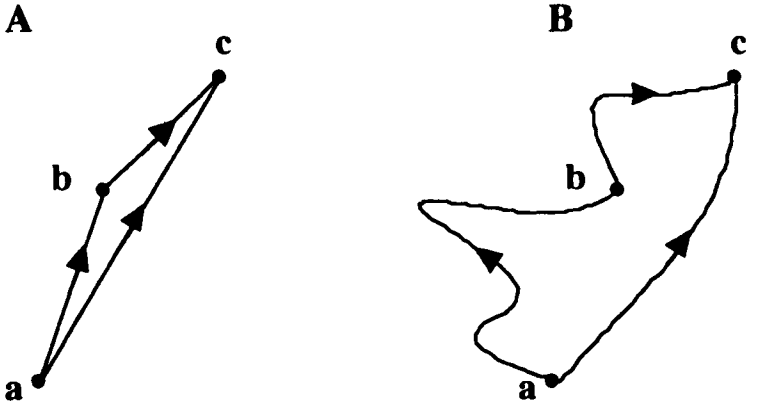
To gain insight into the matrix theorem, we treat thinking as a process of forming connections between some logical elements **a**, **b**, **c**,... . In logical terms, forming a connection between elements **a** and **b** entails that **a** implies **b**, which can be visualized as an implication arrow line going from **a** to **b**:



Suppose we also established an implication connection between **d** and **c**. If **b** = **d**, we can join the two implications into a single one, getting a serial reasoning where **a** implies **b** and **b** implies **c**:



There is a short-cut which resembles the rule for vector addition: we can go directly from **a** to **c**, instead of traversing the point **b**:



A resemblance to the addition of vectors should not lead one to start to think geometrically: the connections in cognitive logical space are essentially **topological** in nature and **gauge-invariant**. Instead of A we could very well have drawn B. We are not interested in the geometrical properties or distances, but we only wish to know which element is connected to which. What is of major importance is that the short-cut in the triangle of links is the fundamental feature of the intelligent operations. It commonly manifests itself in logical reasoning, in concept formation, memorization, summarization, abstraction, intuition and many other operations of the brain. Even though consciousness may, and often does, lose track of all intermediary steps leading to the interiorization of a particular thought, and we may not even be able to

actualize them explicitly, the intelligent inference one way or the other fundamentally relies on the omnipresent universal rule of the syllogism:

$$((X_1 \rightarrow X_2) \wedge (X_3 \rightarrow X_4) \wedge \ldots \wedge (X_{n-1} \rightarrow X_n)) \rightarrow (X_1 \rightarrow X_n)$$

Without this inference rule, the intelligent ability to solve problems, if not the whole fabric of consciousness and the mechanism of understanding the world and ourselves would break down. Because thinking typically tends to minimize the path from X_1 to X_n, a syllogism can be meaningfully interpreted as **the least action principle** in logic. Without loss of generality it will be sufficient if we restrict our analysis to the basic case of just three logical elements:

$$((x \rightarrow y) \wedge (y \rightarrow z)) \rightarrow (x \rightarrow z),$$

from which the general syllogism law is easily derived by induction.

Prior to understanding the thinking brain physically, we must be able to understand it mathematically, independently of whether the language of the brain itself is mathematical or not. Having understood the key thinking operation phenomenologically, we now will seek an answer to the fundamental question:

What is the mathematical structure of intelligent thought?

This is a critical question one has to answer meaningfully and in technical terms prior to being able to put human thought into precise mathematical equations. In science one canhardly finds a more complex and intractable problem. Despite various attempts at deciphering the code of the thinking brain, the fundamental mathematical structure of the intelligent thought has remained a mystery.

The enormous complexity of this problem has defeated some and inspired others. John von Neumann was led to believe, although later abandoned the idea, that the laws of the cognitive brain are not mathematical in nature and therefore noncomputable. A century earlier August DeMorgan and George Boole had offered a heuristic, but surprisingly effective, mathematical model of logic, in which the abstract logical operations **AND**, **OR** and **NOT** were identified as the algebraic operations over binary numbers. Their names are immortalized in the DeMorgan duality laws and Boolean logic algebra. However, in the *ad hoc* system of DeMorgan and Boole the conversion from logic to algebra and the connection between logical and algebraic operations do not stem from fundamental first principles. Nor is there a universal mathematical operation in the system: subtraction, product and addition are required, although these can be reduced to two operations making use of dualities linking **AND** and **OR**. Although there are universal and functionally complete monoconnectives **NAND** and **NOR**, these do not have a fundamental mathematical realization.

Boolean logic algebra conveys a sense of simplicity and beauty. George Boole entitled his landmark work *The Laws of Thoughts*, but they are rather the 'thoughts' of a computer than the thoughts of a creative mind. As far as the intelligent brain is concerned, Boolean logic, despite its many attractive

features, cannot be considered fundamental. In a quest for the *mathematics of thoughts* we have to begin from square one. Our goal is to identify and to describe the universal thinking operation mathematically. Moreover, assuming that logical thinking obeys some mathematical laws, we hope to determine these laws through logical considerations alone. In persuing this goal we want to rely on the properties of thinking as such, and not on any other insight or conjecture which mathematical intuition may offer.

We now show that the syllogism rule hides within itself the fundamental mathematical principle of thinking. The logical string

$$(i \to n) \wedge (m \to j)$$

can be contracted to

$$(i \to j)$$

given the condition that

$$n = m.$$

Treating the logical symbols as tensor indices, and lowering and raising the indices in accordance with the tensor summation rule we can write:

$$\to^{i}_{k} \to^{k}_{j} = \to^{i}_{j}$$

When the covariant and contravariant indices coincide, the syllogism inference becomes computable. Introducing the summation sign explicitly, we get the composition

$$z_{ij} = \sum_{k} x_{ik} y_{kj}$$

which one recognizes as the usual rule for matrix multiplication: the product $(i, n)\cdot(m, j)$ exists if and only if $n = m$. Expressing the syllogism law in tensor form, we rediscover from the empirical result of logic the familiar rule of matrix multiplication. The transitivity of implication leads to the matrix principle at the foundation of logical thought. We have thus established the following principle of major importance:

Matrix multiplication is the fundamental inference operation of logical consciousness.

The matrix principle reveals another important fact. The working of the intelligent brain is based on massive superposition, and matrix theory in a natural way introduces parallelism into logic. Matrix multiplication, as the fundamental model of the mechanism of the thought process, takes on a special significance, if one does not overlook the fact that the matrix product AB = C is, in fact, a mutual coherent superposition. Each element of the matrix C is the ordered linear combination and particular entanglement of elements of the A and B:

$$\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix} = \begin{pmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} \end{pmatrix}.$$

HEISENBERG'S MATRIX MECHANICS

Matrix foundation of logical thinking is not the first time we rediscover matrix multiplication from empirical results. The same thing happened to Werner Heisenberg, who in the 1920s developed matrix quantum mechanics, apparently without knowing 'what a matrix is'. It will be instructive to recall the logical steps leading to this. Typically, observable physical quantities are represented by functions on phase space. If one assumes this classical picture, then the set of energies emitted by an atom should form a semigroup inside the real numbers; that is, the sum of two emitted frequencies would also be one. However, that is not the case. Experiments show that the frequencies are confined to a small number of terms ν_{ij} where i, j are in some discrete index set, so to conform to the observed quantization of possible energies of an atom. The frequencies comply with the Ritz combination principle:

$$\nu_{ij} = \nu_{ik} + \nu_{kj} ,$$

where the set of frequencies does not form a group but rather a groupoid. A groupoid is like a group in having associative multiplication, but unlike a group, only for certain pairs of elements the product exists. Studying Ritz's combination principle and the transition elements, Heisenberg arrived at matrix quantum mechanics. He found that if physical quantities such as position and momentum are represented by matrices which obey commutation relations, then experimentally observed values of physical quantities will correspond to the eigenvalues of these matrices. The algebra of observables then follows the usual rule for matrix multiplication. Trying to describe the motion of multiple electrons in atoms, Heisenberg reinvented a system of multiplication using arrangements of numbers called matrices. One is not so much surprised by this fact - history does not repeat itself without reason - than by the fact that Heisenberg was unaware of the theory of matrices.

There is striking similarity between quantum algebra and the syllogism inference. Matrix operations which play a fundamental role in quantum physics are fundamental for logic as well. Both the atomic transitions and logical inferences follow the rule of matrix multiplication. Although this is not yet a guarantee that the brain thinks simply by changing the quantum levels, such a conjecture is compelling and must be thoroughly investigated. Is it plausible that the thought mechanism and quantum mechanics of an atom are particular realizations of one fundamental matrix principle? Could then a general theory unifying the laws of thoughts and the laws of the quantum be developed? The attempt to obtain such a theory requires us to replace geometrical quantities, in the role of the descriptor of a state, by topological quantities, not as explicitly dependent on the spacetime coordinates and therefore more abstract.

THE CONVERSION THEOREM

The matrix principle, a fundamental **mathematical** principle of logical thinking, is closely aligned to the conversion postulate, a fundamental **physical** principle of the thinking brain. Logical thinking is a matrix algebra based on implication. Quantum field physics is an algebra based on the annihilation and creation operators. Apparently, the creation and annihilation operators, that are fundamental to quantum theory, are also essential for logic, allowing us to derive logic from *first principles*. To obtain a fundamental theory of consciousness we need to identify a transformation which finitely converts one algebra, with implication and converse implication, into another, with annihilation and creation:

$$(\rightarrow,\ \leftarrow) \overset{?}{\Leftrightarrow} (a,\ a^*).$$

The group symmetry which accomplishes this exchange constitutes the essence of the conversion principle. In accordance with the postulate, any quantum field theory with annihilation and creation can be naturally reformulated as a logical calculus and vice versa [Ref 94]. The conversion process involves exponential mapping between the Lie groups and algebras which is a generalization of the numeric logarithm, changing multiplication into addition. Implications which are at the heart of cognitive logic, reveal themselves as the exponents of quantum field theory. Specifically, the annihilation a and creation a^* operators expand in the following self-terminating **finite** series:

$$e^{a} = \rightarrow, \qquad e^{a^*} = \leftarrow,$$

where the equalities are **exact** equalities and not an approximation. A remarkable sequence of cancellations occurs that eliminates all potentially infinite nonlogical terms, yielding a finite answer. The fundamental converse of this quantum logical operation is the logical logarithm, also **finite**, which recovers the second-quantized field operators:

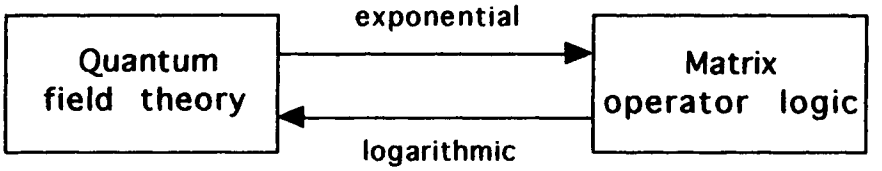
$$\ln \rightarrow = a, \qquad \ln \leftarrow = a^*,$$

a fact which is evident in the logarithm series expansion

$$\ln(I + a) = \frac{a}{1} - \frac{a^2}{2} + \frac{a^3}{3} - \frac{a^4}{4} + \ldots$$

In the phase space of quantum mechanics the Fourier transform takes us from one canonical coordinate to another. Unlike in quantum mechanics, the conversion theorem suggests the existence of a sort of a logical 'hysteresis', in which the direct and reverse transformation are not uniform. It takes a *wave* to cross from a^* and a to inference logic but it is the *logarithm* which brings us back.

The abrupt finite collapse of the conversion series is the logical analogue of the quantum mechanical measurement and elevates the conversion postulate to a fundamental principle of the quantum theory of consciousness. According to this scheme two nontrivial and beautiful mathematical physical processes underlie the working of the logical brain:



The annihilation and creation field operators can also be generated in a purely 'physical' fashion as the two square roots of the matrix vacua:

$$\sqrt{\mathbf{0}} = a, \qquad \sqrt{\mathbf{0}} = a^*.$$

Due to the finite collapse, the matrix powers of implications are solved **exactly**. Thought is self-potent. The unique property of an implication is that being raised to the power of itself, the implication remains invariant:

$$\rightarrow^{\rightarrow} = \rightarrow, \qquad \leftarrow^{\leftarrow} = \leftarrow$$

which entails the exact equality of the powers and the field exponents, one of the critical result of this study:

$$\rightarrow^{\rightarrow} = e^{a}, \qquad \leftarrow^{\leftarrow} = e^{a^*}.$$

Whenever the annihilation and creation processes converge to zero, a logical system must be reduced to identity. This is confirmed by the limits

$$\lim_{a \to 0} \rightarrow = \mathbf{I} \qquad \text{and} \qquad \lim_{a^* \to 0} \leftarrow = \mathbf{I}.$$

According to the matrix principle the implications are the key operators of thinking systems, just as the annihilating and creation operators are fundamental for the quantum field. Both are closely intertwined and distinguished by special and unique properties. The dual operators obey the commutator invariance:

$$[\rightarrow, \leftarrow] = [a, a^*]$$

which is the only antipersymmetry in matrix logic. Acting as the spin unitary transform, it converts the field exponent into the inverse:

$$\sigma e^{a} \sigma = e^{-a}, \qquad \sigma e^{a^*} \sigma = e^{-a^*}.$$

The rates of change of the implication thoughts as a function of a^* and a are the constant fixed points:

$$\frac{d\rightarrow}{da} = \frac{de^a}{da} = e^a = \rightarrow, \qquad \frac{d\leftarrow}{da^*} = \frac{de^{a^*}}{da^*} = e^{a^*} = \leftarrow.$$

Swapping **logical** functions and **quantum** arguments, and again making use of the conversion theorem, we derive the rates of change of the field operators a^* and a as a function of implication:

$$\frac{da}{d\rightarrow} = \frac{d\ln\rightarrow}{d\rightarrow} = \frac{1}{\rightarrow} = \rightarrow^{-1}, \qquad \frac{da^*}{d\leftarrow} = \frac{d\ln\leftarrow}{d\leftarrow} = \frac{1}{\leftarrow} = \leftarrow^{-1}.$$

These rates are the inverses of the fixed points:

$$\frac{da}{d\rightarrow} = (\frac{d\rightarrow}{da})^{-1}, \qquad \frac{da^*}{d\leftarrow} = (\frac{d\leftarrow}{da^*})^{-1}$$

and can be obtained simply by the reversal of the sign of the annihilation and creation operators or with the aid of the antipersymmetric spin unitary transform,

$$\frac{d\rightarrow}{da} = 1+a \quad \overset{\sigma}{\Leftrightarrow} \quad \frac{da}{d\rightarrow} = 1-a$$

and

$$\frac{d\leftarrow}{da^*} = 1+a^* \quad \overset{\sigma}{\Leftrightarrow} \quad \frac{da^*}{d\leftarrow} = 1-a^*$$

where

$$1+a = \frac{1}{1-a}, \qquad 1+a^* = \frac{1}{1-a^*}.$$

The quantum brain cannot alter matrix logic; fundamental logic can reverse itself via the brain.

Combining the cosine and sine matrix series

$$\cos a = I - \frac{a^2}{2!} + \frac{a^4}{4!} - \ldots, \qquad \sin a = a - \frac{a^3}{3!} + \frac{a^5}{5!} + \ldots,$$

we determine that transitive matrix implications are the finite quantum **operator waves:**

$$\mathbf{IMPLY} = \cos a + \sin a, \qquad \mathbf{IF} = \cos a^* + \sin a^*$$

This is similar to the **Euler**'s formula $e^{ix} = \cos x + i \sin x$, but differs from it because of the absence of the imaginary factor. The operator derivative of the matrix cosine we take by modulo, so that the derivative simply swaps the cosine and sine. A 2-step syllogism is a quantum phase counter

$$\rightarrow^2 = (\cos a + \sin a)^2 = \cos 2a + \sin 2a,$$

which is finitely reaffirmed by the theorem for double superquantized 'angles', and by induction one proves

$$\rightarrow^n = \cos na + \sin na.$$

When the brain computes a particular chain of thoughts, it chooses a particular n, recursively codified by the operator phase. A logical thought is a superquantized phase which leaves no traces. Just as a quantum system is

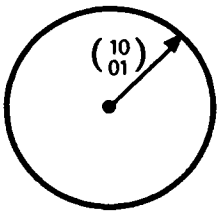
unchanged when all wavefunctions are multiplied by the same phase, if a thinking system is in an eigenstate, multiplying the eigenstate by an arbitrary phase will produce no observable effect:

$$e^{na}|\text{eigenstate}\rangle = |\text{eigenstate}\rangle.$$

It follows that the universe of a thought is bounded by the hyperdimensional unit **sphere**

$$\cos^2 a + \sin^2 a = I^2 + a^2 = I$$

with the matrix 'radius' $\begin{pmatrix}1&0\\0&1\end{pmatrix}$ which does not have a simple intuitive spatial image. The closest analogy of a matrix radius is a worldtube in string theory. Matrix radius is not simply a line but an extended object in matrix space with at least four degrees of freedom:



Combining the matrix and conversion theorems, we explicitly shown that an implicative thought is a **4**-dimensional phase, a rotation in four-dimensional space, which in a natural way connects the theory of consciousness with the space of relativity. However, there is even deeper and more intriguing possibility. Since the number of dimensions **n** of a sphere exceeds the number of dimensions **m** of its radius by at least one:

$$n \geq m + 1,$$

logical thought may naturally extend in **5**-space. Physicists are looking for signs of the **5**th dimension in general field theory. It is quite possible that consciousness provides the direct evidence of the fifth dimension [Ref 79].

In logic we are familiar with the DeMorgan dualities:

$$\langle p|\wedge|q\rangle = \overline{\langle \bar{p}|\vee|\bar{q}\rangle}, \qquad \langle p|\vee|q\rangle = \overline{\langle \bar{p}|\wedge|\bar{q}\rangle}.$$

According to the general scheme, connecting the logical and quantum brain, the conversion process can be naturally treated as the generalized DeMorgan transformations:

$$\langle p|a|q\rangle = \overline{\langle \bar{p}|\rightarrow|\bar{q}\rangle}, \qquad \langle p|a^*|q\rangle = \overline{\langle \bar{p}|\leftarrow|\bar{q}\rangle}.$$

Consequently, a single fundamental operation exists, connecting the physical and the mental aspects. Such a universal operation is the duality transform, which uniformly converts the quantum field operators into logical operators

$$\overline{\theta a} = \rightarrow, \qquad \overline{\theta a^*} = \leftarrow,$$

and vice versa:

$$\overline{\theta \rightarrow} = a, \qquad \overline{\theta \leftarrow} = a^*.$$

The dual operators share a common eigenstate,

$$\rightarrow|0\rangle = 1|0\rangle = |0\rangle, \qquad a|0\rangle = 0|0\rangle = |0\rangle,$$
$$\leftarrow|1\rangle = 1|1\rangle = |1\rangle, \qquad a^*|1\rangle = 0|1\rangle = |0\rangle,$$

hence commute and thus can be known quantum-mechanically simultaneously:

$$[a, \rightarrow] = 0, \qquad [a^*, \leftarrow] = 0,$$

which is a unique situation for the logical and quantum field operators a^* and a. The commutation entails that if the brain is in a logical eigenstate, then it can naturally revert to a state with the complementary eigenvalue.

Finally, the conversion principle is manifest at the highest level of cognitive logical operations. In matrix logic the covariant intelligent inferences, **Modus Ponens** (the method of affirming) and **Modus Tollens** (the method of denying), are cross-couplings of the dual logical and quantum operators:

$$\overline{\langle x|\rightarrow|y\rangle\langle x|a^*|y\rangle} \equiv 1 \qquad \overline{\langle x|\leftarrow|y\rangle\langle x|a|y\rangle} \equiv 1$$

Modus Ponens **Modus Tollens**

The conversion postulate is a fundamental logical duality theorem. Duality emerges as an underlying unifying concept which is essential for the understanding of a complete undifferentiated brain-mind reality. It will take us to the farthest edges of theoretical physics and to the advanced theories of the brain, some of which are so new that many physicists have only a vague idea of them, if at all. Fearfully and almost instinctively, physics refrains from the study of the mechanism of consciousness, setting it aside as if it were incompatible with the scientific method. In spite of a clear understanding that logic is causal, for too long physicists have failed to realize that physics is logical and logic therefore should not be viewed simply as an abstract construct but as a fundamental structure at the core of physical theory. A convergence of the logical and physical methods is responsible for the fact that logical theory is beginning to reflect in its structure the fundamental principles of physics whose laws in turn reflect the fundamental laws of logic. The conversion theorem warrants serious scientific investigation of the phenomenon of consciousness. The time has come to examine it in a fundamental framework and to seek specific predictions for experimental verification. A verification test of the thinking brain can be formulated on the basis of the fundamental invariance of implication, taking into account that cognitive elements are nonlocal hyperdimensional waves formed as a coherent superposition of the renormalized sinusoid matrix 'harmonics':

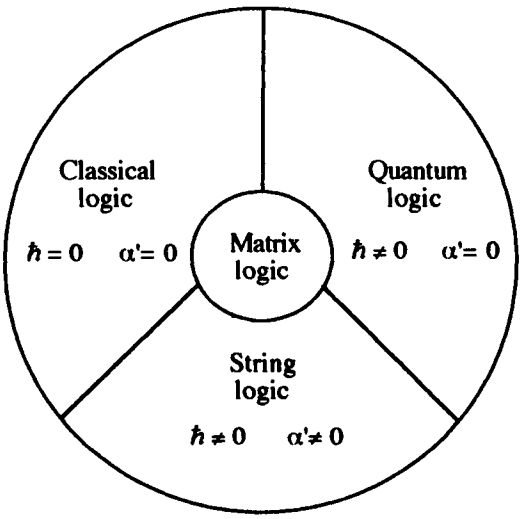
$$\sin a^*, \ \sin a, \ \sin a^* \sin a, \ \sin a \sin a^*.$$

Renormalization theory makes it possible properly to handle infinities of quantum fields which are absorbed into free parameters of the theory, so that it gives finite predictions for all observables to all orders in perturbation

theory. In logic, calculations done with a perturbation expansion eliminate all infinite terms if the vacuum operators are taken as the arguments of the expansion.

No strings attached?

Whereas in quantum theory quantized fields propagate in spacetime, in string theory we have only a supporting two-dimensional field theory that encodes the information. A change from a classical trajectory to a wavepacket converts classical logic, $\hbar=0$, to quantum logic, $\hbar\neq0$, but a replacement of the trajectory by a worldtube changes one-dimensional logic into two-dimensional logic. Matrix theory, which is a tensor product of two-dimensional logics, naturally obtains the fundamental meaning of an underlying logic of the theory of superstrings:



Gauge symmetry is of paramount importance in much of modern physics. Is the gauge principle is relevant to the intelligent brain, which is after all a very large but finite quantum field theory? The purpose of this study is to demonstrate that the laws of logic are not a matter of our free choice but fundamental in origin, and just as quantum particles are fundamental solutions to gauge symmetry in physics, so intelligent thoughts are fundamental solutions to gauge symmetry in logic.

MATRIX STATISTICS

There are deep, and often overlooked, connections within the triad: scalar, bosonic, Abelian, on one hand, and within the triad: matrix, fermionic,

non-Abelian, on the other. Matrix algebra is generally noncommutative, and the advent of m(atrix) theory has revealed a fundamental noncommutativity in the foundation of logic. In contrast to conventional scalar logic, which by all accounts must be considered to be bosonic:

$$\text{true}\bullet\text{false} = \text{false}\bullet\text{true},$$

the corresponding logical matrices do not commute:

$$\mathbf{TRUE}\bullet\mathbf{FALSE} \neq \mathbf{FALSE}\bullet\mathbf{TRUE},$$

a fact which plays a critical role in identifying new logical statistics. Matrix Statistics stands apart from Bose-Einstein and Fermi-Dirac statistics, the two fundamental statistics describing the *physical* universe. Our key claim is that the description of the *mental* universe, the logical mind, requires Matrix Statistics. Further still, we intend to show that the two known quantum statistics are the eigenstate limits of general and supersymmetric Matrix Statistics.

Quantum mechanics overturned the traditional notion of symmetry. In addition to bosons, interchange of which sends the state-vector to itself, physical experiment determined the existence of fermions. If two fermions are interchanged, the state-vector does not go to itself but to minus itself. Thus, in quantum physics a swap of particles and fields is 'mysteriously' governed by the factors $\Phi = \pm 1$:

$$\psi(a)\psi(b) = (\pm 1)\,\psi(b)\psi(a).$$

with $\Phi = +1$ for bosons, and $\Phi = -1$ for fermions. The mystery can ultimately be traced to and resolved within the matrix approach. Taking a unified view, we uniformly derive the factors identifying a statistics as the matrix square root of identity $\Phi = \sqrt{\mathbf{I}}$:

$$\Phi = +\mathbf{I},\ -\mathbf{I},\ \mathbf{N},$$

where the root $\Phi = \mathbf{NOT}$ determines the new logical statistics of the brain:

$$\mathbf{TRUE}\bullet\mathbf{FALSE} = \mathbf{N}\bullet\mathbf{FALSE}\bullet\mathbf{TRUE},$$

Since $\mathbf{NOT}^2 = \mathbf{I}$, changing over the logical states twice, we recover the initial product

$$\mathbf{TRUE}\bullet\mathbf{FALSE} = \mathbf{N}\bullet\mathbf{N}\bullet\mathbf{TRUE}\bullet\mathbf{FALSE}.$$

Never before in history of logic has a statistics been used in a fundamental way. A logician unfamiliar with physics may better understand the essence of Matrix Statistics in terms of conventional logical operations. Implication, the key multiplicative matrix operation, underlying transitive thought, follows the rule of Matrix Statistics. When the complementary antecedent and consequent are changed over in the implication, the negation factor is needed to retain the equality:

$$(0 \rightarrow 1) = \mathbf{N}\,(1 \rightarrow 0)$$

which is exactly the structure of the Matrix Statistics. Significantly, the factor $\Phi = \mathbf{NOT}$, controlling the statistics, is the key logical operator of negation which gives the statistics a fundamental character. The m(atrix) principle

naturally takes account of classical, quantum and cognitive processes. Revealing the existence of one universal general statistics, it brings different statistics under the one umbrella of the square root of identity, and is the most natural way towards a grand unification. This proposal goes as follows. In a manner quite unlike anything we have encountered thus far in quantum physics, the cognitive brain supersymmetrically mixes bosonic and fermionic fields. We have a false vacuum, which is unstable. When and if logical supersymmetry is broken down, the stable Bose-Einstein and Fermi-Dirac subsymmetries emerge, being the two eigenstate limits of the general Matrix Statistics. In addition to bosons and fermions we must look for fundamental matrix particles, which are neither bosons nor fermions but partly both.

The implications of this scheme are far-reaching and profound, and not only for identifying a missing physics of the mind. Supersymmetry is commonly associated with high-energy physics on the order of the Planck length, which is far beyond anything attainable with present-day technologies. If Matrix Statistics is realized by the brain, this suggests a possibility of supersymmetric states at the low-energy end, which may give us new clues in the hitherto unsuccessful search for a Theory of Everything. There are precedents. There was a time when the state of superconductivity was associated exclusively with temperatures in the vicinity of absolute zero, and few have thought about the existence of a 'hot' supercurrent at room temperature.

The selection of a particular root of matrix identity selects a particular physics. The quantum particles with integer spin will obey Bose-Einstein and those with half-integer spin Fermi-Dirac statistics. Most importantly, a statistics dictates the number of particles that can occupy a quantum state. While there is no limit on the number of bosons in a state, no two fermions can occupy the same quantum state. The latter property is known as the Pauli **exclusion principle**. The question naturally arises what sort of exclusion or inclusion distinguishes Matrix Statistics. Experimental studies in cognitive science are inconclusive but show that the human brain can handle about **5±2** mental elements simultaneously. In the context of m(atrix) theory this problem takes on a very different and fundamental character. Matrix Statistics must answer the question: how many nibbles or *logical particles* are allowed in a cognitive state?

SUPERSYMMETRY

The possibility of deriving Bose-Einstein and Fermi-Dirac statistics from general Matrix Statistics indicates its potential applicability for supersymmetry, which is the ultimate proposal for the unification of all particles and forces in physics. Traditionally quantum theory kept bosons and

fermions apart. Forces are described by bosons which are quanta with integer spin. Such are the photon, the quantum of electromagnetic field, the Yang-Mills field or the graviton, the hypothetical unit of gravity with spin **2**. As opposed to forces, particles, such as quarks and leptons, are fermions which are distinguished by the half-integer spin **1/2, 3/2, 5/2**, etc. Unitary symmetry may shuffle nucleons or quarks among themselves, but bosons and fermions do not mix. Supersymmetry has changed all that. Supersymmetric equations interchange bosons and fermions while remaining intact. A multiplet of supersymmetry consists of an equal number of fermions and bosons. In supersymmetric theory, all particles have superpartners or **sparticles**. The gravitino, for example, is the supersymmetric partner of graviton. The supersymmetric partners of the lepton and the quark, respectively, are the slepton and the squark.

Unfortunately supersymmetry is based on a bizarre kind of number system, a calculus of supernumbers, in which most of the operations we take for granted in ordinary algebra fail. If **a** and **b** are supernumbers, then

$$\mathbf{a} \cdot \mathbf{b} = -\mathbf{b} \cdot \mathbf{a}$$

and for $\mathbf{a} \neq 0$ the product still can vanish:

$$\mathbf{a} \cdot \mathbf{a} = 0.$$

If these were ordinary numbers, such relations necessarily would collapse the system to nil, rendering it useless and physically meaningless. Not so with supernumbers, which yield a self-consistent and nontrivial system, albeit artificial and counterintuitive one.

This counterintuitive element, however, disappears at once if we revert to Matrix Statistics, that is, change over from the scalar to the matricial dimensions. Suddenly, perplexity gives way to the utmost clarity. What seems to be quite weird and strange for ordinary numbers is obvious and elementary for matrices. The matrix equality

$$A \cdot A = \mathbf{0},$$

where $A \neq \mathbf{0}$, states nothing but simple fact that A is a divisor of zero. Likewise there is nothing extraordinary in the Lie matrix multiplication:

$$[A, B] = -[B, A].$$

where the change of order of the multipliers in the commutator causes the negative sign to appear.

Looking back at all the blind alleys and false starts in the theory of consciousness, we see that Nature does care about common sense after all. It is not reality but our current mode of thinking which requires a radical alteration. Matrix Statistics is a most natural and adequate language of supersymmetry. The future of postquantum and postrelativistic physics, with their hitherto unfulfilled promise of ultraunification, depends to a large extent on our ability to derive testable predictions beyond the Standard Model from Matrix Logic.

INFORMATION IS PHYSICAL PHYSICS IS LOGICAL

There is one basic cause of all effects.

Giordano Bruno

DUALITY PRINCIPLE

Duality is the existence of two equivalent formulations of a single theory, with a symmetry exchanging antecedents and consequents. The duality laws hold many secrets, one of them being the fundamental mechanism of consciousness. Even though the conversion postulate and the discovery of the finitely halting series and logarithms have provided a proper theoretical framework for the study of the unknown dualities, we still have to learn a great deal more about duality before the secrets of the thinking brain are fully unravelled.

Eugene Wigner, analyzing the role of conscious observers, came to the conclusion that *physics is incomplete* without consciousness, which has the last word in collapsing a quantum state vector. The Schrödinger equation does not apply to consciousness and must be replaced by a nonlinear theory which might explain a resolution of quantum superposition into actual alternatives. According to Kurt Gödel *logic is incomplete*, requiring infinite addition of new axioms to avoid inconsistencies in a formal system. It cannot be complete and consistent at the same time. The Gödel theorems tells us that the logical mind is fundamentally incomplete. They don't tell us *why*. In this study we will seek an answer by joining physical and logical theories, which amounts to introducing physical operations into logic and information primitives into physical description. In the resulting unified theory brain-mind is treated as one undivided whole, controlled by the duality principle. This include a premiss that logic is incomplete without additional physics, in particular without quantum mechanics, provided that the latter is a universal physical theory. And if it not, as repeatedly emphasized by Einstein, we still have a first approximation whereby quantum uncertainty on one hand and the incompleteness of logic on the other join together in circular causation. It is in this sense that we suggested before that Gödel's theorems can be interpreted as a kind of quantum-mechanical statements [Ref 88]. Provided quantum mechanics is incomplete without logic in the same way as logic is incomplete

without quantum mechanics, can we understand the intelligent brain by joining these two *incompletenesses* of Wigner and Gödel together? If treated jointly, wouldn't they compensate each other's weaknesses, leading to the adequate theory of the brain?

There is a fundamental connection between the logic of nature and the nature of logic. A project aimed at unraveling the essence of this connection should amount to the development of a scientific theory of conscious mind. For many centuries the study of the mind has been and remains in many respects a subject of philosophy. Lately, the situation has changed dramatically. In the unsettling area between the two philosophical counterpoints, Descartes' dualism and Spinoza's monism, a scientific concept of the mind is taking shape. After wasting much time arguing which platform, dualistic or monistic, is more suitable for the understanding of the mind, we no longer have to choose. Neither dualism nor monism succeeded in withstanding the test of time and the scrutiny of scientific inquiry. The advance of exact sciences has shown a way to make sense of the inner sense. Among many, often conflicting, theories a view is emerging that the answer to the puzzling phenomenon of the thinking brain lies in duality. It must be emphasized that **duality** is an integral concept which naturally combines the dualistic and monistic approaches and which should not be confused with **dualism**.

The principle of duality, which we will explore in this study, has a long and distinguished history in mathematics and physics. It has played an important role in the development of physical theory, epitomized by Maxwell's equations and the wave-particle principle in quantum mechanics. Duality is an old idea, and one finds it very surprising that many philosophers and researchers in brain science continue to give preference to one approach over the other, monistic over dualistic or vice versa, while the principle of duality allows both, seemingly antagonistic, models to coexist in harmony.

To trace the origin of the duality principle one perhaps must start with Desargues (1591-1661), the French mathematician and engineer, who in a work published in 1639, founded the discipline of projective geometry, largely misunderstood and neglected until the 19th century. Desargues was influenced by perspective in art and struck by the fact that the projection of a conic is also a conic. He assumed that parallel lines meet at an ideal point (infinity) and like Kepler he considered the parabola to have a second focus at infinity. More generally, projective geometry can be defined as the study of those properties of surfaces and figures that are unchanged under central projection. The central projection of a given set of points in one plane onto a second plane is the set of points produced by lines through a fixed point and through the given points intersecting the given plane.

Duality is the connection between lines and points or between planes and points in solid geometry. Since a line can be defined by two points and a point by the intersection of two lines, an important aspect of projective geometry is the existence of dual theorems, in which points and lines are interchanged. In this sense, the line and the point are said to be dual elements. Similarly,

connection of points by lines and intersection of lines to give points are dual operations. A statement in which the name of each element is replaced by its dual element and the description of each operation is replaced by the dual operation leads to a dual theorem. The principle of duality in projective geometry states that if a theorem is true, its dual is also true.

What is the relevance of the duality principle for consciousness? Although we are well aware of the existence of a fundamental connection between the logic of the physical world and the logic of the *mental* world, the exact mechanism by which the brain projects one logic into the other has remained largely a mystery for a very long time. How does one pass from a physical state (the fact) to a logical statement (the idea) and conversely, how does one convert a logical statement into a physical state of the brain? To understand the mechanism of consciousness in scientific terms we must understand this relation between the logic of facts and the logic of ideas. The facts are absolute, the ideas can be and often are wrong and relative. How does one identify a central projection which prevents ideas from going astray? Is consciousness a central projection?

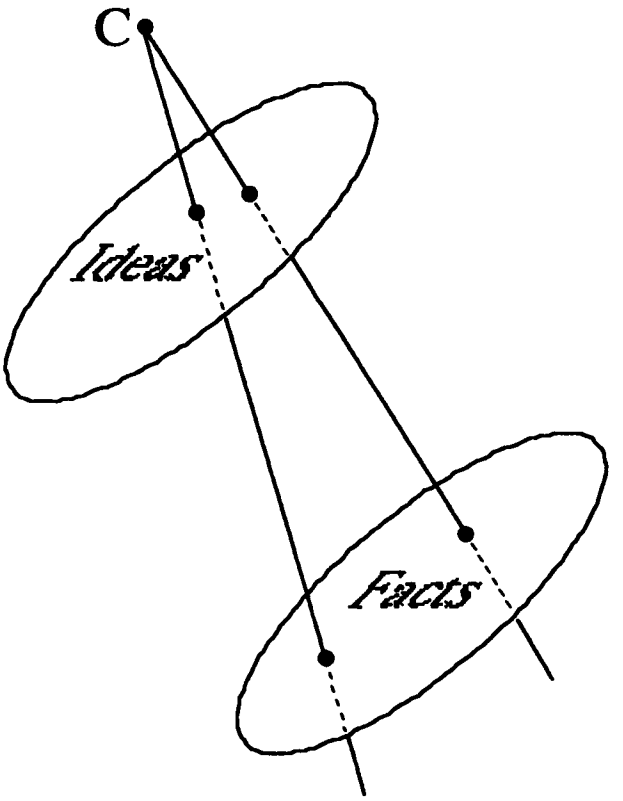


Fig. 2
A cognitive 'central projection' connects facts and ideas.

The relation between facts follows the laws of physics. The relation between ideas follows the laws of logic. How it is possible that the relations between ideas, which we deduce through the pure faculty of the mind and mathematics, have an exact and often fundamental projection to the relation between the facts? Is consciousness a map, a fundamental code, connecting the two systems? A mapping from facts to ideas we understand much better than the reverse mapping from ideas to facts. The principle of duality, applied to logic, brings new clarity into the difficult subject of consciousness, opening new avenues for the study of the connection between the physical brain and phenomenal experience.

PROJECTIVE LOGIC

There is a simple duality in classical logic: the axioms of Boolean algebra are symmetrical with respect to interchange of • and +, and 0 and 1. If some logical expression is valid, then the expression obtained by swapping • and +, and 0 and 1, is also valid. In matrix logic the duality principle gains new ground. Similar to geometry where 2 lines identify a point and 2 points identify a line, in matrix logic 2 scalar logical values determine a logical state:

$$x \otimes \bar{x} \Rightarrow |x>,$$

while 2 logical states, interacting via an operator, which can be the identity, determine a logical value (a scalar):

$$<x|\, \mathbf{L}\, |y> = z.$$

The principle of duality in projective logic is just like in projective geometry. We can change from logical scalars to logical vectors, because what is true for Boolean scalars is true for matrix logical states. A statement in which each element is replaced by its dual element leads to a dual statement or dual theorem. If a theorem is true, its dual is also true. What, however, distinguishes projective logic is that its duality is imperfect. While any Boolean truth has a dual in matrix logic, there exist matrix theorems without a scalar equivalent. Most significantly, two interacting logical states can produce not only a logical scalar <x|y> but also what we call *abstraction*, for a reason to be explained later. The *abstraction* has no analogues in conventional logic and is realized as the tensor outer product of the logical states:

$$|x><y| \Rightarrow \text{logical matrix}$$

In a sense the *abstraction* is a thing in itself. Whereas the products of states yield logical scalars or matrices, the product of abstractions is again an abstraction, which remains such until it is traced over to give a scalar:

$$\mathbf{Tr}\ |x><y| = <x|y>.$$

Mind needs physics. Physics needs logic

Projective logic stands in direct parallel to projective geometry. To achieve a scientific theory of consciousness it is not enough to gather information about how the brain does its job. It is not enough to know the states of brain. Likewise, there can be no adequate understanding of the mind which relies solely on the logical machinery of manipulating statements. The fundamental theory must embrace the states of the brain and the statements of the mind as one integral whole. A new paradigm which makes it possible is the duality connecting logical states and logical statements.

We have learnt much about how to connect different logical dualities between themselves and physical dualities between themselves. The question how to interconnect these dualities into a unified and consistent picture and in this way to join the mental and the physical realms remains open, largely because we know very little, if anything, about the actual mechanism of the brain-mind interface. This situation is further aggravated because whatever knowledge about the physical processes in the brain may be available, it still cannot tell us what cognition is and how the mechanism of thought actually works. Clearly, a scientific theory of the brain has to incorporate information and logical aspects into the physical description of the brain. Just as the chemistry of DNA is more than just a chemistry but also a transmission of genetic information, a quantum action in the brain is concomitantly a logical action, the two being fundamentally entwined with each other.

Reflecting on the fact that neither physics nor logic alone can confront and unravel the unique phenomenon of cognition, projective logic calls for the unification of physical and logical primitives in a single formalism, just as they are fundamentally unified in the inner working of the brain. According to this approach the key to the understanding of the brain lies in duality. In physics, duality essentially means the invariant interchange between the electric E and magnetic H aspects of the theory or between weakly and strongly interacting fundamental non-Abelian fields.There is also wave-particle duality in quantum mechanics. In logic, duality concerns the invariant swap between the basis sets of connectives, $\{\mathsf{N}, \wedge\}$ and $\{\mathsf{N}, \vee\}$, according to the De Morgan rules:

$$\wedge = \overline{\mathsf{N} \vee \mathsf{N}}, \quad \vee = \overline{\mathsf{N} \wedge \mathsf{N}}.$$

One can regard E (or $\wedge$) as fundamental, and then H (or $\vee$) appears as dual, and vice versa. Both 'physics to physics' and 'logic to logic' duality mappings leave physical theory and logic theory unaltered: the equations change but have the same form. But what about 'physics to logic' and 'logic to physics' cross-mappings? Aren't precisely these sorts of dual cross-mappings to be found at the root of the cognitive operations of the brain? Wouldn't such cross-mappings provide a foundation for the elusive brain-mind interface?

Consciousness is self-reference. In a self-referential logical system one must be able to extend a logical state not only in an orthogonal basis of true and false, but in an arbitrary basis of other logical states. In other words, **a logical state must depend not only on the probability of states but**

on states themselves. We know quite well how to procure a logical operation by logical means. Logic tells us explicitly how to combine some logical statements p and q, or the logical states $|p\rangle$ and $|q\rangle$, into a logical statement r or state $|r\rangle$. For that there is a certain truth-table in Boolean logic or the *logical* operator **L** in matrix logic:

$$\langle p| \mathbf{L} |q\rangle = r.$$

But can we express a logical operation physically? A positive answer to this question entails that there exists some *nonlogical* physical operator Ç capable of mixing p and q to deliver the same outcome value r as the logical operation above does:

$$r = Ç\ (p, q)$$

Let the operator Ç be a set of the coefficients c, $\bar{c}$, such that

$$Ç\ (p, q) = c\,|p\rangle + \bar{c}\,|q\rangle = |r\rangle.$$

The coefficients are normalized linearly:

$$\bar{c} + c = 1,$$

as opposed to the quadratic rule of normalization in quantum mechanics. At first glance the operator Ç looks like an ordinary probability, but a close scrutiny reveals that in general

$$c, \bar{c} \notin [0, 1],$$

which can only be the case if the coefficients are negative or greater then unity. Hence, generally, the physical composition requires a nonlogical element, even though the outcome of the composition is necessarily a classical probability:

$$r \in [0, 1].$$

The duality theorem

If the logical values α and β are dual:

$$\alpha = \langle p| \wedge |q\rangle,$$

$$\beta = \langle p| \vee |q\rangle,$$

then their superpositions are complementary:

$$|\alpha\rangle = c\,|p\rangle + \bar{c}\,|q\rangle,$$

$$|\beta\rangle = \bar{c}\,|p\rangle + c\,|q\rangle.$$

Proof

The dual logical values α and β are explicitly calculated as:

$$\alpha = pq$$

$$\beta = p+q-pq$$

Summing up the results, we get:

$$\alpha + \beta = p+q.$$

Summing up the superpositions

$$\alpha = c\,p + \bar{c}q,$$

$$\beta = \bar{c}p + c\,q,$$

we derive the same result:

$$\alpha + \beta = c\,p + \bar{c}q + \bar{c}p + cq = (c + \bar{c})p + (\bar{c} + c)\,q = p+q.$$

Because in general

$$2 \leq \bar{c},\ c \leq -1,$$

the superposition of logical states cannot be obtained by logical means alone. Even though the coefficients are jointly normalized to unity, they are neither classical nor quantum probabilities, taking values outside the unit interval. Understanding of these values, uninterpretable in terms of traditional probability, will lead us to introduce a fundamental concept of **improbability**.

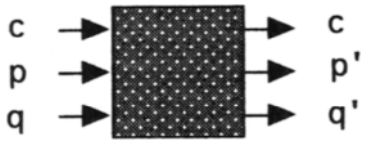
The duality theorem unexpectedly provides a bridge between the logical superposition principle [Ref 91], which is a counterpart of the quantum superposition principle, and reversible logic [Ref 78, 22], which is a counterpart of quantum relativistic field theory. While irreversible logic is essentially macrophysical and information about the intermediate steps of computation is irretrievable, by its very definition consciousness must keep track of (remember) thinking operations, and in this sense it is reversible, hence unitary micrologically. Given a map for a direct operation, we are often interested in determining an inverse map. In the theory of thinking the issue frequently arises whether to use rules for forward or backward inference, in other words, whether deduction should be driven by putting questions or by the available facts.

We are looking for general laws of the consciousness phenomenon, no matter how it is accomplished, with underlying belief that it is ultimately governed by fundamental physical principles. The irreversible logic is based on noninvertible primitives, reflecting aspects of macroscopic physics. In contrast, the reversible logic of the intelligent brain reflects microphysical aspects of consciousness. As we will show further in relevant sections, the logical superposition principle

$$|r\rangle = c\,|p\rangle + \bar{c}\,|q\rangle$$

and projective logic play a decisive role in understanding the reversible mechanisms in the brain and is instrumental in the topological concept of consciousness, which is the central theme of this study.

In addition to data lines reversible logical gates require an extra element. The Fredkin reversible gate has a triple of inputs, adding the control bit c, to a standard pair of inputs p and q:



When the value of control bit is 1 the data follow a predestined path, but when the value is 0, the outputs are swapped. In reversible logic all

computations are ultimately reduced to conditional routing of information bits. In strong affinity to the physical laws of conservation, the bits can be moved in the course of computation, but their total number remains intact, and can be treated as an integral constant of motion of the gate. The number of output ports always equals that of input ports.

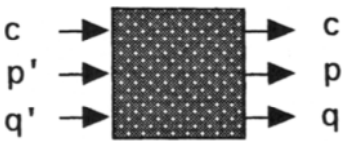
If the outputs of the gate mix the inputs in accordance with the logical superposition principle:

$$c = c$$

$$q' = \bar{c}\, q> + c\, p$$

$$p' = c\, q + \bar{c}\, p,$$

then in the discrete limit the gate obeys the fundamental conservative dynamics. When the output is fed through the gate, the gate retraces itself backward, giving in the output the original input: the gate coincides with its own inverse:



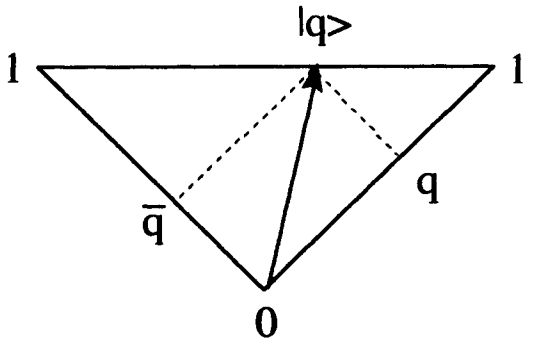
Reversible logic connects the study of intelligence to fundamental principle of physics. The logical superposition principle was proposed long time ago, but until now there has been a major lack of the understanding of its implications for a fundamental theory of logic. Physicists commonly think in terms of the complex-weighted superpositions, with the squared moduli of the complex amplitudes representing actual and normed probabilities:

$$|c|^2 + |\bar{c}|^2 = 1$$

If consciousness could directly perceive different quantum alternatives without magnifying them to the classical level, then it would be subject to a nonquadratic rule of normalization:

$$c + \bar{c} = 1.$$

Logical superposition, however, confronts us with a serious problem. Superposition of logical states can violate the central axiom of probability theory which demands that probability does not exceed the unity and is strictly positive. Cognitive logical elements will be confined to a space of normed logical states, where any logical vector erected from the origin ends up on the *horizon* which is a straight line $[1, 1]$ as shown below:



The logical triangle is surrounded by the forbidden area where values are not confined to the unit interval of probability. This is in strong resemblance to the worldcone which separates causal timelike and acausal spacelike events in relativity theory. Relativity is known to run in conflict with the finite size of elementary particles that cannot pass through the origin of the Minkowski space without penetrating into the acausal area or absolutely remote spacetime. The logical mindcone has strikingly similar features which make it almost irresistible to connect the fundamental mechanism of logical consciousness with the fundamental notions of relativity.

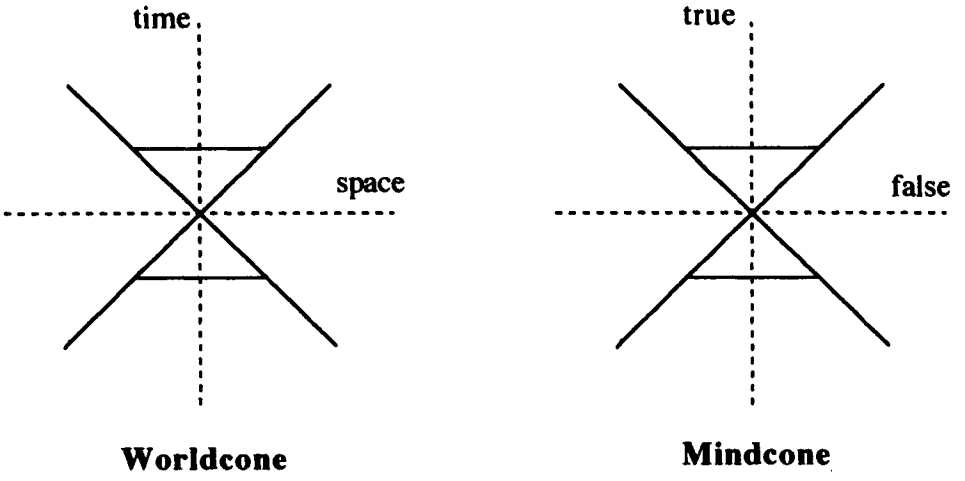
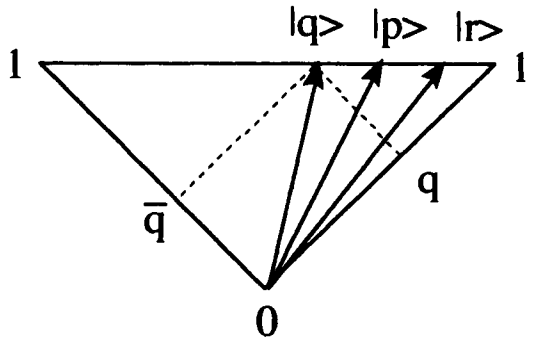


Fig. 3
Pseudo-Euclidian logical interval (false^2-true^2) naturally reduces to the linear interval (false-true), with isotropic worldlines in the worldcone corresponding to the logical undecided in the mindcone.

Suppose we wish to expand the logical state |q> as a superposition of the states |p> and |r>:



Since $q \notin [p, r]$, logical superposition will not be attainable by logical means alone. The factors c and $\bar{c}$ are jointly normalized but are not probabilities and would necessarily take the states outside the logical worldcone.

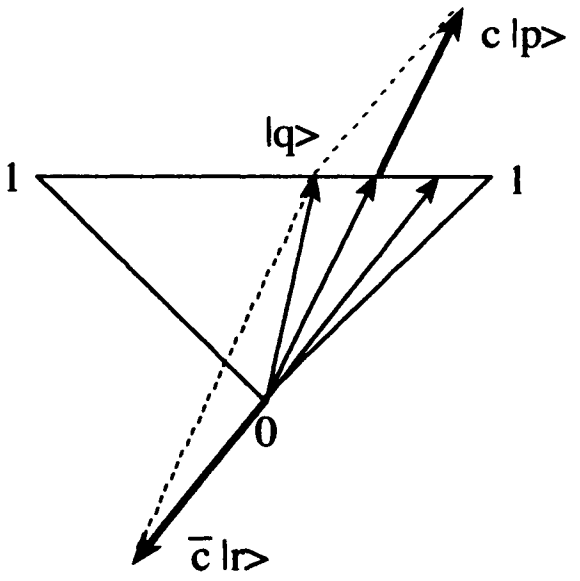


Fig. 4
Bold arrows extend outside the domain of classical probability.

To reiterate the result of this section: The duality principle forcefully indicates that logic needs physics. The idea that mind needs brain or consciousness needs matter is far from trivial. We are aware only of one part of logic, the *cognitive* logic. Over the logical horizon, outside the logical mindcone, there resides another major part, which is termed virtual or *nonconscious* logic [Ref 88]. Understanding of this unexplored and fundamental part of the mind brings abstract logical theory into direct contact

with quantum relativistic physics and is important for deciphering the mechanism of high-level intelligence.

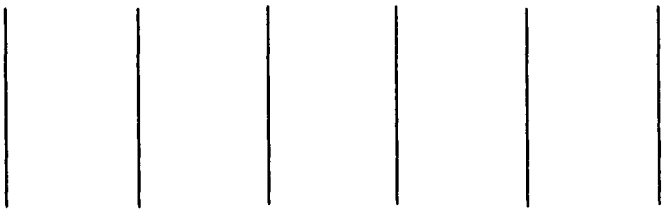
THE DIMENSIONS OF THE MIND AND THE THIRD QUANTIZATION

Information is what determines space and time.

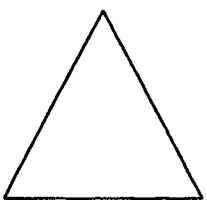
David Bohm

One-dimensional classical logic is a scalar, and grossly inadequate to deal with the complexities of creative thinking. To grasp the essence of consciousness we need to rise to the challenges of matrix dimensions.

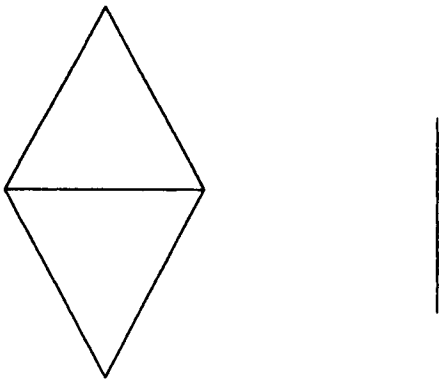
Most of us familiar with a remarkable and beautiful logical problem: Given six matches one is asked to construct 4 equilateral triangles,



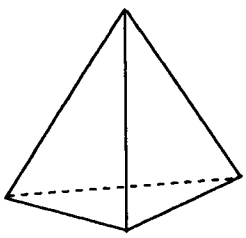
with the sides equal to the length of a match:



On a desk, in *flatland*, one quickly constructs a triangle with three matches, than a second triangle with the remaining three. The next typical logical step is to let two triangles share a common side, which saves us an extra match:



But here the confusion begins. There is only one extra match with which to construct the two more triangles required. Whichever way one attaches the extra match to the figure, the two remaining triangles cannot be built. The flat thinker finds it utterly impossible, and it seems that no other saving can be achieved to make up for the deficit of available sides. I have seen few people solve the problem themselves. And how simple and astounding the solution looks, if you can recall your feeling the first time you were confronted with it. Instead of thinking in a two-dimensional plane, jump into three-dimensional space. By adding an extra dimension, the pyramid is built with the extra match connecting two vertices and making for the two missing equilateral triangles at once:



Following this line of reasoning an attempt was made in 1970s to consider the extra fifth *information* dimension as a possible approach to the problem of the mind [Ref 79]. Earlier, in 1920s the mathematician Theodor Kaluza had examined a five-dimensional field theory - one of time plus four of space -, followed by Oskar Klein and Uri Rumer in 1950s who attempted to combine electromagnetism and gravitation in 5-space, and even considered the extra dimension to be physical. Suggesting the existence of an extra dimension, physical or abstract, these works anticipated the modern gauge and string theories. Although multidimensional theories, including Kaluza-Klein theory, primarily concern pure physics, the evidence is mounting for their relevance to the fundamental theory of logic and consquently for the logic of the thinking brain. M(atrix) logic is opening up an window of opportunity for discovery of a fundamental theory of the brain, in which the space and time

coordinates are replaced by the matrix logical coordinates, the matrix dimensions.

The idea of describing spacetime coordinates by matrices instead of numbers was introduced in matrix logic in 1986 [Ref 88], then a decade later in 1996 it was explored in quantum field theory of strings by Banks, Fischler, Schenker and Susskind [Ref 6]. When space and time coordinates are identified by matrices which, in contrast to ordinary numbers, generally do not commute, spacetime becomes a fuzzy concept, quite similar to what happened to the coordinates of classical Hamiltonian phase space when quantum mechanics emerged. Connecting matrix logical coordinates with spacetime is an important step towards the third-quantized grand unified theory. Since the early attempts of superunification it has been expected that joining relativity with quantum will lead to spacetime itself becoming ill defined. A landmark contribution in this direction was made by Snyder in 1947, who conjectured that spatial coordinates may take only quantized values

$$x, y, z = \pm l_0, \pm 2l_0, \ldots$$

where l_0 is the elementary length.

Just as the components of the spin of a quantum particle do not commute,

$$M_x M_y - M_y M_x = i\hbar M_z,$$

Snyder's space coordinates X_i obey the relation which is similar to the uncertainty relation of quantum mechanics, except that the action quantum $\hbar$ is replaced by Planck's length l_0, the smallest physical distance:

$$[X_i, X_j] \sim l_0$$

For $l_0 = 0$, Snyder's quantized space is reduced to a standard theory in which the particle's coordinates commute. Otherwise, the spectra of possible values of the coordinates is discrete and it is not possible to determine the precise values of all coordinates simultaneously, just as in quantum mechanics we cannot know at once the value of all of spin components. Measurement of the coordinate X introduces uncertainty into the measurement of Y or Z. In this sense Snyder's theory is nonlocal, because its main postulate forbids localizing the particle exactly in all three spatial coordinates simultaneously.

The matrix logical approach to the third quantization, which will be introduced below, is analogous in spirit to Snyder's; a fundamental difference, however, is that we replace the elementary length l_0 with the fundamental time t_0:

$$[X_i, X_j] \sim t_0.$$

This implies that uncertainty in matrix logical coordinates is measured in units of time. Even though relativistic spacetime was already widely known when Snyder introduced his idea, the theory takes no account of the time coordinate. This is surprising, but not so surprising, since then, and to this day, quantum mechanics did not have a time operator. Although Heisenberg's uncertainty principle formally applies equally well for coordinate and momentum

$$\Delta x \Delta p \geq i\hbar,$$

and for energy and time

$$\Delta E \Delta t \geq \hbar,$$

we know only how to convert x, p and E into quantum-mechanical operators, but not t. Raising time to the status of a dynamical observable is an important goal of this study. The understanding of the logical brain goes hand in hand with the understanding of time.

There is a mathematical science of space, geometry. Shouldn't there also be a mathematical science of time? Geometry connects points in space, spatially. Logic connects points in time, causally. In the same manner as geometry supports space, we suggest that matrix logic provides a mathematical framework for a theory of time.

In physics time is of fundamental importance. In a way everything what physicists do is to study time and whenever a new level of understanding of time is reached, a paradigm shift occurs in the theory of physics. Relativity theory is the most outstanding example. There are those who reject the reality of time, and those at the other extreme, like Whitrow, for whom time is primary and even more fundamental then space [Ref 108]. Modern physics considers time to be a real attribute of the universe and not an illusion of the mind. However, the problem of time is full of unanswered questions. In quantum mechanics the observable quantities should be represented by operators, but for time such an operator has not been found, which is a major drawback. Time enters into physical equations of motion simply as a parameter.

For consciousness time is of paramount significance. **Thought runs forward, understanding runs backward.** In a spectacular but poorly understood way the brain manipulates time, allowing us to access the present and the past and to probe the future. We know very little about this extraordinary function of the mind. Such effects as the déjà-vu illusion, Wilder Penfield's open-brain experiments on conscious subjects, first reported in 1950s and the backward-in-time referral observed by Libet et al [Ref 45] indicate that something is seriously wrong when we try to impose the standard concept of physical time on the thinking brain.

Somehow, and quite asymmetrically, the actual world has three spatial dimensions and one time dimension. Topologically time is contained in spacetime and the topological rank of spacetime is higher then the rank of time. If space X is topologically contained in space Y,

$$X \subset Y$$

and Y in X,

$$Y \subset X$$

then X and Y have same topological rank. Space X is topologically contained in space Y, if X is homeomorphic to some subset of Y. When topological inclusion is only in one direction, then the topological rank of one space is higher then the rank of the other. Two spaces may have same topological rank without being homeomorphic: for example, a closed unit interval and an infinite line. Nevertheless if X and Y have same topological rank, then there exists

$$A \subset X$$

and

$$B \subset Y$$

such that A is homeomorphic to B, and X-A is homeomorphic to Y-B. The topological rank of two spaces could be incomparable. This is the case for a circle and set comprising three lines with a common end (Fig.5).

Fig. 5
Two spaces with incomparable topological ranks.

In the course of this study, we will introduce a 4-dimensional fundamental logical phase space, defined by the matrix logical coordinates

{ **NOT, AND, OR, TIME** },

where **NOT, AND** and **OR** are the three symmetric dimensions and **TIME** is antisymmetric.

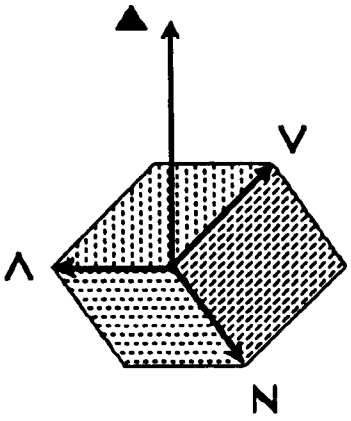


Fig. 6
4-dimensional matrix logical phase space.

We consequently will determine that the uncertainty in symmetric logical coordinates is universally measured in the units of antisymmetric time, according to **the time quantum theorem**:

[NOT, AND] = TIME

[NOT, OR] = TIME

[AND, OR] = TIME

This surprising and remarkable singularity is in contrast with the cyclic orthogonal parity in the Snyder model, and reflects the important feature of matrix logical phase space. Relativity has established a fundamental link between time and motion. The goal of the topological theory of the brain is to link time and consciousness.

There is notable **+2** discrepancy between the logical and quantum dimensions. The critical dimension in which a perturbative field theory is consistent in flat Minkiwski spacetime is **26** for the bosonic string and **10** for supersymmetric string theories. Matrix logical lattice also accommodates **26** bosonic charges (**13** charges + **13** anticharges). But for the supersymmetric code, expanded both as commutator and anticommutator, the critical logical dimension is **12**, which is **2** dimensions more than for strings. To obtain ultra unification, which must account for consciousness in the universe and thus besides quantum gauge symmetries also must include logic, a nonorientable two-dimensional field, which encode information, must be attached to a supersymmetric brane.

CLOSING PANDORA'S QUANTUM BOX

Since 1900, when Max Planck introduced the action quantum $\hbar$ into physics, quantum concept has become the great and stable fashion of 20th century physics. The *problem of black-body radiation*, which physicists initially wrongly believed would soon be gone, signalled the start of the era of quantum mechanics. As a hundred years of quantum mechanics have passed, physics is once again at a turning point. The new 'cloud' on the horizon of physics is the consciousness of the observer, *the grey-brain problem*, which in the 21st century will undoubtedly turn physics around. Those issues which seem to the majority of physicists to be more philosophical than physical, and sometimes are even held in disdain as irrelevant, will become the main and powerful new engine of physical thought. Much work still has to be done, but matrix logic sets up a research programme for the 21st century. In the coming decades we are going to find answers to many mysteries of consciousness, and the extraordinary achievements of 20th-century physics are going to look pretty mundane compared to the discoveries in brain science and the new technologies which they will entail.

The quantum paradigm went a very long way. First we quantized the energy and momentum of particles and determined that whenever a particle is

confined to an atomic structure its energy spectrum is no longer continuous but breaks up into quanta

$$E_n = \hbar\omega(n+\tfrac{1}{2}), \qquad n = 0, 1, 2, \ldots$$

The numbers n are positive integers or 0, the latter indicating the unavoidable presence of the essentially quantum ground state oscillations. In this **first quantization** the particle is described by a wavefunction, expanded in a complete system of basis functions or eigenstates

$$\psi = \sum c_i \psi_i, \qquad \sum |c_i|^2 = 1.$$

Although the energy of a particle is quantized, the particle moves in classical fields, considered to be continuous.

Next came the **second quantization**, where not only particles but fields, too, underwent quantization and the discrete nature of spacetime was accentuated. Wavefunction ψ, which in the first quantization is a fictitious object acted upon by operators of observables, now itself becomes the operator ψ, which is applied to more general quantum states:

$$\psi = \sum \psi_i a_i , \qquad \psi^* = \sum \psi_i^* a_i^*$$

where a_i^* and a_i are the creation and annihilation operators, the important new objects of quantum field theory. The operators ψ^* and ψ respectively increase and decrease the number of particles in the system. We have a quantized field where the numbers n in the expression for energy now can be determined as the eigenvalues of the population operator, the product of creation and annihilation $a^* a$:

$$H = \hbar\omega(a^* a + \tfrac{1}{2}),$$

and any state can be obtained by acting with the creation operator on a vacuum state. The form of the operator functions ψ^* and ψ in the second quantization is analogous to the expansion $\psi = \sum c_i \psi_i$ of the first quantization: it is as if the expansion is quantized for the second time, which explains the term *second quantization.*

The first and second quantizations having been explored, we now must prepare ourselves for a new quantum upheaval. Not only particles, fields and spacetime are discontinuous, our *descriptions* must be quantized too. This new quantization concerns information, the aspect of reality we are all well aware of but still poorly understand. Quantum-mechanical formalisms make use of wavefunction, which is in itself a continuous function while the quantities it refers to are not. In the information quantization we have to overcome a subtle and nontrivial conceptual hurdle and quantize wavefunction for a third time. At the heart of the scheme are the conjugated cross-products between the first and the second quantizations which convert the complex amplitude ψ_i into the real probabilities p_i, which sum up to unity:

$$\psi_i^* \psi = \sum \psi_i^* \psi_i a_i = \sum p_i a_i , \qquad \psi_i \psi^* = \sum \psi_i \psi_i^* a_i^* = \sum p_i a_i^*.$$

In a sense the information quantization emerges as a quantum mechanics in which complex probabilities cancel out. Most significantly, the fundamental

time operators, which are missing in the theory, are then derived as a commutator of the first and the second quantizations:

$$\blacktriangledown = \psi\psi^* - \psi^*\psi = \psi\psi^* - \psi^*\psi = [\psi, \psi^*]$$

$$\blacktriangle = \psi^*\psi - \psi\psi^* = \psi^*\psi - \psi\psi^* = [\psi^*, \psi]$$

This new definition of time as arising out of the interaction of particles and fields will allow us to treat time not simply as a parameter but as a dynamical logical observable.

COGNIZING IS QUANTIZING

The complexities of the quantization of information cannot be fully grasped without a true understanding of the cognizing mechanism of the intelligent brain. Cognizing is quantizing. To substantiate this claim we require to bring together a wide range of scientific fields previously viewed as separate and unrelated. This major unification effort takes the brain to the forefront of scientific enquiry. Just as quantum mechanics and relativity theory were the focus of the 20th-century science, the thinking brain is rapidly becoming and is destined to be the central issue of the science of the 21st century. As we try to keep pace with the bewildering speed of scientific progress, the objectives of brain science are rapidly changing. Following the advent of quantum theory, quantum ideas found their way into cognitive science, forcing us to start to think quantum-mechanically. At the beginning of the last century Heisenberg and Schrödinger laid the foundations of quantum mechanics. At the dawn of the new century one wonders whether they were not also the first true neuroscientists.

What is cognizing? Phenomenologically we know cognizing all too well. We cognize the world, ourselves, the face of a friend or enemy, a keyboard, a computer program, the idea of sets and millions of other difficult concepts or trivial things. As much as we are aware of the fact that we are *cognizers*, there is little, if at anything, we know about the actual *physical* mechanism of this extraordinary function of the brain. For many centuries man has looked with amazement or frustration into this great puzzle but without many technical results. Now as we cross the border into the next millennium, a critical mass has been accumulated, giving us a realistic hope that finally we can decipher the physical basis of the laws of thought and to unlock the secrets of SELF. The mystery of the mind is beginning to open up. The time is right, and there are important reasons for that. After years of search, much confusion and much hard work, the truth has come close. Having seemed remote and unattainable for so long, understanding of consciousness is within the reach of the methods of the fundamental sciences. We have come to grips with the idea that the information primitives have to be included explicitly in the foundations of physical theory. In reverse, logic can be viewed and studied as physics in a fundamental sense. **Information is physical, physics is logical.** The brain, quantum mechanics, relativity, logic and information are merely the elements of a greater picture, a complete theory the pieces of which

are beginning to emerge. An understanding of time as a dynamical variable, given directly to consciousness, is playing an important role in this development. Consciousness is a mechanism of measuring time, which in the third quantization scheme can be calculated as the commutator:

$$\left[\textbf{1st quantization, 2nd quantization}\right] = \textbf{Time.}$$

The antisymmetric commutator product eliminates the complex amplitudes of wavefunction, replacing them by the actual probabilities.

Emerging on a microlevel as the result of the interaction of particles and fields, time is the third quantization effect. In matrix logic the symbolic expressions which seem logically uninterpretable enter the domain of logic as real quantities representing time. Although this logical time is not a time in the traditional clock sense, it conforms to all formal mathematical rules for ordinary time, which enable one to distinguish between *before* and *after*, and *forwards* and *backwards*. There is no absolute logical time, there is no privileged direction of time. Time is given directly to cognizing systems, and it can be altered, reversed or cancelled altogether. The quantum field definition of time shows that it is determined by the difference between the annihilation and creation processes

$$\text{TIME} = a^{*} - a,$$

and to have a particular direction it must be asymmetrically disbalanced towards creation or annihilation. Since such a disbalance can be determined by the complement of the operator phases

$$\overline{\cos 2a + \sin 2a} = a^{*} - a \quad \text{and} \quad \overline{\cos 2a^{*} + \sin 2a^{*}} = a - a^{*},$$

time can be expressed in terms of the basic logical operations of implications:

$$\blacktriangledown = \overline{\rightarrow^{2}} = \leftarrow - \rightarrow, \quad \text{and} \quad \blacktriangle = \overline{\leftarrow^{2}} = \rightarrow - \leftarrow .$$

To understand consciousness we ought first to understand time. We must begin to think about time not in abstract but in physical terms, just as we think about energy or space in physical terms. Where there is no interaction there is no time. Interaction means that there is a cause and effect, the elements we require to determine the flow of time. In this respect time is a measure of interaction: the stronger the interaction, the shorter the interval of time. This agrees with the energy-time uncertainty relation, $\Delta E \Delta t \geq \hbar$.Taking this line of reasoning, we can conclude that consciousness is self-time. By that I mean a new understanding of time in cognizing systems as arising from self-measurement. The fundamental time 'measures' the information distance between two different logical states. But the cognizing system is a topologically closed self-measuring system: the two states are the same state, if only out of phase with itself. But if phase does not show up, in a cognitive system time can be excluded from the system, stopped or even become negative. Time can be viewed as an emergent quantity due to the interaction between the particles and fields. We must give time an actual reality, subject to observation by the brain. Somehow we do not think about time in physical categories, the way we think about energy or momentum. Just as energy is a capacity to perform physical

work, time is a capacity to perform *information work*, i.e. to obtain, to derive and to generate information. The important conclusion which follows from this is that **time must be measured in information bits, not in seconds.** The information bit gains the status of a fundamental unit, joining the distinguished family of c and $\hbar$. This understanding of time in terms of bits 'counted' by the brain, instead of seconds, given by clocks, is a major liberation for our intuition. An object can be in the same place at different instants of time, but our intuition revolts when we think about an object being in different places at a given instant of time, although that is what quantum nonlocality frequently demands. However, reformulating the problem in terms of information creates no such conceptual difficulties: information can encoded nonlocally in different places. For example, memory is a complex interaction of various parts of the brain, which cannot be exclusively localized in any of the parts.

The theory of the time operator which we put forward in this study treats time as a complex quantity. This, in turn, entails that complex numbers, which until now were considered to be a pure mathematical abstraction, may have actual existence in consciousness. **Then another way of saying that the mind manipulates time would be to say that the mind actually manipulates complex numbers**, which is possible only if logic is freed from the constraints of Hermitian physics. To understand why consciousness can be regarded as a logical system controlling complex numbers, we must understand the underlying operation of the thought processes, the comparison operation, which is in direct relation to time. Logic studies comparison mathematically and in this sense it is a mathematical science of time. If we want memory to be conserved, but cognitive time to have a nontrivial dynamics, we have to have an imaginary unit present. As opposed to quantum physics, in which complex values cancel out by conjugation when we calculate the actual probabilities, in logic the conjugation rule does not apply because probabilities are given directly and not in the concealed form of complex square roots. We consequently cannot do away with the imaginary number. Logic is fundamentally non-Hermitian. Seeking a new place for time in consciousness, we are confroned with the obstacles that are not due to technology but which are fundamental in origin.

QUANTUM COMPUTER

In a strange twist of fate the fundamental problem of the intelligence code of the brain directly overlaps with the problem of the quantum computer. As a matter of fact the very idea of quantum computation first emerged some thirty years ago in the framework of quantum-mechanical studies of the brain's high-level codes and not within computer science itself. [Ref 78]. Then,

as they to a large extend still are, computer designers and programmers were in quest for precision and certainty. The ultimate computer - the intelligent brain - works quite differently, taking advantage of the quantum-mechanical fluctuations and entanglement - the features which a computer designer might see only as undesirable and detrimental to computation.

When we look into the brain with its many billions of operational elements we are amazed how this extraordinarily complex marvel, confined to a cosmologically very tiny spatial volume effectively takes-in the whole universe, intelligently and in real time implementing sophisticated programs which naturally avoid exponential explosions. By contrast, many computer programs display quite weird behaviour. A sufficiently large computation on a computer typically demands an amount of computational steps or time which grow exponentially with the size of the problem. Although at the start a computation may proceed at a reasonable pace it often suddenly collapses, either running out of storage or failing to complete an operation in a realistic amount of time.

What kind of logic makes it possible for the brain to operate without exponential explosions? If the laws of the atom determine the laws of the brain shouldn't we look for a quantum solution to the puzzle of the brain and the intelligent quantum computer? The quantum codes are not in the cipher, nor in the key, but in their interaction [Ref 91]. Until recently mathematical logic and computer science in general based their theoretical conclusions and practical applications on a full reliance on the concepts of classical Boolean bits. The familiar 1's and 0's, standing for true and false, respectively, are the 'elementary particles' of information used by the conventional computer. There are those who passionately insist that our thoughts are nothing more than very long binary numbers of 1's and 0's. Anyone who has experienced a totalitarian society may disagree with mathematics: the sum of zeros is a frightening number. But a fundamental flaw of this either-black-or-white philosophy has been revealed explicitly with the development of matrix logic, in which superposed information bits were first indentified [Ref 88] and then, several decades later, 'rediscovered' in quantum-mechanical computing. Just as Boolean logic forms the basis of conventional computers, matrix logic has led to a computer based on quantum bits [Ref 74, 78 82, 85]. Portraying logical connectives as matrix operators acting in the two adjoint spaces of logical states we demonstrated deductively that a logical bit exists not only in the form of the separable Boolean bits $|1\rangle$ and $|0\rangle$ but also can form coherent superpositions, the superposed bits or **superbits**

$$|S\pm\rangle = |0\rangle \pm |1\rangle.$$

Before theoretical physicists began to realise the full potential of coherent superposition for computation, the puzzling quantum bits had already been well identified in matrix logic, causing a major conceptual turmoil. Although physicists had known about coherent superpositions since the dawn of quantum

mechanics in 1920s, until matrix logic came along and explicitly revealed the fundamental role of superbits for a general theory of logic, no-one had the vision or insight to introduce the quantum bits into computer science. This surprising and unexpected result, initially quite confusing, showed that nonclassical coherent superpositions, emerging at the intermediary steps of computation, are the hidden but organic aspect of logic. **The matrix of classical logic conceals within itself a quantum matrix**, which provides for a significant additional boost in computer performance. The deductive and unintentional character of the discovery of superbits in matrix logic, as opposed to the postulation of quantum bits in quantum information, gives matrix logic the status of the theoretical foundation of quantum computing.

It has taken a long time from Boole's 0's and 1's to matrix logical superbits, but to a theoretical physicist this discovery should indeed not come as a surprise. Since Schrödinger introduced his celebrated cat, coherent superpositions have been at the heart of quantum mechanics. According to the superposition postulate a quantum system resides in a coherent mixture of all possible states, reduced to a single eigenstate in the act of measurement. If the information bits are quantum states, the concept of superposed bits may come out naturally. It seems quite strange that nobody noticed that for so long, not even John von Neumann with his expertise in both computer science and quantum mechanics. There are several important reasons why identification of the superbits has had to wait until the development of matrix logic. First of all, the main concern of physicists is with physical phenomena and not with logic. Secondly, the relentless quest of computer science for precision has distracted logicians and computer designers from the domain of quantum mechanics, which is fuzzy and imprecise. It is also easier to conjecture and postulate than to deduce superbits in an exact mathematical fashion. But it is this deductive character of the discovery of superbits in matrix logic that places it in the forefront of quantum computing. In matrix logic coherent superpositions are not optional but a mathematical consequence which passes various consistency tests. The concept of superbits has liberated computer theory from the rigid constraints of classical information and become the major promise of computer science for the 21st century. Quantum superpositions, which were considered as detrimental noise at the beginning of the computer era, have turned out to be its new promise. The quantum computer derives its new powers precisely from the processing of superposed bits, which makes it more efficient than a classical computer in such applications as intelligent computing, prime factorization and search in large databases. However, it is still poorly understood that the **quantum computer can be introduced in a purely deductive fashion, as a *physical answer* to some fundamental questions emerging in the field of *logic***. By exploring the bizarre properties of the quantum world, we could construct a computer with undreamt-of intelligence power. The advantage of quantum computers will not only be, as many believe at present, in some "killer applications", such as

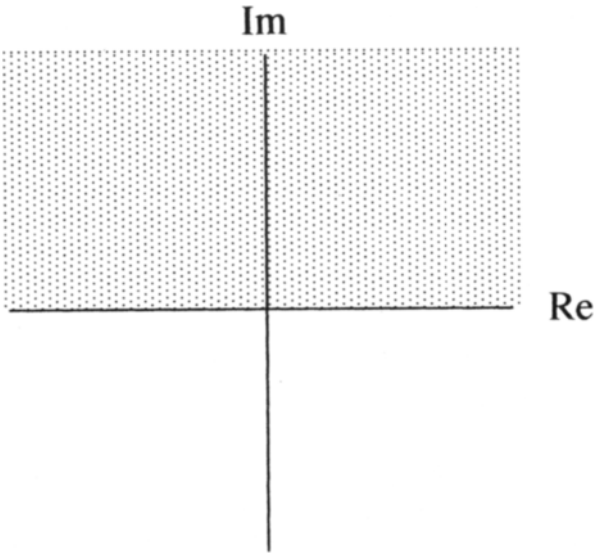
supersecure coding, speedup of computation or quantum parallelism but in a genuinely intelligent computer.

REALITY AS A COMPLEX NUMBER

The inclusion of information primitives in physical theory narrows the gap between the physical and logical components of reality. However, in each case we still have separate sets of equations: the field equations describing physical phenomena and the logical formulas describing thinking processes. There is no common solution. Although cognitive and physical 'worldlines' do cross in reality, they do not cross in the theory. To redeem this flaw and to establish a common solution, I propose here to describe complete reality as a complex number

$$z = \mathrm{Re}(z) + \mathrm{Im}(z),$$

where the real term represents physical phenomena, and the imaginary term consciousness·



This proposal is more than just an exercise in symbolics: the imaginary term represents the actual reality of consciousness. Our failure to connect the thought processes to physics and the difficulty we encounter in grasping the cognizing mechanism arise from the fact that the eigenvalues of the time operator are complex numbers. This should automatically put cognizing phenomena outside the framework of experimental physics if not outside physical theory altogether. What I want to stress, however, is that thought

processes do not lie outside physical theory, but outside the physical theory which we know. A more general theory must provide room for consciousness, which means that it must be able to accommodate the imaginary eigenvalues of the time operator, the complex term of reality. It is clearly seen in the complex plane that the conscious 'dimension' (**Im**) is orthogonal to the physical 'dimension' (**Re**):

$$\langle \text{conscious} | \text{physical} \rangle = 0.$$

The action of the time operator is equivalent to multiplying the complex plane by a complex unit which rotates the plane $\pi/2$, swapping cognitive and quantum observables, which perhaps can best be seen by considering the eigenequation for the operators of comparison:

$$\blacktriangle |q\rangle = \lambda |q\rangle.$$

Solving the characteristic polynomial we obtain the eigenvalues which are the imaginary units:

$$|\blacktriangle - \lambda \mathrm{I}| = \lambda^2 + 1 = 0,$$
$$\lambda^2 = -1,$$
$$\lambda_{1,2} = \pm i.$$

The application of the comparison operator to an eigenstate is equivalent to expanding the latter by the imaginary unit $\pm i$:

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = i\begin{pmatrix} 1 \\ i \end{pmatrix} = \begin{pmatrix} i \\ -1 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} i \\ 1 \end{pmatrix} = -i\begin{pmatrix} i \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ -i \end{pmatrix}.$$

which induces a $\pi/2$ rotation of a complex plane. Hence, the eigenstate and the product state are orthogonal:

$$(1\ i)\begin{pmatrix} i \\ -1 \end{pmatrix} = 0, \qquad (i\ 1)\begin{pmatrix} 1 \\ -i \end{pmatrix} = 0.$$

Significantly, the pair of eigenstates of the time operator is not orthogonal, as one would normally expect it to be, but self-orthogonal:

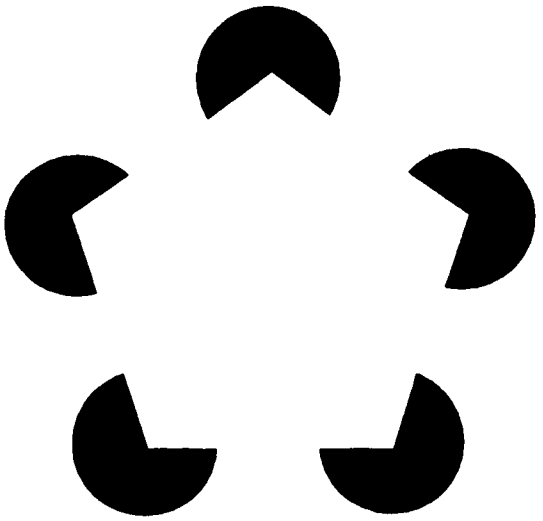
$$(1\ i)\begin{pmatrix} 1 \\ i \end{pmatrix} = 0.$$

The tensor product of the eigenstates is the spin operator σ, the complex square root of which gives the diagonalized time:

$$\sqrt[i]{\sigma} = \mathbf{TIME}_{\,\mathrm{diag}} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix}.$$

The illustration below may allow one to get some insight into how consciousness works, adding imaginary elements to reality. Consider the five black segments which are real objects which we can fully identify by real numbers:

If these segments are rotated and ordered differently, a frail but definite pentagon jumps out of the picture. The pentagon exists as an 'imaginary' quantity which is added in by your consciousness.



Where does the pentagon came from? Where does it exist? It is certainly not there, if we turn away from the picture or let it be analysed by a physical laboratory detector. By manipulating physical quantities we have created a pure cognitive quantity which can be described by complex numbers. This artistic example vividly demonstrates that the complex nature of consciousness is nevertheless rooted in and correlated with the code of physical objects.

Even though the focus of our discussion is on complex numbers, it must be emphasized that a more general model of thought can be obtained in terms of hypercomplex numbers: **quaternions** and **octonions**. The nonassociativity observed in cognitive operations leads to the conclusion that thoughts are octonions, the only nonassociative division algebra we know. The octonions, appearing in the thought process, could have a different physical realization. But if we confine an octonion to a set of electrons of a single atom, then the octet of electrons in the atom of oxygen is the unique choice one can make. What is interesting is that the self-dual nature of the intelligence code also leads to an oxygen atom as a suitable stable doubly even code [Ref 91, 93].

It must be stressed that the idea of giving imaginary values actual existence, typically denied to them in innate physics, originated in matrix

logic. The proposal that total reality is a complex number is equivalent to the proposal that it is a matrix, which has fundamental implications. In view of the correspondence

$$1 \sim \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad i \sim \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

the algebra of complex numbers can be represented by the (2×2) matrices of the special kind:

$$z \sim \begin{pmatrix} x & y \\ -y & x \end{pmatrix},$$

$$\det z = x^2 + y^2$$

which are equivalent to the complex numbers $z = x + iy$, where x and y are real. Likewise the hypercomplex algebra of quanternions is equivalent to (4×4) matrices of the form

$$Q = \begin{pmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \end{pmatrix},$$

$$\det Q = (x_1^2 + x_2^2 + x_3^2 + x_4^2)^2$$

Making use of matrices, one can design a physical algebra in which the imaginary values are not present explicitly. Most significantly, the unbroken topological phases $.H$ and $H^{\cdot}$, which are central to our analysis, are the Hadamard complex numbers in matrix form:

$$.H = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \qquad H^{\cdot} = \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}.$$

Hadamard matrices and codes have been extensively studied computationally, up to the orders exceeding 200. However, to my best knowledge, the fact that the prime Hadamard matrices are in fact complex numbers has not been realized in the mathematical literature explicitly.

Generally, a Hadamard matrix H_n is an (n×n) square matrix with entries +1 and -1, satisfying the condition

$$H_n \tilde{H}_n = \tilde{H}_n H_n = nI$$

where I is the (n×n) unit matrix, and n is 2 or multiple of 4.

The rows and columns of H_n are orthogonal, and multiplying any row or column of H_n by -1, as well as exchanging rows or columns, forms another Hadamard matrix of the same order. The existence of Hadamard matrices of a higher order is an important question. If there exists a Hadamard matrix H_n, there exists a Hadamard code, with $2n$ codewords of length n and minimum distance $n/2$, obtained by binarization of the rows of H_n and $-H_n$: the +1's are replaced by 0's and -1's by 1's. For the Hadamard complex number matrices H_2, representing the unbroken topological phase, this produces the underlying matrix logical code:

00, 01, 10, 11.

Logical time can be manipulated topologically, taking advantage of the bivalence of exponential expansion of the logical energy operators. Given the unbroken topological phase and its transpose, $.\mathbf{H}$ and $\mathbf{H}^{\cdot}$, satisfying the condition of suprasymmetry

$$.\mathbf{H} + \mathbf{H}^{\cdot} = .\mathbf{H} \bullet \mathbf{H}^{\cdot}$$

the two-term exponential expansions of the phases identify the two directions of time:

$$e_2^{-.\mathbf{H}} = \mathbf{I} - .\mathbf{H} = \blacktriangledown, \qquad e_2^{-\mathbf{H}^{\cdot}} = \mathbf{I} - \mathbf{H}^{\cdot} = \blacktriangle.$$

The three-terms expansion of the suprasymmetric phase is logical too, but gives a vacuum:

$$e_3^{-.\mathbf{H}} = \mathbf{I} - .\mathbf{H} + \frac{.\mathbf{H}^2}{2!} = \mathbf{0}, \qquad e_3^{-\mathbf{H}^{\cdot}} = \mathbf{I} - \mathbf{H}^{\cdot} + \frac{\mathbf{H}^{\cdot 2}}{2!} = \mathbf{0}.$$

In the general scheme of things the significance of the complex and the hypercomplex numbers is far from clear. The idea which we pursue in this study concerns the hitherto unrecognized fact that the physics of the logical brain and more generally the physics of life are quaternionic. Interestingly, attempts have also been made to construct quaternionic quantum mechanics [Ref 1]. In matrix logic the quartric identity

$$\blacktriangle^4 = \mathbf{I}$$

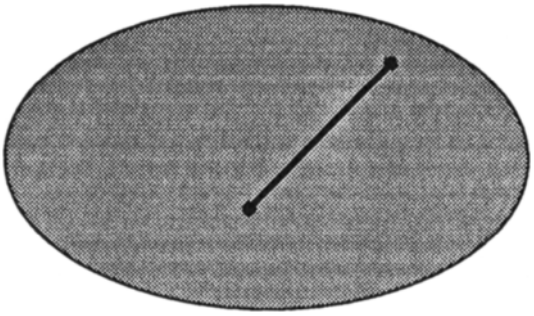
gives rise to 5-fold operator symmetry, which is the exclusive and essential symmetry of life. Living systems evolve (grow) in time, which is a complex matrix dimension. Although quaternions and octonions were discovered a long time ago, their role for a fundamental theory of the brain is only now coming into focus. There is a precedent. Physics made wide use of negative numbers, but it was not until Dirac unravelled the secrets of the *physical* vacuum that the negative number obtained actual physical reality. A similar process is now occurring with the complex numbers, whose actual reality we intend to derive from the properties of the *logical* vacuum.

NONCONVEX PROBABILITY

Brain science in its development followed in the footsteps of physics and mathematics. As neuroscience progresses, the tables seem to be turning: physics, the queen of sciences, may prepare itself to the bite bullet and start learning from brain science. To a great extent this reversal is due to the new understanding of the concept of probability, which is universally viewed as a convex. Traditionally information has been described in terms of probability distributions which are continuous and differentiable functions. Therefore to quantize information we have, counterintuitive as it might sound, to quantize probability, which in all accounts since Laplace, Borel and Kolmogorov and to this day has been considered to be an analytical function without singularities.

However, in the course of this study we will demonstrate that the notion of **probability can and should be extended to the domain of singular functions**, a conceptual shift with repercussions far beyond probability theory.

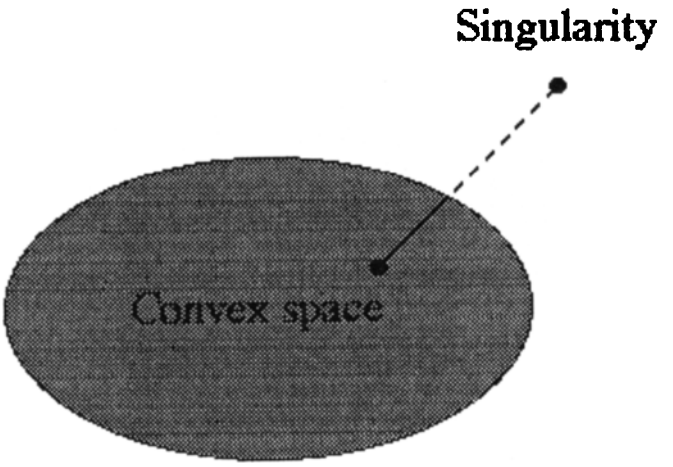
Modern physics and other scientific disciplines employ the concept of convex probability. However, when this concept is carried over into brain science, we are confronted with a major difficulty because matrix consciousness is nonconvex, and cannot be understood in terms of traditional probability. A topological space is defined as a convex if a straight line joining any two points on the space is contained entirely in the space:



Probability space is commonly viewed as a convex space in which we allow only homeomorphic continuous transformations. These can bend and stretch but not tear or break the space. The notion of convexity should be understood in a wide sense. Besides convex sets and functions in the traditional sense, convexity theory has contacts with many area of mathematics, and in this study we intend to connect it with logic.

In this study the classical concept of probability undergoes a reform. It turns from a descriptive quantity into an observable physical quantity. We argue that to achieve self-consciousness it is necessary to step out of the realm of convex probability into that of improbabilities. Improbability is a quantity whose components we define outside the unit interval of classical probability. These components can be negative and greater then unity, which make them uninterpretable in the framework of traditional probability theory. Although negative probabilities have been considered previously by Wigner and others [Ref 107], what is new in our approach, and what was not recognized in these mentioned works, is that we show that it is possible to treat probability as tensor. This tensor approach allows one to tackle coherent superposition classically and without invoking the complex probability amplitudes of wavefunction.

The thought process involves singular points which lie outside the classical convex space and which cannot be connected continuously with any point in the state space. Consequently, the state space of consciousness is nonconvex, and a quantum leap is required to reach a singularity point:



Even though a discrete jump is needed to reach a singularity point, we invite no quantum mechanics to assist in this transition. This is a new nonclassical quantization but without quantum mechanics.

The physics of thoughts is a peculiar one. We all may agree that the equations of this new physics cannot be expressed in such units as centimetres and grams or ergs, etc., all those units to which ordinary physics refers. If we chose to do so, we would simply come down to the biophysics of the brain, not the physics of thinking as such. The latter is information physics or infophysics which refers to logic, which in turn refers to truth-values, bits and probabilities. The units of measurement which we put into the foundation of infophysics are probabilities which were never before thought of as actual physical units but merely as mathematical objects. It is very different thing to say, for example, that we have probability 1/3 or we have 1/3 probabilities. This is a major departure from the abstract classical understanding of probability, and one of the goals of this study is to demonstrate that consciousness violates the laws of convex probability and is therefore nonlocal. Just as quantum mechanics violates Bell's inequality and some of its predictions are not compatible with a realistic local theory, logical consciousness violates the law of classical probability and some of its actual inferences are not probabilities but improbabilities. Although one expects a normed probability to emerge as a composition of normed probabilities, in logic it can also emerge as a composition of improbabilities. There are topologies for which the probability normalization is violated and logical values are greater than unity and negative. An ordinary probability state is created making use of neither probability nor complex amplitudes, but improbabilities. Since convex probability is local, while improbability is not, there are strong indications that a general cognitive composition is nonlocal.

Consider the implication tautology

$$x \rightarrow x \equiv 1.$$

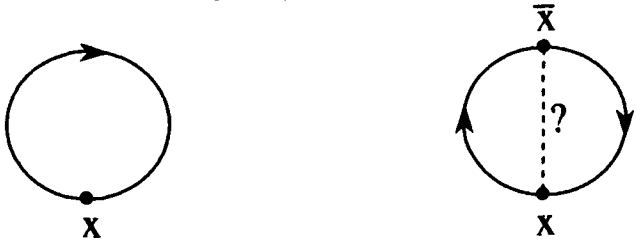
In accordance with the matrix principle, it is always possible to break it down and to insert an intermediary variable which transforms (factorizes) the tautology into the inference syllogism

$$(x \rightarrow y) \wedge (y \rightarrow x) \equiv x \rightarrow x$$

The laws of classical probability work here adequately. But if we choose $y = \bar{x}$, the tautology will change into the contradiction

$$(x \rightarrow \bar{x}) \wedge (\bar{x} \rightarrow x) \equiv 0$$

giving rise to the Liar paradox where the otherwise natural $(x \rightarrow x)$ does not work. The linear structure of the syllogism closes into a loop in which an unknown interaction, mediating the process, must be introduced:



Our goal is to identify this unknown interaction in terms of the formalism of cognitive diagrams, the logical analogue of Feynman diagrams. In doing so we intend to show that the consciousness mechanism connects momentarily matrix logical states to a vacuum. Self-reference polarizes the vacuum. The Liar paradox is a vacuum polarization in which a system is out of phase with itself. It apparently requires a significant modification of logical theory which should include nonlocal mechanisms. In view of the Einstein, Podolsky and Rosen *gedanken* experiment and Bell's theorem many have argued that quantum mechanics does not provide a complete description of reality. But logic is no less weird then quantum mechanics. In each and every mental riddle, the Liar paradox is hidden.

LOGICAL QUANTUM NUMBERS

The energy spectrum of a free quantum particle is continuous, but when a particle is confined to an atomic structure, the energy will assume only discrete values corresponding to eigenstates of the system. The electronic orbits in the atom are characterized by the principal quantum number n, the orbital quantum number l, and the magnetic quantum number m which are the integers satisfying the following inequalities:

$$n = 1, 2, \ldots, \qquad l = 0, \ldots, n-1, \qquad -l \leq m \leq l.$$

To each combination of these numbers n, l, and m there corresponds a particular atomic orbit, with a particular wavefunction describing the configuration of the electronic field of the atom.

Given the fact that the thinking brain is quantum-mechanical at bottom, it is possible in principle to characterize each ongoing state of the brain by a certain number which is a set of quantum numbers of its constituent atoms. More meaningfully we can identify the logical states of the brain with the *logical quantum numbers* $\lambda = -1, 0, 1, 2$, a fundamental new concept which parallels the concept of quantum numbers in atomic physics [Ref 89]. The logical quantum numbers were determined as eigenvalues of the corresponding logical operators and put in correspondence to the atomic quantum numbers n, l and m. The key logical numbers are the synergy or spinergy $S = 0, 2$, the Boolean amplitude $A = 0, 1$, and the logical phase number $P = -1, 0, 1$. The relation between these numbers reproduces the relation between the atomic quantum numbers:

$$S = 0, 2, \qquad A = 0, S-1, \qquad -A \leq P \leq A,$$

where the synergy corresponds to the principal quantum number, the Boolean amplitude to the orbital number, and the logical phase number to the magnetic quantum number. The logical amplitudes **0** and **1** naturally correspond to a pair of antiparallel and parallel spins-1/2:

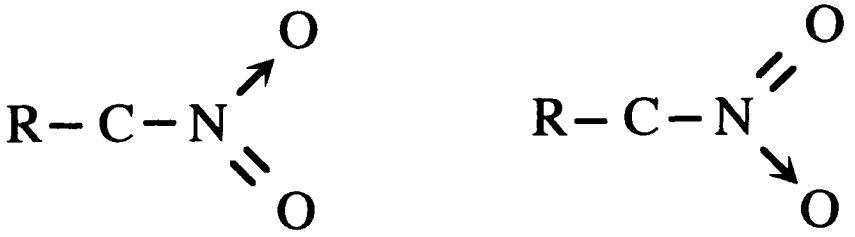
$A=0$ $\qquad\qquad$ $A = 1$

In addition to discrete amplitudes cognitive logic contains continuous spectra which are embedded between the two discrete eigenvalues **0** and **1**.

Coordinate bond

Suppose we accept the factoid that the thought processes reflect the quantum-mechanical nature of the atoms the brain is made of. Since the synergy number S jumps from **0** to **2**, missing out the level $S = 1$, how can we account for the absence of one synergy level? An attempt to explain the missing synergy level $S = 1$ in the framework of molecular neurochemistry leads one to attach special importance to the covalent coordinate chemical bond. In the logical brain the essence of chemical bonding appears to be expressed even more clearly then in quantum chemistry itself. The coordinate

bond between atoms involves a ***donor*** atom with a saturated outer orbit, whose pair of electrons do not take part in the formation of a covalent bond, and an ***acceptor*** atom which has a vacant orbit without electrons. When a molecule is formed these two atomic orbits correspond to the two molecular orbits. Two electrons impregnate the lower binding orbit, forming a stable bond between the atoms. Coordinate bonds manifest for nitrogen and phosphorus atoms. A nitrogen atomcan form a covalent bond with a carbon atom, a double covalent bond with an oxygen atom and a coordinate bond with another oxygen atom. The latter, having one empty $2p$-orbit, acts as a donor. The two resonance nitrogroups, where the coordinate nitrogen-oxygen bond is denoted by the arrow, are shown below:



For a chemical bond to be useful logically it must be integrated into the molecular system of the brain by a (bio)logically plausible mechanism. Biological supramolecules, like DNA and proteins, are information molecules and have the capacity to assemble themselves into a structure. The ***on-off*** switching of a coordinate bond can give such structures the two synergy levels, the empty and the doublet, required for covariant logical inferences. Lack of the bond leaves no valence electrons on the atom, which is energy dislocation or the exciton. When the contact between the atoms emerges or dissolves the exciton moves back and forth, concomitantly with the bond being formed or broken. We have a quantum switch of two electrons which can be plugged in or pulled out. In the living brain there is a certain rate at which coordinate bonds form and decay. The action potentials and subvoluntary neurocurrents alter the rate towards the extension of the lifetime of coherent states, as is required for cognitive thinking.

While the Boolean states $|1>$ and $|0>$ are stable singlet monostates, the superposed logical states $|S_+>$ and $|S_-> $ are metastable and must be reinforced. Although coherent superpositions spontaneously emerge in the brain matter, the role of cognitive logic is to prolong the lifetime of the metastable states to macroscopically significant time intervals. In conditions of oscillatory behaviour, which is commonly found in the brain, the coordinate bond provides a cyclic attractor needed for achieving synchronized and coherent cognitive actions. Changeover between $S = 0$ and $S = 2$ will alternate the system between two orthogonal coherent superposition states. The oscillatory neurocurrents (action potentials) in the range between **50** mV and **100** mV

induce the formation or dissolution of the coordinate bonds, inserting or pulling out the pairs of coordinate electrons. In this scheme the coordinate bond emerges as a promising candidate for a natural element in which the physics of complexity - the physics of consciousness and logic - meets congruently the simplicity of basic quantum chemistry.

Consequences for logic

The eigenvalue problem is as fundamental for logic as it is for physics. Classical treatment logic is at fault in ignoring the operator nature of logical connectives, which ultimately leads to the identification of the latter as logical quantum numbers. The logical quantum numbers **0** and **1** are naturally connected with the Boolean false 0 and true 1, but the puzzling logical quantum number $S = 2$ that appeared in certain matrix logical expressions was initially very confusing [Ref 89] but soon found a meaningful and important interpretation. Classical logic operates with the binary bits 0 and 1, and obeys the involutive rule for logical addition:

$$1 + 1 = 1,$$

which prevents other values, beside 0 and 1, from appearing. Adding truth to a truth produces no observable effect:

OR	0	1
0	0	1
1	1	1

However, general logical discourse must often take account of fuzzy truths, in which the contributions of identical elements cannot be disregarded but accumulate. How to take the discourse outside the domain of classical logic to the more general domain of intelligent additions? Here the logical quantum number $S = 2$ comes into play. By exploring it we can construct the nonbinary truth-tables which accommodate the fuzzy additions for the on-diagonal summators and the off-diagonal counters:

Summators:

$$<p|+|q> = <p\begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}q> = p + q$$

AND + OR	q	$\bar{q}$
$\bar{p}$	0	1
p	1	2

$$<p|\updownarrow|q> = <p\begin{pmatrix}2 & 1\\ 1 & 0\end{pmatrix}q> = \bar{p} + \bar{q}$$

NOR + NAND	q	$\bar{q}$
$\bar{p}$	2	1
p	1	0

Counters:

$$<p|\rightarrow^2|q> = <p\begin{pmatrix}1 & 2\\ 0 & 1\end{pmatrix}q> = \bar{p} + q$$

IMPLY + OFF	q	$\bar{q}$
$\bar{p}$	1	2
p	0	1

$$<p|\leftarrow^2|q> = <p\begin{pmatrix}1 & 0\\ 2 & 1\end{pmatrix}q> = p + \bar{q}$$

IF + ON	q	$\bar{q}$
$\bar{p}$	1	0
p	2	1

Weakening the strictly binary alphabet into a continuum allows one to employ new logical operators, increasing the expressive power of logic. The introduction of nonbinary logical numbers enlarges the volume of admissible logical operations. A natural trade-off between fuzzy logical values and the operator power of logic provides new clues to understanding the creative intellect which tolerates ambiguity and imprecision. The less Boolean the logic, the more logical operations, previously forbidden, become admissible.

DOES THE MENTAL VIOLATE THE LAWS OF PHYSICS?

After many upheavals, many breakthroughs and paradigm shifts physical theory is once again in turmoil. This time around the problem is not found in physics itself but is caused by an attempt to include consciousness in a general scheme of covariant physical law. Mental phenomena are the product of the brain, which is a physical system and as such must be tied to the laws of physics. But there is nothing in known physical theory which could dictate the

mind. Physics does not '*think*'. We are led astray in trying to apply the ordinary concepts of time, space and energy to consciousness. *Free will* and the Hamilton least action principle cannot be reconciled. Voluntary action and physical entropy inevitably run into conflict. Because the Heisenberg uncertainty principle allows for local violation of causality, one might hope that there is room to accommodate consciousness quantum-mechanically. However, the singularities one observes in cognitive logic indicate that the thought process will not agree with the linearity of the Schrödinger equation. The laws of unitary quantum evolution break down for consciousness.

Quantum mechanics has struck a powerful chord in the human psyche. Almost a century after its birth this wonderchild, parented by Bohr, Planck and Einstein, remains an intellectual powerhouse. But the end of the dominant position of the quantum paradigm may be in sight: the main reason for that is human consciousness. It is delivering a blow to quantum theory which is even greater that the EPR paradox and Bell's theorem. Searching for the answer to the question whether the mental violates the laws of physics we must clearly distinguish two different approaches, one seeking a solution within the existing laws of physics and another outside it. There is no shred of experimental evidence that consciousness violates the laws of physics. However, there is no evidence that the existing physics can support the phenomenon of consciousness either. This leads us to a search for a third alternative solution. In order to accommodate the fund*amental* mental level we requires a new physics or infophysics [Ref 91, 92] whose equations must explicitly include information primitives. Infophysics is built up using a matrix logic which allows for a straightforward connection with the matricial formalisation of quantum physics. Such a unified theory depends on the experimental confirmation of both the logical properties of matter and the quantum nature of the brain. In the presence of such a unified theory the information may be codified at the atomic level. Information must enter physics at a fundamental level. A proper theoretical framework for infophysics has emerged only recently, and as is usual with new ideas, it faces many obstacles, one of which is high specialization. Physicists typically dislike the poorly defined problems of bioscience, while neuroscientists often lack proper insight into physics and information theory. Computer scientists excel with programming and formal languages but are not at ease with the fuzzy language of the intuitive brain. Logicians neither predict nor lead us out of the quagmire of the paradox, but follow, merely interpreting the experimental facts.

Even though consciousness cannot be confined to the framework of ordinary physics, the idea that the cognizing effect is the result of a violation of a law of physics may sound outrageous. Not so if one draws a line between the laws of physics and the laws of Nature. Violations of the laws of Nature have never been and never could be observed. In physics, however, when new facts are discovered, the existing theory expands, improving the laws of the previous theory. Consciousness does not violate the law of Nature; it simply expands it. The mind does not violate the laws of known physics because it is

simply not part of them, and one of the goals of this study is to join both into a general theory. Irrespective of the success of this project, any scientific solution to consciousness must be expressed mathematically. **Mathematics is a court from which there is no appeal.** It cuts short lengthy pleadings to intuition that are so typical of philosophical analysis. Mathematics deals with symbols. A symbol is understood in terms of other symbols to which the symbol refers. The purity of this relation begins to break down dramatically when a symbol refers to itself. The paradoxes of self-reference introduce uncertainty resembling the uncertainty of quantum theory, and motivates us to treat logical symbols as observables represented by operators. Following this line of reasoning we will replace the ordinary logical scalars ***true*** and ***false*** by the singular operators **TRUE** and **FALSE** which then become subject to the application of various operations of operator algebra, most importantly to the coherent compositions

$$\mathbf{FALSE} \pm \mathbf{TRUE}.$$

We then show that these can be portrayed as quantum field commutator and anticommutator respectively:

$$\mathbf{FALSE} - \mathbf{TRUE} = [\mathbf{FALSE}\ ,\ \mathbf{TRUE}\ ]$$
$$\mathbf{FALSE} + \mathbf{TRUE} = \{\mathbf{FALSE}\ ,\ \mathbf{TRUE}\}$$

The computational use of quantum field theory allows us to naturally reformulate it as a logical calculus and vice versa [Ref 91, 44]. Just as the orthogonal Boolean bits 0 and 1 form the basis of classical logical thinking, the coherent superpositions form the basis of general cognitive thought based on quantized bits. What is significant for us is that **TRUE** and **FALSE** and their symmetric and antisymmetric field products are the funda*mental* knowledge operators, which self-compute the corresponding eigenstates while acting on an arbitrary logical state. When we calculate the eigenvalues of **TRUE** and **FALSE** we find out that these are the classical bits ***true*** and ***false***, which is consistent with classical logical theory and gives no reason for concern. However, when the respective eigenstates are calculated, the semiclassical nature - half-classical, half-quantum - of the operators **TRUE** and **FALSE** unexpectedly reveals itself. The operators are 'torn apart' between the classical and quantum realms: while one of the eigenstates is Boolean, the other is a coherent superposition. Since these basic logical operators are non-Hermitian, a general thought process cannot belong to the family of quantum observables: **cognitive thought is a non-Hermitian quantity**. Accordingly, infophysics adds a new non-Hermitian physics to the existing Hermitian physics.

This is a radical idea, without competing counterparts, even more so, because a profound conceptual chain reaction follows once one accepts this proposal. The Hermiticity of an operator is in fact a condition of symmetry, and symmetry is the fundamental concept of physics, determining the laws of conservation. Since Noether proved her celebrated theorem, we have known that the laws of symmetry have priority. For example, a quantum system whose decay may be allowed by energy considerations will be prevented from

decay if, say, the parity symmetry will be broken as the result. Since the basic logical operators **TRUE** and **FALSE** are non-Hermitian, a breakdown of some symmetry law will occur in the thought process. But this in turn must cause the violation of some conservation law. Willingly or unwillingly we come to the unsettling conclusion that non-Hermitian consciousness should disobey some law of conservation. Since the French Academy stopped considering proposals for *perpetuum mobile*, the belief in conservation laws in physics has been unshaken. Without the laws of conservation the world simply would not make sense. A physicist instinctively objects to violations of the conservation laws. When parity violation in weak interaction or matter loss in black holes occurs, we immediately look for a larger system, a combined symmetry and quasars, respectively, where such violations cancel out. For consciousness such a larger system does not exist. In quantum field theory, when parity is violated, we can restore the balance by considering antiparticles moving backwards in time. In logic, if parity breaks down, as it does for consciousness, there are no antiparticles to re-establish the symmetry. Antiparticles are particles moving backwards in time. But thought is meaningless if runs backward.

In modern physical theory there are two fundamentally different kinds of quantities, the observable quantities, represented by Hermitian operators:

$$\langle \Im\psi|\varphi\rangle = \langle\psi|\Im\varphi\rangle\,, \qquad \Im = \Im^{+},$$

and unobservable ones, represented by non-Hermitian operators:

$$\langle \aleph\psi|\varphi\rangle \neq \langle\psi|\aleph\varphi\rangle\,, \qquad \aleph \neq \aleph^{+}.$$

The Hermiticity of operators is essential because in the laboratory we express all the results of a physical measurement in real numbers. The eigenvalues of a Hermitian operator are always real, which prevents the results of physical measurement from being recorded in the form of the 'fictitious' complex numbers. However, it is somehow overlooked that the eigenvalues of a non-Hermitian nonsymmetric operator are not necessarily complex and can also be real. Under greater scrutiny, we can also refine and 'improve' the notion of a Hermitian quantity. In quantum physics there are in fact two kinds of non-Hermitian quantities, ones which are **absolutely** non-Hermitian, like the creation and annihilation operators, a^* and a, and other quantities which are **conditionally** non-Hermitian. For example, the momentum of a quantum particle is a physical observable represented by a Hermitian operator which we can measure, but because it does not commute with the position operator, it cannot be fully known simultaneously with the coordinate. Then we can freely choose a partially non-Hermitian representation for the momentum, accounting for the fact that in an experimental set-up in which the coordinate is also the subject of observation, there is an uncertainty region where the momentum is a conditionally unobservable quantity. In general, for a physical quantity which cannot be measured simultaneously with a Hamiltonian we can always choose some non-Hermitian representation. While laboratory devices detect Hermitian quantities, providing no reading on non-Hermitian ones, for consciousness this

situation is turned upside-down as Hermitian and non-Hermitian elements swap roles [Ref 93]. This offers a plausible explanation why our brain is capable of 'measuring' thoughts but not the Hermitian physical quantities like energy or momentum. In reverse, physical devices 'recognize' a Hermitian operator, but are 'blind' to a cognitive thought which is non-Hermitian.

Another debilitating factor for understanding the physics of consciousness is infinite regress. SELF which attempts a model of itSELF must construct a model of SELF, attempting a model of itSELF, which contains a model of SELF, constructing a model of itSELF, and so on to infinity. We do think, in spite of the fact that infinite recursion should make thinking impossible. It appears that in virtue of infinite regress one should not be able to cognize a symbol or oneself in finite time. This makes thinking and consciousness not possible, which is, as we know, is not the case. For the finite brain infinite recursion is not a plausible option, which forces us to make use of approximations. Is this approximation unavoidable? Can we escape the infinities and define a logical action exactly by finite means? Can the brain short-cut without approximations? It could, if for example, an infinite series halted by itself, intrinsically arresting the infinite regress. Such finitely self-terminating series are not known in classical logic but were discovered in matrix logic. Satisfying the conversion principle, a quantum field theory with the operators of creation and annihilation finitely collapses to the logical implication calculus.

In matrix logic any function whose arguments are logical operators or their isomery, is or approximates to being a logical operator or isomery. In order to keep the matrix elements of operators *logical* or binary in the series expansions, we retain the first two linear terms of the expansion

$$e^{-L} \approx I - L,$$

and disregard the remaining terms contributing undesirable fractions. Although we will make use of the exact equality sign, such two-terms approximation is what the matrix exponential usually stands for in this study. But one must keep in mind that generally the series are the infinite sums or the operator waves:

$$e^{L} = I + L + \frac{L^2}{2!} + \frac{L^3}{3!} + \ldots + \frac{L^n}{n!} + \ldots$$

A fundamental exception from the general rule occurs when the argument of an operator wave is a logical vacuum, hence nilpotent:

$$L^n = 0.$$

This is the case with the exponentials of the creation a^* and annihilation a operators, reduced to the two finite linear terms and the vacuum term, contributing nothing to the power series expansion:

$$\exp(\text{field operator}) = \text{finite terms} + \text{vacuum}.$$

The computational use of quantum field theory reveals that series expansion of the creation and annihilation operators is finite, halting at the second term, as all further powers cancel themselves out. What is of fundamental significance

is that the resulting equalities are **exact** equalities, comprising two terms only, which are the transitive implications:

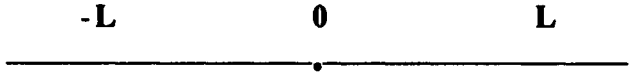
$$e^{a} = I - a = \text{IMPLY}, \qquad e^{a^*} = I - a^* = \text{IF}.$$

Logic is tied up to physical degrees of freedom, and therefore to the laws of physics. As was pointed out by Wigner, everything that happens in a finite volume of spacetime must be exactly analyzable with a finite number of logical operations. The present theory of physics is not like that, and neither will be the corresponding theory of consciousness. This we cannot allow. **We can think about infinity but we cannot think with infinities.** One must prevent space from going down into infinitesimal distances, wavelength to be infinitely great or terms to be summed in infinite order. Furthermore, there is Turing's computational unsolvability of the halting problem. It is not possible to write a program which will tell for all programs whether a program will or will not halt. But if the programs are the universe of all actual matrix physical waves, simply the Universe, then matrix-logical consciousness can "solve" the halting problem intrinsically. Any "program" whose argument is a nilpotent Lie algebra will self-terminate, and we are forced to consider an alternative to the Turing-machine model of computation.

The two-term expansion of a field operator is **exactly** a logical operator. However, excluding nilpotent operators, two-term expansion is an approximation and as always one must be careful with approximations. For the synergy operators this first approximation gives the signed unitary logical evolution operator of negation:

$$e_2^{-H} = I - H = \text{NOT}, \qquad e_2^{-1} = I - 1 = -\text{NOT}$$

where **H** and **1** are logical **Lagrangian** and **Hamiltonian** respectively. The corresponding matrix degree of freedom retains parity:



A breakdown of the parity symmetry would entail singularization of the matrix dimension which loses one-half as visualized below:

The series expansion of the synergy operators is distinguished by being integer logical in the second approximation too, now uniformly yielding the unitary identity:

$$e_3^{-H} = I - H + \frac{H^2}{2!} = \text{YES}, \qquad e_3^{-1} = I - 1 + \frac{1^2}{2!} = \text{YES}.$$

Depending on the degree of approximation a logical system can affirm or negate. In the two-term approximation, the parity of the matrix dimension,

reflected by the opposite signs of NOT, cancels out in the second three-term series, indicating information singularity. Independently of a particular form of the synergy operator the three-term exponentiation of tautology is invariably the identity:

$$e^{-\text{TAUTOLOGY}} = \text{IDENTITY}.$$

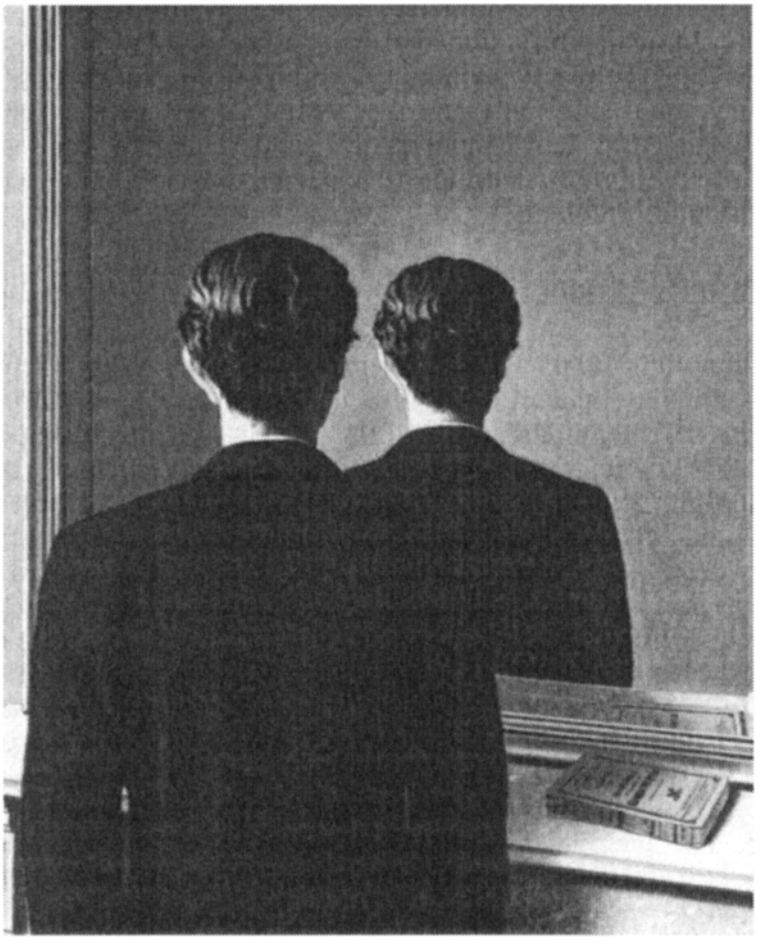
In conditions of singularity the energy operator acts equivalently to matrix vacuum:

$$e^{-0} = \text{YES},$$

This is consistent with the conversion postulate and the matrix principle where the annihilation and creation exponents of implications are exactly the vacuum powers:

$$\rightarrow^{a} = \rightarrow^{0}, \qquad \leftarrow^{a^*} = \leftarrow^{0}.$$

Taking stock of these results, one arrives at a hypothesis that cognitive degrees of freedom are the degrees of freedom of vacuum. Can matrix-logical consciousness harness the non-Hermitian properties of the vacuum through the detection of the ground state oscillations? The vacuum forces are not just virtual, in fact they have been recently detected and measured. When a pair of conducting surfaces bound the vacuum, only modes with wavelengths shorter than the distance between the surfaces can appear, and the exclusion of the longer wavelengths results in a tiny geometry-dependent force between the conductors. Future experimental study of the structure of the physical vacuum may provide new clues to consciousness. The theoretical analysis has led us to conclude that consciousness is an information vacuum singularity violating parity symmetry. There is a fundamental science of space, geometry, and there is a fundamental science of matter, group theory. In this study we suggest that there is also a fundamental science of consciousness, topology. Consciousness is nonorientable and in this respect it 'violates' the laws of 'orientable' physics. In actual three-dimensional space 'orientable' means 'bilateral'. In a laboratory a topological surface has two sides and information is collected by orientable 'bilateral' measuring devices which have the input and output 'sides'. The left and the right vector products are symmetrically defined, and a mirror functions properly, swapping the left and the right. But when we consider consciousness, we have to imagine a world from which the mirror symmetry had been banned. Somehow it must perform rotations that appear impossible to our geometrical brain. Consciousness is a singularity, which can meaningfully be treated as a topological 'defect' with one side, in which mirror does not revert, and self-measurement becomes possible. The backreaction for a system with a single side, which has no other 'side' to go to, is routed back to the system, in this manner inducing the effect of self-awareness. The non-Hermitian universe of consciousness is very much like the bizarre mirror in René Magritte's *Reproduction Forbidden*, although it is not what the painter had in mind.



René Magritte. *Reproduction Forbidden*, 1937, oil on panel.

Parity symmetry is responsible for the symmetry of the left and right attributes of matter. Prior to the discovery of the chirality of the neutrino field, we saw no reason for the nonequivalence of the left and right. That is in spite of the well-known fact of chirality in living systems. While in the innate physics there is a balance between left and right molecular isomers and

crystals, biological molecules always inexplicably curl left. There are possible projections to the macrolevel: the majority of us are right-handed. There are the left and the right brains with asymmetric functions. Living systems consistently violate parity symmetry, reaching its ultimate degree in the thinking brain. In quantum field theory the conservation law is recovered with the combined charge-parity symmetry. But consciousness fundamentally appears to have one 'side' only and is essentially asymmetric. While a fully symmetric balanced system cannot evolve, and sooner or later falls into a steady state of equilibrium, consciousness is endlessly in motion, in a state of permanent disequilibrium.

Towards a new physics

The quantum relativistic principles have a distinctly information character. Within the limits of the uncertainty principle the laws of physics can be violated locally. When the quantum field reaches the macroscopic level we consider these violations to be lost due to decoherence of pure states into a statistical mixture. But in complex information systems local microphysical violations might not in fact be lost but accumulate. Then a single quantum theory can give rise to many valid logical systems, including those which reject the theory itself. **Logic is larger than quantum mechanics.** It is possible to express logic through quantum operations. The basis sets {**NOT**, **ON**} or {**NOT**, **OFF**} can be extended as commutators or anticommutators. But **YES** or **IF** exist only as a anticommutator, while **AND** and **OR** cannot be expressed quantum-mechanically at all. After a hundred exceptionally successful years of quantum physics we have become indoctrinated to believing in the limitless power of the wavefunction. **But there is a life after quantum mechanics.** The phenomenon of the thinking brain forces us to consider another fundamental paradigm, one which is neither classical nor quantum [90]. To explain consciousness infophysics must look for a new framework beyond existing physical theory. We conclude that such a new framework is provided by topology. There are different form of energy and information: classical and quantum, physical and biological. There are is also **topological** energy and information which appears not to be constrained by finite speed of propagation of interactions. Topological properties are 'tachyonic' and could propagate instantly. This might be clear from the following gedanken experiment, close in spirit to the EPR quantum paradox. Consider a two-dimensional strip universe with both ends extended to an absolutely remote area. If someone at infinity twisted and glued the ends of the strip the entire universe would **instantly** change from orientable to nonorientable. In these topological phase transitions I see the seeds of a new physical theory which should provide the basis for consciousness. In laboratory physics we understand by taking things apart; to understand the brain we must put things together.

THE BRAIN IS GEOMETRICAL THE MIND IS TOPOLOGICAL

Physical law should have mathematical beauty.

Paul Dirac

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TOPOLOGICAL CHARGES

Particles and fields are the solutions to the fundamental equations of physics. Thoughts are the solutions to the fundamental equations of logic. The existence of the thinking brain 'proves' that **common** solutions do exist, and our underlying hypothesis is that these solutions are **topological**. In accordance with this idea, **the key goal of this work is to provide a topological solution to consciousness**. The language of topology is the new language for brain science, as well as for physics. Since Einstein the majority of physicists believe that physical forces can be explained using pure geometry, if necessary, the geometry of higher dimensions. The main thrust of this study is to show that as much as geometry is essential for understanding the physical universe, topology is essential for understanding the phenomenon of consciousness [Ref 78]. The development of geometry preceded the development of topology, and due to historical reasons and education our concept of the world, including the brain, was and continues to be primarily geometrical. However, looking at a moving amoeba or considering the liquid flexibility of a developing embryo, one gets a strong feeling that for living matter and for biology in general the concepts of geometry are not enough. Bioscience clearly lacks insight into some unknown laws of topology which can explain a wide range of life phenomena.

Geometry is concerned with the properties of figures in space and with the properties of space itself. Originally, it started as a practical subject in ancient Egypt and Babylonia, used in surveying and building. The ancient Greeks realized that the properties of geometrical figures could be deduced logically from other properties. Pythagorus proved his celebrated theorem, and around 300 BC Euclid produced one of the most famous texts in the whole of mathematics, *Elements*. Despite difficulties with the fifth postulate, the Euclidean geometry of *Elements* survived as an unquestioned canon until the non-Euclidean geometries were discovered. Prior to that, in 1637, Descartes in his *La Géométrie* introduced into mathematics the fundamental principles and techniques of coordinate geometry, where points could be represented by numbers and lines by equations. This gave mathematicians a new analytical tool with which to attack geometric problems algebraically. The 19th century saw major advances in geometry. Cayley developed the algebraic geometry of **n**-dimensional spaces and Lobachevsky non-Euclidian geometry. Finally, in 1854, Riemann put forward a view of geometry as the study of any kind of space of any number of dimensions. The important development of the past two decades has been noncommutative geometry, which extends the geometry to the context of noncommutative rings.

A notion of invariant distance is essential for geometry. A set of points is a metric space if there is a metric ρ which gives to any pair of points x, y a nonnegative number $\rho(x, y)$, their distance or separation, and is such that

$$(1)\ \rho(x, y) = 0 \quad \text{iff} \quad x = y,$$

$$(2)\ \rho(x, y) = \rho(y, x)$$

$$(3)\ \rho(x, y) + \rho(x, z) \geq \rho(x, z)$$

Choosing a pseudo-Euclidian signature $(+ + + -)$ for a metric we obtain the geometrical model for special relativity, known as Minkowski space. Relativity theory rejects the idea of space and time as separate entities, and is extended in four dimensions compared with the three dimensions of ordinary space. Space and time are joined into a single concept of spacetime in order to describe the geometry of the universe, effectively reducing the study of physics to the study of geometry. In special relativity spacetime is flat, much as space is flat in Euclidian geometry. General relativity is concerned with gravitational effects of matter, which cause spacetime to curve: massive objects produce distortions and ripples in local spacetime, and the motion of bodies are then dictated by the curvature. The curved spacetime is described by means of Riemann geometry.

A description of spacetime in terms of Minkowski and Riemann geometries and the fundamental link between geometry and physical laws in general gained greater clarity after Noether in 1917 proved a theorem showing that the conservation laws of physics are in fact consequences of more fundamental laws of symmetries. The conservation of energy and momentum follow from the symmetry (isotropy) of time and space. The conservation of electrical charge follows from a symmetry of a particle's wavefunction. In general, we say that particles such as the electron and proton carry Noether charges, the attributes that are maintained because of geometrical symmetries. **But the attributes and properties of objects may also stay invariant under topological deformations.** The corresponding conservation laws are topological as opposed to conservation due to geometry. Unlike the geometer, who is typically concerned with questions of congruence or similarity, the topologist is not at all concerned with distances, shapes and angles, and will for example regard (this has nothing to do with the decline of moral values) a wedding ring and a tea cup as equivalent, since either can be continuously deformed into the other. A set, together with sufficient extra structure to make sense of the notion of continuity, is called a topological space. More formally, a set X is a topological space if a collection T of subsets of X is specified, satisfying the following axioms:

(1) the empty set and X itself belong to T

$$\varnothing \in T \quad \text{and} \quad X \in T$$

(2) the intersection of two sets in T is again in T

$$X \in T,\ Y \in T \quad \Rightarrow \quad X \cap Y \in T$$

(3) the union of any collection of sets in T is again in T.

$$X \in T,\ Y \in T \quad \Rightarrow \quad X \cup Y \in T$$

The sets in T are called open sets and T is referred to as a topology on X. For example, the real line R_1 becomes a topological space if we take as open sets those subsets U for which, given any $x \in U$ there exists $\varepsilon > 0$ such that $\{ y \in R_1 : |x-y| < \varepsilon \}$ is contained in U. A similar definition is valid in a metric space, but it is not in general required of every topological space that it should be metric.

A well-known correspondence exists between algebraic geometry and physical objects. A space gives rise to a function algebra; a vector bundle over the space corresponds to a projective module over this algebra; cohomology can be read off as the de Rham complex; and so on. We will take steps to establish a different type of correspondence, the correspondence between the elements of logic and the elements of topology. Since our main objective is to show that the laws of topology hold the key to the laws of the thinking brain and that the information physics of the consciousness is rooted in topology, the focus of this study is not the *neurological brain*. What we want is to understand the *topological brain* and its intelligence-supporting logic. Many attempts to explain the cognizing phenomenon and to understAND consciousness neurophysically lead to a dead end. No knowledge about the neural or biophysical processes in the brain can satisfactorily answer the *hard* question: what is the actual mechanism of consciousness? Those who try to answer this fundamental question in the mechanistic framework of the interaction of neurons, the brain's electricity, neurochemistry or quantum mechanics are often as unproductive as those who offer purely philosophical, spiritualistic or theological explanations only. Somehow human thought, even though connected to processes in the brain matter, seems to be intractable, almost immaterial. Abstractions, on the other hand, often have great physical power. Words and thoughts alone can induce measurable changes in the brain, can alter the state of consciousness, with the amplified effects being visible to the naked eye in the state of hypnosis, for example.

As we mentioned earlier the laws of conservation in physics are consequences of corresponding symmetries: the conservation of energy follows from the symmetry of time, the conservation of momentum is due to the isotropy of space. These attributes and others like mass or the charges of elementary particles are conserved due to geometric properties, and can be defined as metric charges. **Mental or logical attributes are maintained not as geometric but as topological objects.** It may happen that the field line of a logical exciton ties a knot in cognitive space which cannot be smoothed out. As a result, it is prevented from dissipating and will behave much like a particle. A parallel example from physics is a magnetic monopole - the isolated pole of a magnet - which has not been detected in nature but shows up as twisted configurations in field theory. In the traditional view, particles such as electrons and quarks, which carry geometric charges, are seen as fundamental, whereas particles such as magnetic monopoles, are derivative

particles, to which we can assign topological charges. What is important is that a topologically nontrivial field configuration, such as soliton, exchanges roles with ordinary quanta. To describe consciousness one does not really need spacetime, or more radically, does not really have spacetime any more, but just a tensor product of two-dimensional topologies, much as with strings where one does not have a classical spacetime but only the corresponding two-dimensional theory describing the propagation of strings. Worldlines are replaced by worldsheets, the interaction vertices in the Feynman diagrams are smoothed out, and spacetime exists only to the extent that it can be extracted from that two-dimensional field which encodes information.

Although we are all familiar with notion of thoughts, in reality we never observe an isolated thought in particular locations of the brain. It is everywhere and nowhere. A thought for the brain is like a neutrino for the universe. The organization of the brain is distinguished by extraordinary plasticity, with one region of the brain smoothly taking the role of the other if the need arises. Following an immediate reflex, one is tempted to connect thoughts with quantum nonlocality. But there is a more fundamental concept, the concept of the topological charge, which brings greater clarity to the question of the nonlocality of thoughts. To understand that we must understand a key difference between topological and Noether charges. A topological charge is a knot which is essentially nonlocal. It is a defect on a field line which characterises it as a whole. A geometrical Noether charge, in contrast, is local. It can be localized in a particular spacetime point, to a degree allowed by the uncertainty relation. We can in principle localize an electron in the brain, but we cannot, even in principle, localize a thought. When a thought emerges, a (topo)logical knot is tied up, and the knot by its very definition is a spatially extended object. This (topo)logical approach to the problem of consciousness offers a new understanding of the phenomenon. Nature obeys mathematical laws, but while for the physical brain these laws are primarily geometrical, both in the commutative and noncommutative spaces, for the cognitive brain the underlying mathematical theory is essentially and fundamentally topological. We pursue this viewpoint to an even greater extreme: geometry **cannot** be used to describe logical consciousness. Thought is essentially a topological effect, connected to the brain by means of duality, much as the magnetic monopole, a collective excitation, is related to the dual electric charge. In the actual brain there are Noether charges and these are converted into (topo)logical excitons that move freely through the neuronal medium, decaying into their constituent parts and recombining back. A (topo)logical exciton emerges as a fundamental quantum of consciousness, forming coherent waves that run through the brain matter. However, unlike electrons, (topo)logical excitons do not carry mass across the brain. They carry topological energy, and in spite of almost classical propagation regime, their spectra remain highly coherent, because a coherent superposition of true and false underscores the very existence of a (topo)logical exciton.

Application of this model to the brain-mind duality offers a fundamental explanation of consciousness. **It suggests that there exist two equivalent formulations of the logical brain** in which the roles of geometric charges and (topo)logical charges are reversed, just as we exchange electric charge and magnetic charge in field theory. In such a dual picture of the brain either charge, (topo)logical or geometrical, can be taken as elementary, and then a dual charge arises as derivative. In quantum field theory a fundamental particle with charge e is equivalent to a solitonic particle with charge $1/e$. This leads to a vast mathematical simplification. For instance, in the theory of quarks we can hardly make any calculations when the quarks interact strongly. But monopoles in the theory must interact weakly, and by doing calculations with a theory based on monopoles one automatically get all the answers for quarks.

The duality principle, applied to the problem of the thinking brain, provides a promising theoretical framework. For a very long time we have been struggling to understand the intractable mechanism of consciousness which somehow converts physical to mental and mental to physical. The duality between (topo)logical charges and Noether charges removes the impediments to understanding how the thought process is able to induce controlled changes in the brain matter. When we think, the brain transforms (topo)logical charges, which are fundamental primitives of thoughts, interacting weakly. When such a transformation is completed we automatically gain the answers for the 'strongly' interacting neurological brain.

A charge is a measure of the strength of an interaction but physical and logical charges obey opposite laws of attraction and repulsion. Identical Noether charges, like those of two electrons, repel,

while identical logical charges gravitate towards each other and merge:

$$x \text{ AND } x = x \quad - \quad \text{the absorption law}$$

The opposite physical charges, like those of an electron and a proton, attract,

but the opposite logical charges are mutually excluding and repel each other:

$$x \text{ AND } \bar{x} = 0 \quad - \quad \text{the contradiction law}$$

No two identical Fermi particles, such as an electron or proton, can ever be in the same quantum state, but 'logical fermions' would not follow the Pauli exclusion principle.

This example touches on fundamental aspects of the brain-mind duality, which connects the strong coupling of one theory with the weak coupling of another. **Consciousness is a topological effect: the brain decides geometrically; the mind decides topologically.** Topology is not a matter of choice but is fundamental. Consequently, there are two dual theories of the brain: the geometrical theory which we used until now and a topological theory whose underlying principles we intend to formulate. When the brain is described in terms of the Noether charges, the dual (topo)logical charges emerge as derivative [Ref 77]. Quite symmetrically one can choose the (topo)logical charges to be fundamental, and then to treat the biophysical electrophysiological brain as derivative,

$$\textbf{(Topo)logical charge} \quad \Leftrightarrow \quad \frac{1}{\textbf{Noether charge}}.$$

The notion of topological charges as the physical basis of consciousness naturally leads to the notion of topological waves or currents which carry the charges. The charges are nontrivial dynamical topological configurations that exchange with ordinary quanta. A (topo)logical current propagating along a closed information loop (knot) manifests itself as the thought process. The knot may have various configurations, but a particular geometry of the knot is irrelevant, as long as it retains the same (topo)logical charge. The (topo)logical currents are effectively isolated from the outside universe and cannot be subjected to ordinary physical measurement. The most we can achieve with state-of-the-art Hermitian devices is to measure the dual Noether currents, and the attempt to do so is made indirectly when we measure the electrical and neurochemical activity of the brain in the laboratory. However, a (topo)logical charge maps to a corresponding Noether charge and vice versa. Making use of this duality we can influence the (topo)logical current and with it the inner content of consciousness. This effectively was done in the classical experiments of Wilder Penfield, performing neurosurgery on conscious patients: electrical stimulation of the brain can induce virtual cognitive states and memories which the subject treats as real. In the Penfield effect the Noether charges are converted into (topo)logical charges but there must be and there is the reverse transformation whereby the (topo)logical charges are mapped onto electrical Noether charges of the brain. There is no mystery in the ability of the mind to influence the physical states of the brain, an idea which strangely enough continues to be treated as theological and nonscientific in some circles. But anyone who doubts that the mind acts on matter, exerting control over the physical brain, may recall, for example, the experiments carried out with paralyzed patients. Patients who couldn't communicate at all were taught to write sentences on a computer screen via electrodes implanted in their brain by

changing their brain waves at will. Signals from an EEG are used to control a simple switch. The patients learn to make their cortical potential more negative or positive, in this manner moving a cursor up or down on a computer screen, writing messages. This system, being based on simple binary choices, has limited scope, but there is no doubt that neuroscience one day will make use of the **'thought translation machines'**, for example to exert fine control over devices that simulate muscles, allowing paralyzed patients to regain control over their bodies.

LOGICAL VACUUM

One of the basic tenets of quantum physics is that all particles have positive energy. According to Dirac the negative levels are filled up and by definition the vacuum has zero energy. This positivity of energy imposes fundamental limitations on what can and what cannot occur in physics. If we were able to produce objects with negative energy, which is less than vacuum, then we could generate exotic field configurations in which time is bent into a circle. However, this effect, much loved by science fiction writers, has become actuality in quantum field theory with the introduction of antiparticles. A closed vacuum diagram describes the birth of particles emerging out of a vacuum and submerging back into the void. At the fundamental level particles and antiparticles moves freely backwards and forwards in time, violating no macroscopic laws of causality.

In the framework of the two alternative dual theories of the brain, the geometrical and the topological, we now introduce notion of a logical vacuum Ç which carries a nontrivial topological charge. The logical vacuum is not physically empty, but a connection-empty space, meaning that there is no interaction between the elements of the system, and as a direct consequence of that, the system has no deductive or inductive logical (reasoning) capabilities. The closest analogy from physics would be a system of noninteracting particles in quantum mechanics. The logical vacuum could have different degrees, reflected in part by different IQ. Indeed, there are, as we all know, some big but empty heads with a lot of 'intellectual vacuum'.

The presence of a logical connection can be interpreted differently. In the classical picture a connection is a convex link which can be thought of as a line joining two points in a topological space. The fundamental matrix-logical picture is less intuitive: the creation of topological links now signifies the creation of a matrix particle or **nibble**. If the particle is annihilated, submerging into the vacuum, the logical links which were created vanish in concert too. The role of nibbles is similar to the role of elementary particles in physics and to genes in biology. They are fundamental elementary logical

'particles' of thoughts and consciousness. In computational terms a nibble is a 4-bit word, two nibbles constituting a byte or 8-bit word.
We build the matrix-logical vacuum bottom-up, beginning with the ground state void of charges and with four degrees of freedom:

$$X_0 = 0\ 0\ 0\ 0$$

We must think in a very general way. The vacuum can stand for the actual physical vacuum of a relativistic quantum field, or a four-particle interaction, or the four chemical bondsvacancies in the carbon atom.

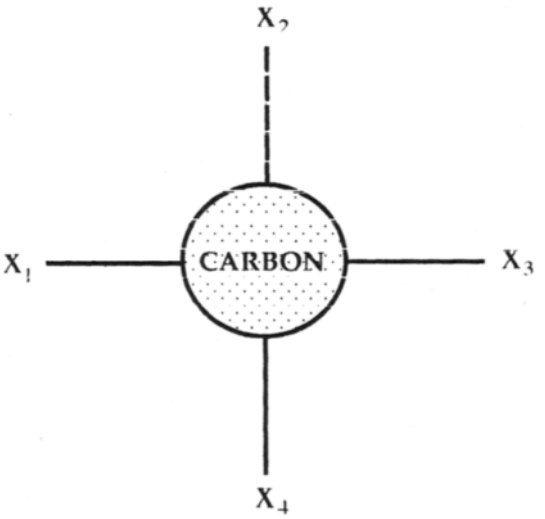
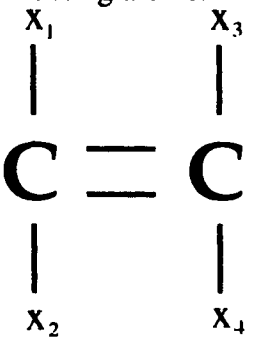


Fig. 6
Four chemical degrees of freedom in the carbon atom.

At this stage the physical nature of logical nibbles is of secondary significance. What we are concerned with is their information structure. A nibble has four degrees of freedom. The product of two nibbles is again a nibble, which for carbon can be modelled by the double-covalent bond, closing up two degrees of freedom in each of the interacting atoms:



One can picture a logical vacuum simply as empty space which tells nothing about its structure, which is in fact very complex. More meaningfully we can present the logical vacuum as being composed of a number of compartments populated by zeros:

0	0
0	0

Apparently, there are different kinds of zeros, the difference between which we must not fail to distinguish. If zeros are taken to be the inner products of orthogonal logical states, we can describe the states of the vacuum in terms of the inner products $\langle 0|1\rangle$ and $\langle S_-|S_+\rangle$, which is a scalar vacuum:

$\langle 0\|1\rangle$	$\langle 1\|0\rangle$
$\langle S_+\|S_-\rangle$	$\langle S_-\|S_+\rangle$

The retrace of the scalar vacuum will result in the outer product matrices, determining the 4 different corners of the vacuum:

$\|0\rangle\langle 1\|$	$\|1\rangle\langle 0\|$
$\|S_+\rangle\langle S_-\|$	$\|S_-\rangle\langle S_+\|$

Because the logical vacuum depends on interaction and communication between logical elements, it is an information system with a complex matrix structure. This becomes evident when we polarize the vacuum. There are different ways of doing so: multiplicative polarization and additive polarization.

MULTIPLICATIVE POLARIZATION

Multiplicative polarization is a factorization of the vacuum into a set of nonzero elements, the divisors of **0** whose product is empty. Such polarization confirms the existence of the **4** self-dual corners in Ç, comprising the octet of matrix nibbles as shown in Fig.7.

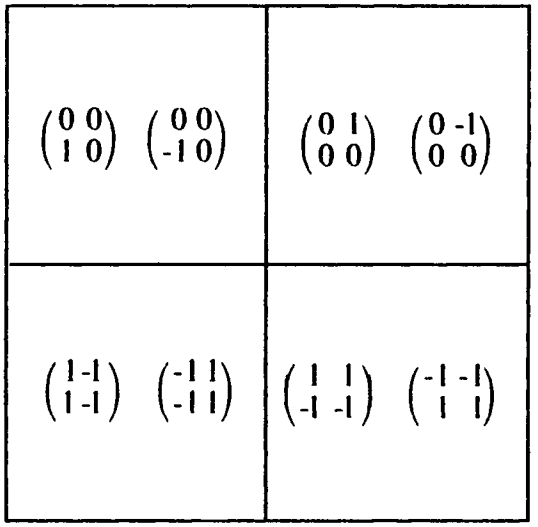


Fig. 7
4 fundamental self-dual moduli of the logical vacuum, comprising **8** nilpotent matrix nibbles; the product of any matrix with itself is the zero matrix $\mathbf{L}^2 = \mathbf{0}$.

The octet multiplet represented by self-dual operators is a *true* logical vacuum characterized by high symmetry which comes from the fact that for each corner of Ç the vacuum multipliers by modulo are the same and can mix without restraint. The square root polarization of the vacuum results in the octet of the singular operators

$$\begin{pmatrix}1&1\\-1&-1\end{pmatrix},\begin{pmatrix}-1&-1\\1&1\end{pmatrix},\begin{pmatrix}1&-1\\1&-1\end{pmatrix},\begin{pmatrix}-1&1\\-1&1\end{pmatrix},\begin{pmatrix}0&0\\1&0\end{pmatrix},\begin{pmatrix}0&0\\-1&0\end{pmatrix},\begin{pmatrix}0&1\\0&0\end{pmatrix},\begin{pmatrix}0&-1\\0&0\end{pmatrix}$$

comprising the asymmetric quadruplet:

$$\begin{pmatrix}0&0\\1&0\end{pmatrix},\ \begin{pmatrix}0&0\\-1&0\end{pmatrix},\ \begin{pmatrix}0&1\\0&0\end{pmatrix},\ \begin{pmatrix}0&-1\\0&0\end{pmatrix},$$

and the (bi)antisymmetric quadruplet:

$$\begin{pmatrix}1&1\\-1&-1\end{pmatrix},\ \begin{pmatrix}-1&-1\\1&1\end{pmatrix},\ \begin{pmatrix}1&-1\\1&-1\end{pmatrix},\ \begin{pmatrix}-1&1\\-1&1\end{pmatrix}.$$

These quadruplets in turn are formed by two dublets that are the transpose of each other

$$\widetilde{\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}, \qquad \widetilde{\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}} = \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix},$$

$$\widetilde{\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, \qquad \widetilde{\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}} = \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix}.$$

comprising two operators which differ only in sign:

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = -\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = -\begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix}.$$

The self-dual corners of the logical vacuum anticommute to identity:

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

A number of important matrix features reveal themselves in the properties of the logical vacuum. Matrices differ from numbers in that the product of two nonzero matrices can be zero, which is not possible for numbers. A true logical vacuum is stable against transposition in the vacuum product because for a self-dual nibble both multipliers are the same. But the product of two different nonzero matrices can also be a zero matrix. This is the case of a *false logical vacuum*:

$$\widetilde{\mathbf{FALSE}} \cdot \mathbf{TRUE} = \mathbf{0}, \qquad \widetilde{\mathbf{TRUE}} \cdot \mathbf{FALSE} = \mathbf{0},$$

which is not stable and whose symmetry will be broken if the multipliers are swapped over:

$$\mathbf{TRUE} \cdot \widetilde{\mathbf{FALSE}} \neq \mathbf{0}, \qquad \mathbf{FALSE} \cdot \widetilde{\mathbf{TRUE}} \neq \mathbf{0}.$$

The components of the false vacuum give rise to the true vacuum additively. Subtracting the corresponding elements of the false vacuum we get the (bi)asymmetric self-dual operators, populating two coners of the true logical vacuum:

$$(\mathbf{FALSE} - \mathbf{TRUE})^2 = \mathbf{0}, \qquad (\widetilde{\mathbf{FALSE}} - \widetilde{\mathbf{TRUE}})^2 = \mathbf{0}.$$

The true vacuum gives rise to the false vacuum multiplicatively. This theorem concerns the commuting product

$$(\mathbf{ON} \cdot \mathbf{OFF}) \cdot (\mathbf{OFF} \cdot \mathbf{ON}) = \mathbf{0},$$

its converse:

$$(\mathbf{OFF} \cdot \mathbf{ON}) \cdot (\mathbf{ON} \cdot \mathbf{OFF}) = \mathbf{0},$$

and its block converse, where the transposition of operators within each block recovers the original product:

$$\widetilde{\mathbf{ON} \cdot \mathbf{OFF}} \cdot \widetilde{\mathbf{OFF} \cdot \mathbf{ON}} = \mathbf{ON} \cdot \mathbf{OFF} \cdot \mathbf{OFF} \cdot \mathbf{ON} = \mathbf{0}.$$

ADDITIVE POLARIZATION AND THE FUNDAMENTAL LATTICE

The additive polarization of Ç yields **80** logical matrix particles which must sum up to **0**,

$$\sum_{i=1}^{80} \mathbf{L}_i = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}.$$

Together with the vacuum itself **0**, these nibbles populate the **81** vertices of the fundamental matrix logical lattice, which is the Boolean **4**-dimensional hypercube extended with the negative coordinate. Explicitly these are the following fundamental nibbles:

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -1 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & -1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$$

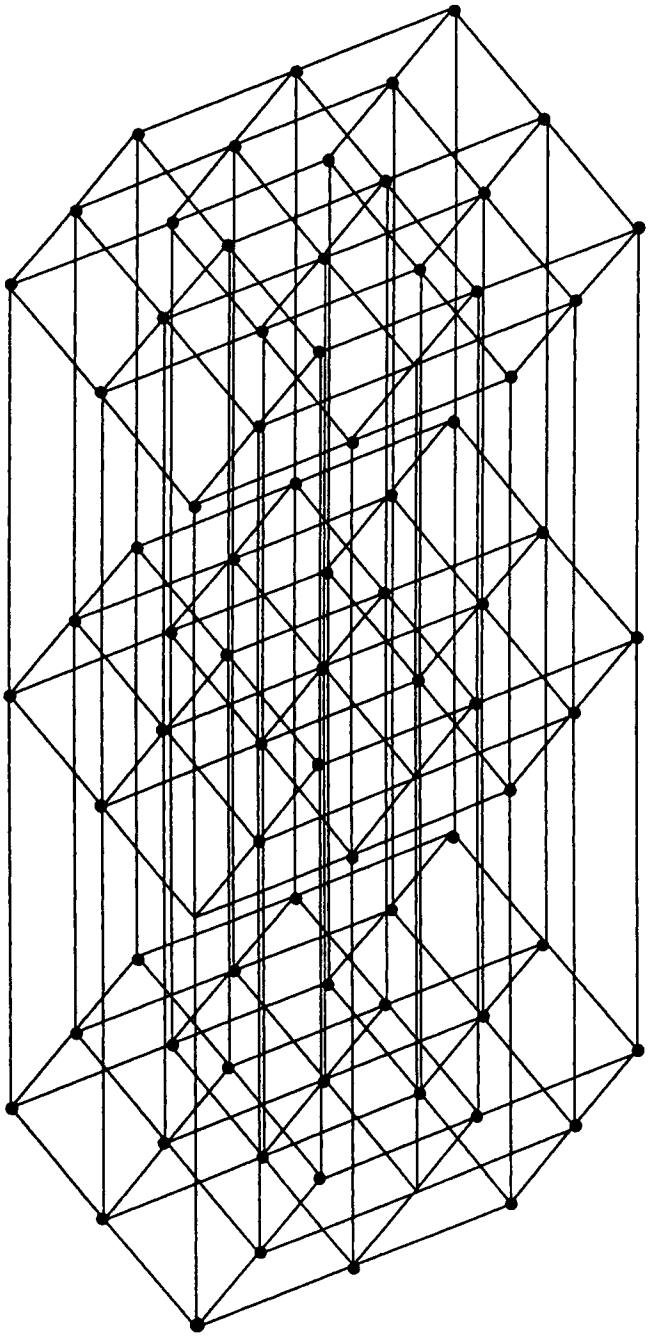


Fig. 8
Matrix logical lattice with **81** vertices, comprising three subsets with **27** vertices each.

The subject of this study is to unravel the complex universe of this lattice. The fundamental set of **80** matrix particles is divided into a subset of **26** Hermitian (symmetric) and a subset of **54** non-Hermitian (asymmetric) matrices, which in themselves constitute the two branches of the additive vacuum, symmetric and asymmetric. Naturally each branch separately sums up to **0**.

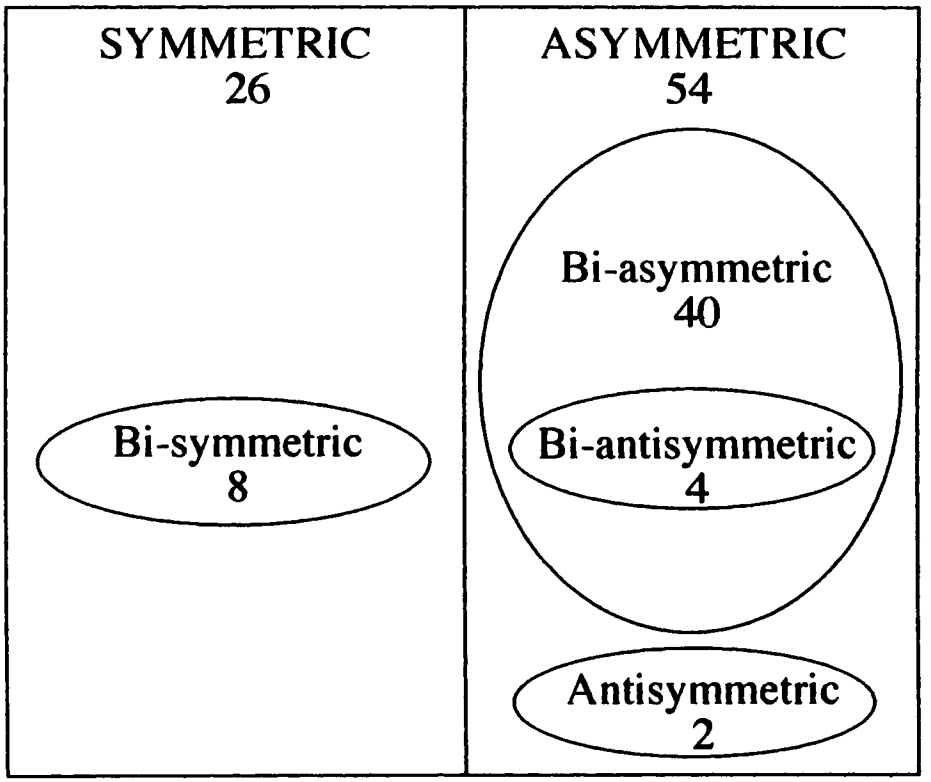


Fig. 9
The content of the additive polarizations of logical vacuum.

Note:
Since

$$n0 = 0$$

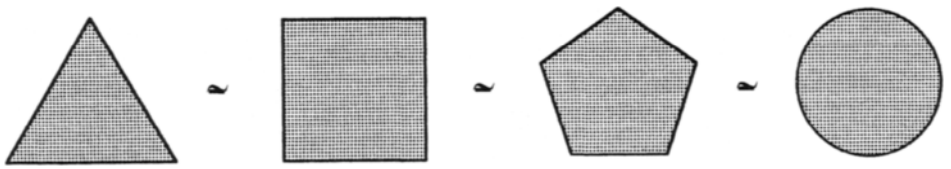
in addition to **80** binary polarizations in Ç we will also allow conformal global isomeries **nL**, where n is typically **2**. The existence of the isomeries **2L** is important for the exponential expansion of logical operators, allowing for three-term linear approximation.

THOUGHTS, KNOTS AND CROSS-CUPS

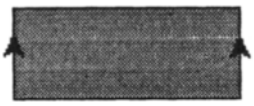
Because (topo)logical charges are maintained in the form of knots, we can treat the fundamental cognitive elements of a brain as topological knots. We cannot observe an isolated thought in particular locations of the brain, just as we never observe an isolated quark. With the increase of the distance between quarks the attraction force does not fade, but counterintuitively increases, forcing the quarks to remain together. There is a general conviction that the confinement of quarks due to asymptotic freedom is an exclusively microphysical effect, without precedents in the macroworld. However, defying logic, molecular science has come up with a sort of antirubber: if stretched in one direction, it expands in all others. The fundamental macroanalogues of asymptotic freedom are to be found in high-level emotions and consciousness. Nature seeks balance. In art and in love 'less is more': familiarity breeds contempt, separation increases attraction. But there is also the dual 'more is less'. In the mind of a beginner there are many possibilities; in the mind of an expert there is none. A slogan, repeated too many times, is irritating, but if it is too new, it provokes hostility and rejection.

The cognitive phenomena which follow the rules of asymptotic freedom come to be better understood in the framework of topology. Topology studies the properties of objects which are consequences neither of size or form, nor of how they sit in space. Where geometry thinks of structures as rigid, topology thinks in terms complete flexibility. For a long time geometry has dominated physical thought. However, a search for the physical basis of consciousness leads to topology, more precisely to new laws of symmetry that are consequences of topological invariance, as opposed to the conservation laws of traditional physics which are due to geometry. The Noether theorem proves that the invariance of a Lagrangian implies a conserved quantity. In a similar vein we intend to show that the invariance of a topological charge implies the Planck constant and the derivation of quantization from first principles.

Notwithstanding its violation of the parity symmetry, the advantage of the topological approach to abstract thought is that it provides a new fundamental measure of equivalence, which is lacking in existing approaches. Systems which are different from a geometrical standpoint and seem to have nothing in common are topologically indistinguishable. For example, a triangle, square, pentagon and circle are topologically equivalent since it is possible to continuously transform one into any of the others:

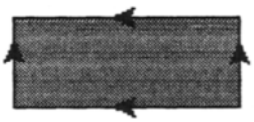


Topology is concerned with the properties of objects that remain invariant under continuous deformation of the object without any puncturing or cuts. If one object can be continuously deformed into another, the two are topologically equivalent. This is the first underlying principle of topology. By bending the surface and gluing the edges one obtains a variety of topologies. If the two ends of a rectangular surface are glued together, the result is a cylinder. If prior to being glued they are twisted, we get a Möbius strip, a surface which has one side and no amount of continuous deformation can change its unilateral topology. If the opposite edges of a rectangle are glued together, the result is a torus. The Klein bottle is a self-intersecting surface with no edges and only one side. It will be formed if the opposite edges of a rectangle are glued together, but one pair is twisted. Even though visual ways of drawing these surfaces appear curved, mathematically it is not actually necessary to perform the twisting or bending. Instead the points of an edge are identified with the corresponding points of another edge which join the edges next to each other.

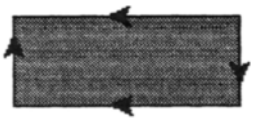


Cylinder

Möbius strip



Torus

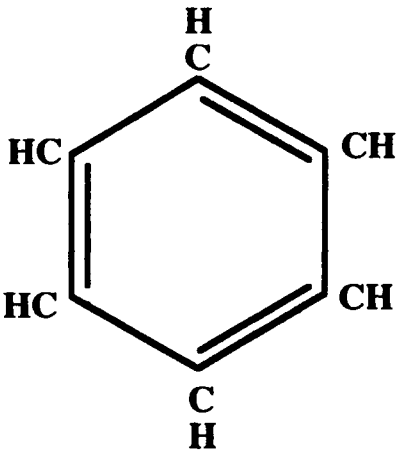


Klein bottle

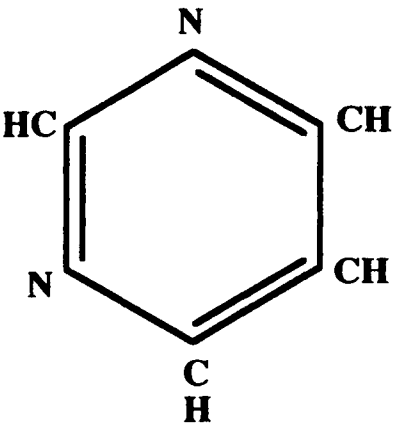
In this study our prime objective is to introduce an entirely new language in brain science, the language of topology. Topology is rich in counterintuitive structures but never before has it been used in a fundamental way in brain science. While the determination of a geometrical equivalence can be accomplished with ordinary physical devices, the determination of a topological equivalence requires intelligence and abstraction. The abstract investigation of a topological manifold enables one to determine whether it is a loop, a string, a membrane, an oriented or nonoriented surface, with no edges or some, and then how many.

Topological properties may not reveal themselves explicitly and sometimes can be difficult to grasp. In this study nonorientable topologies, such as the Möbius strip, a surface with only one edge and only one side, take a prominent place. The surface bears the name of August Möbius, who introduced it in 1858 as a purely mathematical abstraction. Soon after Möbius

made his discovery, in 1865, August Kekule announced a finding that was to become the basis of organic chemistry: carbon atoms can join to form long chains, which can curl around to form closed ring. Kekule identified the structure of benzene C_6H_6 where **6** carbons formed a hexagon ring with a hydrogen atom linked to each carbon. A number of other closed molecular rings were subsequently discovered.



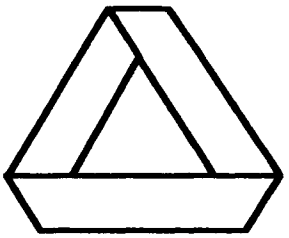
Kekule's benzene ring



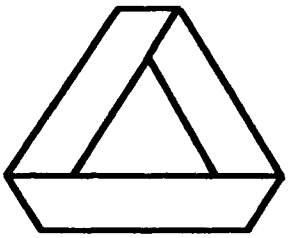
DNA's pyrimidine ring

Benzene is not a Möbius strip, but in the hundred or so years since Kekule understood its topology, more complex molecules, such as proteins, the double helix DNAs and molecules in the shape of a Möbius strip have been observed and synthesized. A proposal for a such synthesis was originally suggested by the author [Ref 87]. In macromolecular biology, the most famous 3-dimensional structure is DNA, with its supercoiled double helix joining two strands and compressing genetic information into a manageable region of space. The supercoiling - coiling of coils - stores topological energy that neuron cells can use, like the energy in a twisted rubber band. The DNA molecule comprises two interconnected strands, forming a molecular ladder which is bent around. If the ends of the molecule are joined to form a loop and the ladder gets a half-twist, the resulting loop is a Möbius strip. If we then break the molecular bonds between the strands, the molecule will remain in one piece, just as a Möbius strip, if being cut down the middle, stays a single convex band but with a circumference twice the circumference of the original [Ref 87].

The unilateral topology of the Möbius strip can be obtained in different ways, by twisting a strip clockwise or anticlockwise, as shown in a 2-dimensional flattened representation of the Möbius strip below:



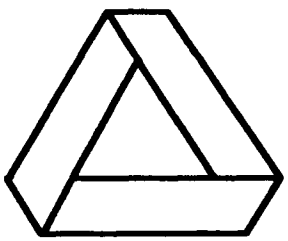
clockwise half-twist



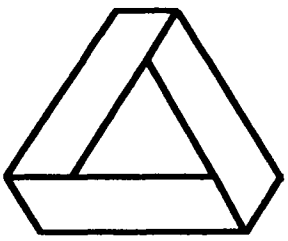
anticlockwise half-twist

We have a pair of Möbius strips that are mirror images of each other, and will refer to the clockwise Möbius as right-handed and to the anticlockwise Möbius as left-handed. Telling a topology from its mirror topology and determining their equivalence or otherwise are important questions. Although in 3 dimensions the left-handed and right-handed strips cannot be continuously deformed into each other, it turns out that in 4 dimensions this can be easily done, a fact with repercussions for the theory of mindware and its underlying matrix logic.

There is an important question which does not seem to have been addressed in the literature. There exists a direct link between the cuts on closed topologies and the theory of knots, which allows one to obtain knots with the desired properties and provides a new way of classifying alternating knots. A closed cut on the left-handed and right-handed strips produces an orientable loop with a length twice the length of the original. However, if instead of twist-1/2 we give a Möbius twist-3/2, as shown below,



clockwise twist-3/2



anticlockwise twist-3/2

then the closed cut will result in an elementary convex knot with 3 crossings. Contracting the manifolds to lines, we get two knots which are mirror images of each other, just like the strips which they came from.

Throughout this study, we developed the theme that the laws of topology provide a conceptual framework in which consciousness can be properly understood and the laws of logic become unified with fundamental physics. Much attention will be devoted to substantiating the idea that consciousness exerts control over the topology of the brain. This question is intimately linked to the question of connecting a continuum to discrete knot states. Topology is

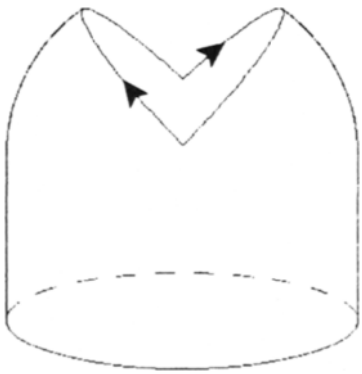
the level where the fundamental laws of quantum mechanics are translated into the laws of cognitive logic, taking away much of the mystery surrounding such confusing questions as where a thought resides in the brain and by what means it is embedded in the matter of the brain.

Topology is defined as rubber-sheet geometry, which is true. But more fundamentally topology can be understood as the science of **nonorientable** objects. It is for these objects that geometry is truly of little use so that we have to introduce topological methods which distinguish topology as a distinct theory. In view of our key claim that thoughts are nonorientable, the use of topology becomes essential for the theory of consciousness. But we can also identify nonorientable objects in strings, quarks, a physical vacuum and in black holes.

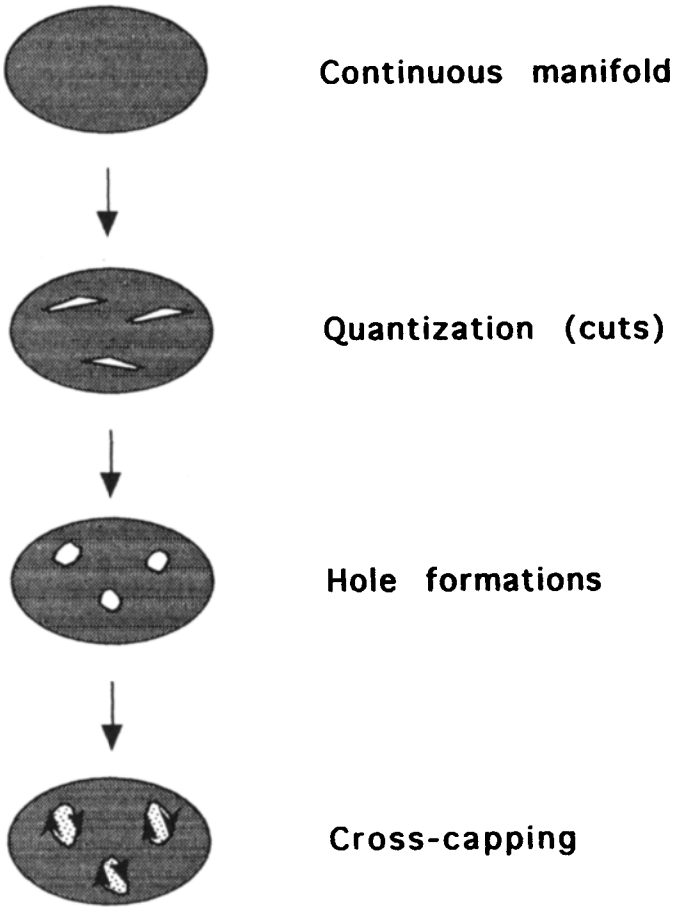
Given that the brain is an orientable manifold, while the objects of consciousness, thoughts, are nonorientable, the questions arise how to 'insert' a nonorientable logical thought into the orientable brain. The problem of the embedding of thoughts in the brain presents itself as a topological problem of connecting orientable and orientable manifolds. Gluing a manifold and a projective plane has a direct bearing on the question of consciousness, whose elements emerge as cross-caps attached to the orientable system.

A projective plane is homeomorphic to the topological space obtained from a disc by identifying the opposite points on the boundary (with twists). To glue the manifold and the plane we puncture them with holes. Then the disc is turned into a ring or equivalently into a cylinder in which the opposite points of one base are identified:

When the identified points are joined in 3-dimensional space, the resulting topological surface, intersecting with itself, is a cross-cap:

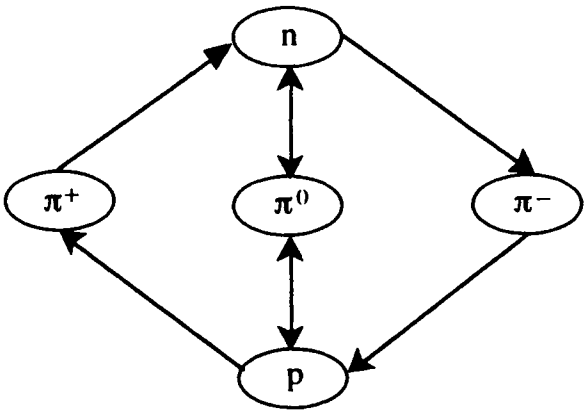


A projective plane with a hole can be inserted into 3-dimensional space without self-crossing since it is homeomorphic to a Möbius strip. The properties of a Möbius strip, such as convexity and nonorientability, are preserved under the closed cut, is easier to grasp than the properties of a cross-cap, but if we wish a hole to be cancelled out literally and not in the abstract sense of identification of points, then we need a cross-cap. It follows that in order to join a projective plane with manifold, we must identify the boundary of a cross-cap with the boundary of a hole or removed disc. This operation attaches (glues) a cross-cap to the manifold. Since prior to gluing, we must remove a disc from the manifold, it would be better to say that we close a hole with a cross-cap or, equivalently, that we insert a Möbius strip in the manifold. We could have obtained the same result by making a hole in the manifold and identifying the opposite points on its boundary.



Cross-capping is a suitable model, describing the insertion of nonorientable elements of consciousness (thoughts) into the orientable topology of the brain.

It is an abstract mathematical model, which must be connected to actual physical processes. Nucleonic cross-capping is one such connection, as we discussed earlier. The idea originated in the context of the fundamental model of the intelligence code, based on the Heisenberg isospin theory [Ref 81]. In this theory the proton and neutron are viewed as two states of same particle, a nucleon, which nuclear interaction does not distinguish. A proton, accepting π^- or emitting π^+, is translated into a neutron; a neutron, accepting π^+ or emitting π^-, is translated into a proton. Absorption or emission of a neutral pion π^0 is identity. The resulting Dedekind structure elegantly accommodates the quantum logic of the true, false and undecided:



In this quantum gate, created by nature, a change-over from proton to neutron and vice versa allows for topological interpretation of isospin in terms of nucleonic cross-capping provided by the pion triplet $\{\pi^+ \pi^o \pi^-\}$. The identification of proton and neutron with the two states of same particle is topologically equivalent to identification of the opposite points in the removed disc. The underlying ternary logic of the isospin can be treated as a cross-cap on the Hilbert space of protons and neutrons in an atomic nucleus.

The wavefunction of a quantum particle is a coherent superposition. It is possible to treat a particle confined to atomic or molecular orbit as an nonorientable knot, having no preferred direction in space, and carrying topological energy. The splitting of the energy level into two levels will correspond to the topological ***CUT*** operation, which can be spontaneous or induced externally, e.g. by application of a uniform magnetic field in the Zeeman effect.

Topology explains many counterintuitive mysteries of quantum mechanics. The topological scheme stands apart both from the Copenhagen interpretation of quantum theory and from the decoherence model. In a modern model of decoherence the macroframe effectively acts as a measuring apparatus. Due to the irreversible coupling of the observed system to the outside world the off-diagonal elements in the density operator ρ vanish: $\rho^2 = \rho$. In our topological treatment of quantum measurement, a particle, prior to measurement, is

topologically nonorientable, its energy levels being degenerate with respect to certain quantum numbers [Ref 79]. For example, we expect for the angular momentum $(2J+1)$-fold degeneracy of the energy levels vs. the magnetic quantum number. In the act of measurement this degeneracy is lifted and the nonorientable topology turns into orientable one.

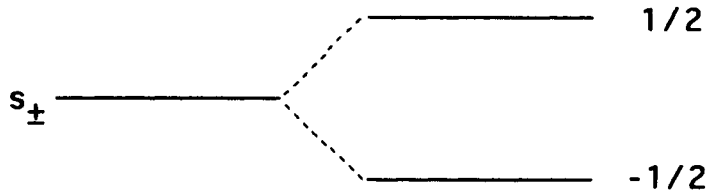


Fig. 10
The resonance splitting of a quantum level into a pair of levels corresponds to the ***CUT*** topological operation.

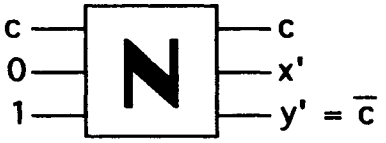
Topological energy and its relation to quantum energy provide clues to the mechanism of logical thinking. The change of topological energy is naturally achieved by a quantum jump from one atomic level to the other. Although in the brain there are no magic fingers to tie up or untie logical knots, nothing can be as easy as to excite or cool down an electron, which, jumping to another orbit, would perform the required topological transformation. In a surprising turn of events Bohr's theory of the atom becomes intertwined in a fundamental way with the topological theory of consciousness. In analogy to Boltzmann's thermodynamic energy $E \sim kT$, the thought process is realized by lowering or raising topological 'temperature', the process which can alter the spectral pattern of the molecules of the brain. Conversely, modification and redistribution of atomic molecular levels will locally change the topological 'temperature', which may support or tamper with the logical thought process.

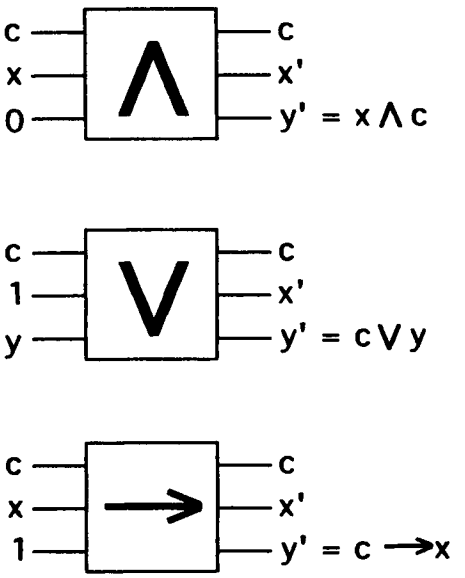
There is also a macroquantum picture, in which the brain is interpreted as an actual macroscopic 'atom' whose 'orbits' are populated with (topo)logical quanta, dynamically changing in resonance mode with corresponding quantum knots. Underneath the bold idea that the laws of the atom hold the key to the laws of the thinking brain there lies even greater promise: the laws of topology hold the answer both to the puzzle of the brain and to the mystery of the quantum.

COGNITIVE CIRCUITS

Reversibility and the conservation laws of physics are closely linked. The question is how a reversible computation for which the output is implicit in the input can generate information. Logical **AND** and **NOT** form the basis set of logical primitives. However, **AND** and other logical operations are irreversible: the information of the inputs is erased in the output. In fact any binary logical operation is irreversible, the pair of inputs not being recoverable from a single output. The information is irreversibly lost and the computation process cannot be traced backwards. Such intrinsic irreversibility can only reflect aspects of macroscopic physics. Given the fact that thought runs forward but understanding runs backward, how can we remedy the deficiencies of irreversible logic? Wouldn't irreversibility deprive the brain of memory, and of keeping track of its own operations, a capability which is so critical for consciousness? To enable the system to operate backwards, one must assume quite literally a microphysical basis for consciousness. Logical computation is irreversible in the macroscopic realm but can obey conservative dynamics on a microphysical scale. This is not merely a technical problem: rather one faces fundamental difficulties, chiefly connected with the second law of thermodynamics. Theoretical as well as technical advances are needed to break the conceptual stalemate. The consciousness effect includes the ability of a logical system to exercise control over its intrinsic topology. The reversible system of matrix gates in which the outputs are identified with the inputs constitutes a self-measuring cognitive circuit [Ref 73], where in accordance with the logical superposition principle, depending on the status of the control bit, the circuit exists either in the nonorientable or orientable processing mode. The thought process manipulates the internal topology of the brain, locally inducing conversion between nonorientable and orientable topologies, which could in principle be detected.

The theory underlying the functioning of the logical brain must explicitly reflect in its axioms certain fundamental principles of physics. Consciousness keeps track of its own operations and is implemented by reversible logical gates, with appropriate initial conditions. Given that causal effects propagate through local interactions, as a starting point we can take the reversible Fredkin gates [Ref 81] which for the standard logical basis set are the following:





Inspired by a topological theory of consciousness we have associated a cognitive circuit with a closed topology [Ref 78], subsequently refined into the unilateral topology of the Möbius type with twists [Ref 79, 83]. Twist of the neuronal connections is an important feature of the architecture of the brain. The swapping and closure of (neuro)logical information lines gives topologically a Möbius, which together with the control bits provides for the logical realization of a cross-cup. These bits control swaps and in this way a changeover between orientable and nonorientable modes of operation. A reversible Fredkin gate, if closed on itself, is topologically a cross-cup, realizing a logical function. The topological closure of the system of gates transforms it into a cognitive circuit.

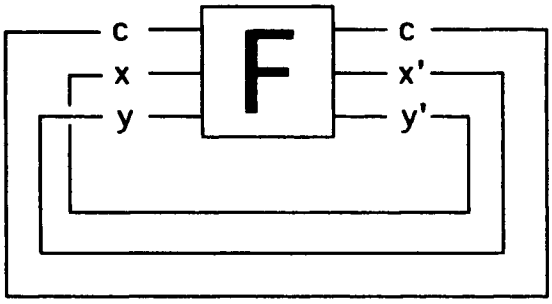
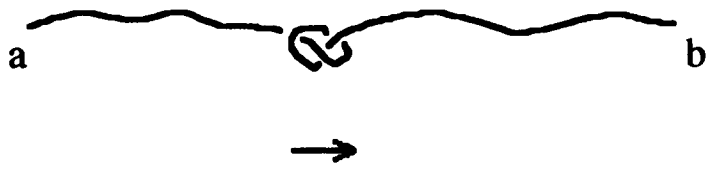


Fig. 11
Topological interpretation of the Fredkin gate closed on itself as a cross-cup for the value of the control bit **c** = 0.

Making use of reversible logic, and a topological interpretation of the Fredkin gate as a logical cross-cup, we obtain a description of the topological brain. The method of logical cross-cups closely mirrors fundamental principles of physics and is in a better position to analyse the cognitive mechanism than traditional models. It constitutes a substantial advance in establishing the correspondence between computation and logical thinking.

TOPOLOGICAL CURRENT

Thinking is a dynamical process. We now understand in principle the logical organization of the cognitive circuit. But we also must understand its underlying physics, which should not be confused with the physics of electrical spikes in neurons. A significant contribution of topology to the fundamental theory of consciousness is the notion of a topological current. When a topological knot, a defect in a set of field lines, cannot be smoothed out, it is prevented from dissipating and in effect behaves like a massless quasi-particle. Moving a knot, without a mass transport, from one point a to another b constitutes a topological current, carrying a topological charge:



Now by identifying the ends of the field line, a=b, one forms a closed knot topology. According to our scheme, when actually realized in the brain matter, the topological current manifests itself as consciousness. A coherent system of thoughts is a system of knots which can tie and untie by natural mechanism. The process relies on information symmetry breaking, which is similar to spontaneous symmetry breaking, exemplified in ferromagnetism. A bar magnet loses its magnetic properties when heated and become symmetric, but when the system cools, the symmetry is broken and its magnetization restored. Cognitive knots operate in a narrow range. When a knot is "heated" the topological charge and consequently the cognitive effect vanish but if the system "cools" and the symmetry is broken consciousness is restored. This scheme will be used to describe the stream of consciousness as a (topo)logical current. The modern view in physics is that electrons are individual fields, while a monopole must be treated as a collective excitation. In a similar manner we treat thoughts as a topological current of collective excitation.

A particle is an isolated object, moving in space between different spacetime points. A knot is interwoven into space itself, and movement of a

knot along a field line can be treated as a movement of the perturbation of space itself. In general relativity, gravity locally distorts the structure of spacetime and so might do the topological knots of consciousness on a microscale. To understand the mechanism through which the fundamental level dictates the behaviour of the brain, we have to address the problem of how the quantum events are to be reconciled with the covariance of logic operations. The goal is to define a reversible measurement through which theory and observation can be compared with each other. The idea of topological current provides clues how the physical level gives rise to the effect of thoughts in the quantum-theoretical brain. It also provides realistic hope of measuring the fundamental processes which dictate the cognitive effects. A fundamental theory of the brain must be consistent with the topology of quantum actions, constantly interrupting its macrological operations and in this way providing the foundation for intelligent thought. Recent advances in quantum information give us considerable hope that we could conduct tests in the laboratory in the not too distant future. A positive outcome could have far-reaching implications beyond brain science, in particular for quantum mechanics.

A topological charge navigating around a closed loop acquires a topological phase, the trivial 1 for nonconvex knots and the nontrivial phase -1 for convex ones. If the brain is in the fermionic limit of Matrix Statistics, then the nontrivial phase will be detected; if it is in the bosonic limit, then the phase is trivial and the topological charges are entangled.

Working with coherent superposition as computational basis states, we must want to perform the exchange operation, by measuring the topological current, using calibrated Noether charges, and to measure Noether charges, using previously calibrated currents. We will suppose that we are able to produce a large supply of knots: the brain can assemble a large reservoir of calibrated flux pairs that can be withdrawn as needed during the course of a computation. The local state has trivial interactions with any flux, and so carries no detectable charge. After producing a current, we can perform a measurement to project out one of the flux eigenstates. But is our topological brain universal? Can it perform any desired cognitive transformation? To address this issue, we have to take into account that we not only must have the possibility of processing topological charges but also of storing them. A length-minimizing topology often occurs naturally in interfaces between inorganic and organic (living) materials, because it minimizes tension. This situation is emulated on the cognitive level where topological and geometrical brains meet. The entropy of a closed system is known to increase until equilibrium. But consciousness puts the brakes on the increase of entropy and the topological complexity of the system can create information. When an ordinary field loop is subjected to the dissipation process, after a while we will find it in an optimal symmetric, circular or elliptic, form which distributes energy optimally, maximizing entropy. But if a knot, which cannot be smoothed out, has been tied on a closed field line, the increase of entropy will

be arrested. The presence of the defect forms the energy barrier, and the more complex the knot that is tied on the field line, the further away the system is from equilibrium or the ground state, with the dissipation and growth of disorder being terminated. Such a system will create negative entropy, which is information. Measuring the current can be achieved by measuring the charge and a controlled self-measurement can be carried out to any desired accuracy.

If a topological theory of the brain is to be consistent with the results of quantum physics, it must account for a unitary evolution. One possibility is to treat topological current as a coherent Bose condensate, in which all quantum particles are attracted to a single state. Examples of these are a superfluid involving spontaneous coherence of massive particles in liquid helium and superconductors, both of which are strongly interacting systems. There are other systems of massive particles, alkali gases and excitons, which demonstrate spontaneous attraction to a single coherent state. The topological excitons with nontrivial phase are bosons, as opposed to Boolean logical states, which are fermions, subject to the Pauli exclusion. One may think about a topological 'gas' of excitons, which, unlike atomic gas, must have a relatively long transient lifetime.

Theoretically there are two different ways we can create topological charges and currents. One way is simply to tie a knot on a closed field line. Another, equivalent, way is to view it as the result of the closed cut of a unilateral surface with helicity equal or greater then 3/2. But first we ought to have such a unilateral topology. The mechanism for converting a bilateral into a unilateral topology is provided by a *logical derivative.*

THE LOGICAL DERIVATIVE

Dynamical equations of motion, such as the Hamilton equations or the Schrödinger equation, is the foundation of physical theory. As far as the dynamical description of the thought process is concerned, logic is no match for physics, lacking the essential conceptual and mathematical tools needed for dynamical analysis. In order to formulate the dynamical equation for the thinking brain, we must examine consciousness not just in phenomenological terms but in the context of fundamental physics, which extends from the elementary quantum level to the cosmology of spacetime and the Big Bang. To

describe the stream of consciousness as a (topo)logical current we need to connect topology to logic in a fundamental way. Technically speaking, we require a dynamical operation for converting an orientable topology into a nonorientableone. Such fundamental operation is the *logical derivative*:

$$' = \{ \perp, \mathsf{T} \ ]$$

where $\perp$ and T are the **TRUE** and **FALSE** logical operators respectively, and the bracket structure $\{, \ ]$ signifies a supercommutator. Logical derivatives exist in the symmetric form of the anticommutator $\{\perp, \mathsf{T}\}$ and the antisymmetric form of the commutator $[\perp, \mathsf{T}]$. The reason for identifying the derivative with the commutator operation becomes clear if one compare

$$\frac{d}{dq}|q> \ = \ \frac{d}{dq}\begin{pmatrix}\bar{q}\\ q\end{pmatrix} = \begin{pmatrix}-1\\ 1\end{pmatrix}$$

with

$$[\perp, \mathsf{T}\ ]|q> \ = \begin{pmatrix}-1 & -1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\bar{q}\\ q\end{pmatrix} = \begin{pmatrix}-1\\ 1\end{pmatrix},$$

observing the fact that the differentiation operator acts on a logical state in exactly the same way as the commutator $[\perp, \mathsf{T}]$. The state is given a half-twist and glued into the nonorientable topology '**true-false**' either by applying the commutator or equivalently by differentiation. Likewise, the anticommutator can be identified with the modulo differentiation, also resulting in a nonorientable topology, which entangles the orthogonal '**true**' and '**false**' into the coherent state '**true+false**':

$$\{\perp, \mathsf{T}\}|q> \ = \begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}\begin{pmatrix}\bar{q}\\ q\end{pmatrix} = \begin{pmatrix}1\\ 1\end{pmatrix}.$$

The two forms of the logical derivative are dual:

$$\{\perp, \mathsf{T}\}\bullet[\perp, \mathsf{T}] = \mathbf{0},$$

and the corresponding topologies are orthogonal:

$$< s_+|s_-> = 0.$$

Whereas for the anticommutator bracket

$$<q|\{ \perp, \mathsf{T} \}|q> \ \equiv \ 1,$$

we will determine that the antisymmetric form of the logical derivative is special in that its average equals the average of the differential operator:

$$<q|\frac{d}{dq}|q> = <q|\ [\perp, \mathsf{T}\ ]\ |q>.$$

TRUE and **FALSE** do not commute and the zero average cannot be obtained with the commutator. But whenever q=1/2, the average vanishes.

The logical derivative, making use of the knowledge operators, is destined to play a very important role in the further exposition, forging fundamental links between logic, topology and quantum mechanics. Throughout this study the logical derivative will be subjected to various consistency tests, passing them successfully.

CUT AND ZIP OPERATIONS

We consider (topo)logical charges to be the fundamental primitives of the cognitive phenomenon represented by knots. The logical derivative is a knowledge operator which, acting on the classical logical calculus, gives rise to a nonorientable topology. In itself logical differentiation is essential but not sufficient for the creation of (topo)logical charges (knots). These are produced when an initial nonorientable manifold is given extra twists to obtain a half-integer but greater than **1/2** helicity. A unilateral topology alone is not enough to create a topological charge, we must have at least a Möbius-**3/2**, which then is ***CUT*** down the middle:

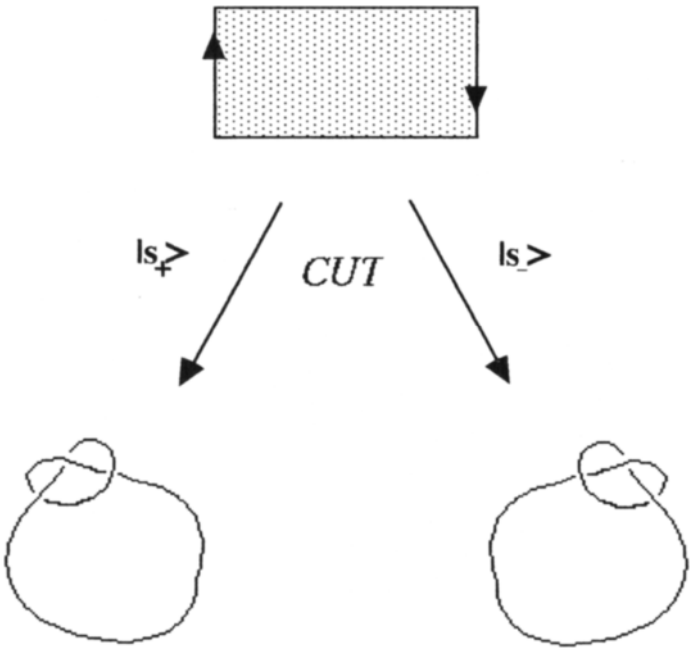


Fig. 12
The left and right elementary knots, resulting
from logical differentiation+*CUT*+contraction.

The second-order logical derivative collapses the nonorientable Möbius loop, both the left and the right, to the origin or the empty state:

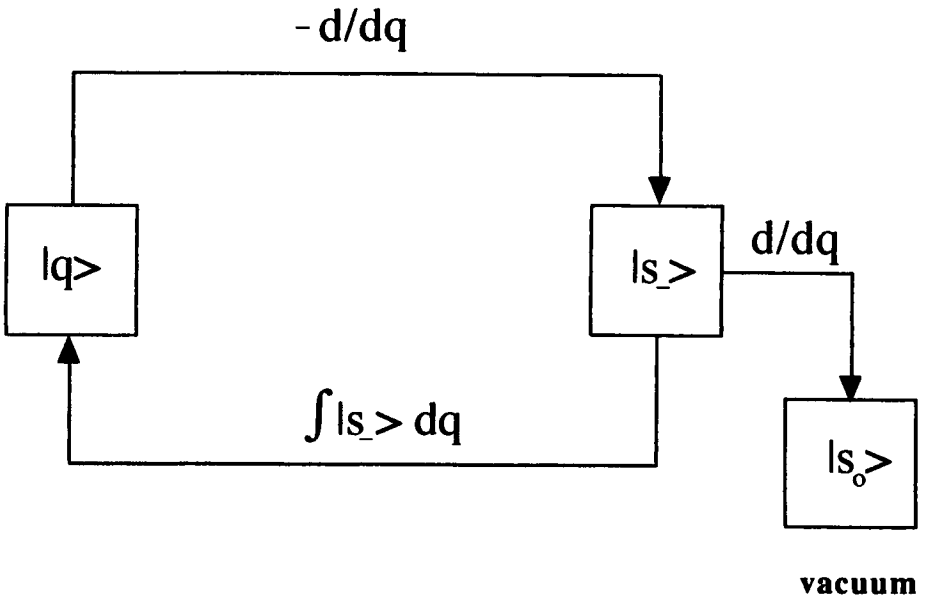
$$\frac{d^2}{dq^2}\text{STATE} = \frac{d}{dq}\text{LOOP} = \text{ORIGIN}$$

Matrix-logically, the origin is the logical complement of the left Möbius

$$|\bar{s}_+> = |0>$$

which indicates a possible way of obtaining the cognitive states by negation of the empty state. This provides for the interpretation of a cognitive state as the after-image of a logical vacuum. Simply negate the void to form a logical Möbius.

Thought carries a topological charge which dissipates under logical integration. A nonorientable state, formed by logical differentiation or negation of the vacuum, can be cancelled out by logical integration or reduced to a vacuum by the second derivative:



We also introduce a ***ZIP*** operation which is the inverse of ***CUT***:

$$ZIP = CUT^{-1}.$$

If logical differentiation is followed by ***CUT***, a (topo)logical charge emerges and may persist in the form of a knot. This process can be reversed by the ***ZIP*** operation and, most importantly, can be brought into a logical resonance state with the corresponding Mobius-**n/2** loop. Maintaining such resonance states provides a dynamical basis for thinking+memory.

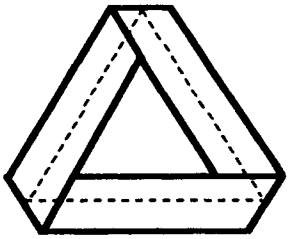
(Topo)logical charges are saved as knots where the logical **false** and **true** condense to **false±true**. Such a 'condensate' can be formed through two different mechanisms: externally or from within. The external condensation is induced by the logical differentiation

$$\{\perp, T\}\ |q\rangle = |s_{\pm}\rangle$$

which produces a nonorientable Möbius-**n/2**, left or right, $n \geq 3, 5, 7, \ldots$ Then a knot is created by means of the operation ***CUT***:

$$CUT\ |s_{\pm}\rangle = |K(n)\rangle$$

which is the closed cut, for example like the one shown below:



With a strip of paper and scissors you can observe the emergence of a knot from a nonorientable manifold.
The operation ***ZIP***, applied to a knot, smoothes out the defect, recovering the original manifold:

$$ZIP\ |K(n)\rangle = |s_{\pm}\rangle$$

In a dynamical system a resonance Möbius state can occur at the equilibrium

$$\{\perp, T\}\ |q\rangle = ZIP\ |K(n)\rangle,$$

where both sides must be synchronized, because logical differentiation can give different knots whilethe ***ZIP*** of a given knot yields only one logical theory. Synchronization amounts to counting the number of the half-twists **n** made prior to the closure of a logical membrane on itself. Depending on the value of **n** a logical condensate acquires different topological properties. In the resonance state between a nonorientable Möbius-**n/2** and the corresponding orientable knot $|K(n)\rangle$ memory and the thought process can be dynamically connected:

$$\textbf{MÖBIUS-n/2} \quad \overset{CUT}{\underset{ZIP}{\Leftrightarrow}} \quad \textbf{KNOT(n)}$$

One of the goals of the present work is to motivate and present a fully topological treatment of consciousness, which agrees with fundamental results of the quantization of matrix logic. In the standard metric quantization of quantum mechanics a classical quantity is coded into the metric of the phase space. Geometric quantization schemes may not explicitly use the metric, but it is nevertheless there. One needs a flat Klauder metric on the classical phase space, or at least on a copy of it, so that one can ascribe physical meaning to the mathematical expressions for quantized classical quantity. The geometric approach to quantization has provided some answers, although to date only a limited class of dynamical systems can be treated consistently with the results of quantum mechanics. The dependence of quantization on the metric and the choice of coordinates is clearly an unsettling state of affairs since nothing physical, like quantization, should depend on something as arbitrary as the choice of coordinates. The concept of topological charges remedies the defects of the geometric quantization scheme. The key idea of topological quantization

is the use of 2-manifolds to create knots on field lines. A field line is tied into a knot, acquiring a topological charge. Traditionally, a knot is defined as a subspace of the Euclidian space R_3 which is homeomorphic to the circle S_1 or a set of circles. To introduce a rigorous rule for logical quantization, instead of treating knots simply as 1-manifolds, we define them as the one-dimensional limit of the homeomorphic contraction of 2-manifolds with twists, subjected to closed ***CUT*** prior to contraction. The induced topological charge is explicitly linked to the Bohr quantization scheme for canonical coordinates

$$\oint pdq = 2\pi\hbar(n+\tfrac{1}{2})$$

where the integral corresponds to a closed contour in phase space at a constant energy value, and $n = 0, 1, \ldots$ is the principal quantum number.

It will be clear that one cannot create a topological charge by differentiating a line but needs a logical membrane, with **true** and **false** 'sides' that merge as result of logical differentiation. The concept of logical derivatives provides a natural link between logic and topology on one hand, and between logic and quantum formalism on the other. Further we will prove the key theorem which explicitly connects the traces of the logical derivative and the quantum-mechanical spin:

$$\mathbf{Tr}\ \Omega\frac{d}{dq} = -\mathbf{Tr}\,\rho\sigma$$

where Ω and ρ are the logical and quantum-mechanical density matrices respectively. In our theory the logical derivative enjoys a status similar to that of the quantum-mechanical derivative in quantum mechanics. Obtained by the differentiation, the nonorientable logical states **false**$\pm$**true**,

$$\begin{pmatrix}1\\1\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}1\\-1\end{pmatrix},$$

are strongly reminiscent of the coherent superposition states in quantum mechanics, and quantum coherent superpositions can actually be understood as topological knots. This re-interpretation puts the logical derivative in close correspondence with the quantum-mechanical derivative, which we consider next.

THE QUANTUM-MECHANICAL DERIVATIVE

The expectation value of a physical observable in the state $|\psi>$, represented by the operator Q, not depending explicitly on time, is

$$<Q> = <\psi|Q|\psi>$$

From here, differentiating with respect to time, we get

$$\frac{d<Q>}{dt} = <\frac{\partial\psi}{\partial t}|Q|\psi> + <\psi|Q|\frac{\partial\psi}{\partial t}\psi>$$

Making use of the Schrödinger equation

$$i\hbar \frac{\partial|\psi>}{\partial t} = H|\psi>$$

we derive

$$i\hbar \frac{d<Q>}{dt} = <\psi|\,[QH - HQ]\,|\psi> = <\psi|\,[Q, H]\,|\psi>$$

from which it follows that the time derivative of the expectation value of the operator Q corresponds, disregarding $i\hbar$, to the average of the following commutator operator

$$\frac{dQ}{dt} = [Q, H]$$

This is the quantum-mechanical derivative, which determines the change of the expectation value of physical observables with time. Its advantage is that it allows us to study the changes of physical quantities without concretization of the particular form of the wavefunction. For instance, the expectation value of a physical quantity will be constant in all states, if its quantum-mechanical derivative equals zero. The quantum-mechanical derivative is zero whenever the operator of the observable commutes with the Hamiltonian, and then the observable is a constant of motion.

The fact with important theoretical repercussions which must be emphasized is that in the same manner as the quantum-mechanical derivative identifies the quantum action:

$$[Q, H] = -i\hbar,$$

the (topo)logical derivative identifies the fundamental logical momentum:

$$[T, \perp] = -\frac{d}{dq}.$$

THE COVARIANT DERIVATIVE

We have used logic to develop matrix theory, and in this respect logic is more general. The fundamental matrix theorem, however, tells us that the laws of logical deduction are dictated by the laws of matrix algebra and in this respect matrix theory pushes logic into a subordinate position. How can logic, which gives rise to the idea of matrices, be a part of the matrix concept itself? Here we are reminded once again of the fundamental self-referential nature of consciousness. In physics we typically study a particle in some reference system: either we fix the reference system and consider a moving particle, or we *fix* the particle and let the reference system move. For self-referential consciousness this distinction cannot be easily done: here a system is simultaneously a reference system to itself; both aspects are inseparable. **It is not possible to fix one and allow the other to evolve.** Both change together as integral whole. Situations of this kind show up in the interaction picture in quantum mechanics and in the formalism of covariant differentiation.

Consider a vector field $\mathbf{V}(\mathbf{r})$ with components V_1, V_2, V_3 in a system of Cartesian coordinates with orthonormal basis $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$. Then in terms of the differentials of the components the differential of $\mathbf{V}(\mathbf{r})$ equals

$$d\mathbf{V} = d(V_j, \mathbf{i}_j) = \mathbf{i}_j\, dV_j$$

where $d\mathbf{i}_j = 0$ since the basis $\mathbf{i}_1, \mathbf{i}_2, \mathbf{i}_3$ does not vary from point to point in a Cartesian coordinate system. Similarly, let a vector $\mathbf{V}$ have covariant components V_1, V_2, V_3 and contravariant components V^1, V^2, V^3 in a system of oblique coordinates with basis $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$. Then

$$d\mathbf{V} = d(V_j, \mathbf{e}^j) = \mathbf{e}^j\, dV_j, \qquad d\mathbf{V} = d(V^j, \mathbf{e}_j) = \mathbf{e}_j\, dV^j$$

where now $d\mathbf{e}_j = d\mathbf{e}^j = 0$ since the bases $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3$ and $\mathbf{e}^1, \mathbf{e}^2, \mathbf{e}^3$ do not vary from point to point in an oblique coordinate system.

But in a system of generalized coordinates the basis is local and basis vectors are vector functions of coordinates

$$\mathbf{e}_j = \mathbf{e}_j(x^1, x^2, x^3), \qquad \mathbf{e}^j = \mathbf{e}^j(x^1, x^2, x^3)$$

It follows that

$$d\mathbf{V} = d(V_j, \mathbf{e}^j) = \mathbf{e}^j\, dV_j + V_j\, d\mathbf{e}^j, \qquad d\mathbf{V} = d(V^j, \mathbf{e}_j) = \mathbf{e}_j\, dV^j + V^j\, d\mathbf{e}_j$$

The differential $d\mathbf{V}$ now has two terms: besides a term describing the change of the components of the vector, it also contains a term $V_j\, d\mathbf{e}^j$ or $V^j\, d\mathbf{e}_j$, stemming from the fact that the basis of the generalized coordinates also varies from point to point.

Since

$$d\mathbf{V} = \frac{\partial \mathbf{V}}{\partial x^k}\, dx^k$$

the partial derivatives of the vector $\mathbf{V}$ with respect to the generalized coordinates x^k will be

$$\frac{\partial \mathbf{V}}{\partial x^k} = \frac{\partial V_j}{\partial x^k}\mathbf{e}^j + V_j \frac{\partial \mathbf{e}^j}{\partial x^k} = \frac{\partial V^j}{\partial x^k}\mathbf{e}_j + V^j \frac{\partial \mathbf{e}_j}{\partial x^k}$$

where the components of the vector $\partial \mathbf{V}/\partial x^k$ $(k = 1, 2, 3)$ are themselves the components of a second-rank tensor called the covariant derivative of the given vector, covariant or contravariant. Thus, the covariant derivative of the covariant vector has components

$$\frac{\partial \mathbf{V}}{\partial x^k}\,\mathbf{e}_i \equiv V_{i,k}$$

while the covariant derivative of the contravariant vector has components

$$\frac{\partial \mathbf{V}}{\partial x^k}\,\mathbf{e}^i \equiv V^i_k$$

With the aid of the Christoffel symbols we can write

$$V_{i,k} = \frac{\partial V_i}{\partial x^k} - \left\{ \begin{matrix} j \\ i\ k \end{matrix} \right\} V_j, \qquad V^i_k = \frac{\partial V^i}{\partial x^k} + \left\{ \begin{matrix} i \\ j\ k \end{matrix} \right\} V^j.$$

Observe that we have a minus sign if the dummy index is a covariant index and a plus sign if it is a contravariant index. It follows that the covariant derivative of a vector field involves not only the rate of change of the field itself, as we move along the field lines but also the rate of change of the local basis. If the

basis does not vary, all the Christoffel symbols vanish, and the covariant derivatives reduce simply to

$$V_{i,k} = \frac{\partial V_i}{\partial x^k}, \qquad V^i_k = \frac{\partial V^i}{\partial x^k}.$$

Because the vanishing terms are due entirely to a local basis which varies from point to point, it is possible to express the Christoffel symbols in terms of the derivative of the components of the metric tensor. The covariant derivative of the metric tensor vanishes; hence the components of the metric tensor can be regarded as constants under covariant differentiation. It can be shown that the covariant derivative of a tensor of rank n is a tensor of rank n+1. In the case of a scalar, which is a tensor of zero rank, the covariant derivative reduces to the partial derivative with respect to the coordinate. Thus the covariant derivative of a scalar is a covariant vector, with components equal to the covariant components of the gradient.

THE UNBROKEN TOPOLOGICAL PHASE

We now introduce the concept of the unbroken topological phase, which has a fundamental connection to coding theory, specifically to the Hadamard codes. Given an orientable surface, we identify its two sides with the two normal vectors

$$\begin{pmatrix}0\\1\end{pmatrix} \text{ and } \begin{pmatrix}1\\0\end{pmatrix},$$

that are orthogonal

$$(1\ 0)\begin{pmatrix}0\\1\end{pmatrix} = 0.$$

For a nonorientable surface, such as a Möbius strip, the two sides join in one,

$$\begin{pmatrix}1\\0\end{pmatrix} \pm \begin{pmatrix}0\\1\end{pmatrix},$$

and depending on the direction of the twist, we obtain the right Möbius or the left Möbius, respectively

$$\begin{pmatrix}1\\1\end{pmatrix} \qquad \text{and} \qquad \begin{pmatrix}1\\-1\end{pmatrix}.$$

which also are orthogonal mirror images of each other:

$$(1\ 1)\begin{pmatrix}1\\-1\end{pmatrix} = 0.$$

Definition.

The unbroken topological phase H(·) is a supersymmetric 4-dimensional manifold, containing the left and right Möbius jointly and is described as the Cartesian direct sum

$$\begin{pmatrix}1\\-1\end{pmatrix} \oplus \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1&1\\-1&1\end{pmatrix}$$

The position of the negative matrix element is arbitrary and can be chosen differently. We see that the unbroken topological phase is represented by any of the Hadamardians H., $\dot{H}$, .H, $H^{\cdot}$. These can be extended in two distinct formats

$$\mathbf{I \pm TIME} \qquad \text{and} \qquad \mathbf{N \pm SPIN}$$

which carry different quark logical charges

$$\mu(\mathbf{I} \pm \blacktriangle) = \tfrac{2}{3} \qquad \text{and} \qquad \mu(\mathbf{N} \pm \sigma) = \tfrac{1}{3}.$$

Bringing a Hadamard matrix into normal form, with the 1st row and 1st column strictly positive, we observe that it has the same sign signature as a relativistic spacetime interval:

$$\begin{pmatrix} + & + \\ + & - \end{pmatrix} \Rightarrow (+ \ + \ + \ -)$$

With an appropriately chosen signature the unbroken topological phase can be written as the sum of the real and imaginary units, which is the complex number

$$1 \pm i.$$

The Hadamard codes are important for the thought process and were discussed early [Ref 91]. The three-term exponentiation of the nonsymmetric Hadamard code .H is empty

$$\mathbf{I} - .H + \frac{.H^2}{2!} = \mathbf{0},$$

but the two-term exponentiation gives the comparison operator. It follows that topological phase determines the time coordinate, and the decay of the phase produces two commuting logical energies **H** and **1**,

$$[\mathbf{H}, \mathbf{1}] = \mathbf{0},$$

which are condensed remnants of the unbroken phase. The corresponding energy subspaces are dual

$$\mathbf{H} \bullet \mathbf{1} = \mathbf{0}.$$

Since the unbroken topological phase is nonunitary:

$$H(\cdot)^{+} \neq H(\cdot)^{-1},$$

it would not obey the law of quantum evolution unless it is brought into unitary format. In this model quantization is an emergent phenomenon, with the laws of topology giving rise to the laws of quantum mechanics. The unitarization of the topological phase creates the condition for unitary evolution and quantum fields. To add a quantum dimension to topology, the quantum factor $1/\sqrt{2}$ is required. Then the unbroken phase is readily converted into an unitary operator,

$$\frac{1}{\sqrt{2}} H(\cdot)^{+} \frac{1}{\sqrt{2}} H(\cdot) = \mathbf{I}$$

and coherent quantum superpositions evolve naturally:

$$\frac{1}{\sqrt{2}} H(\cdot)|\psi> = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} \psi(\uparrow) \\ \psi(\downarrow) \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \psi(\uparrow) + \psi(\downarrow) \\ \psi(\uparrow) - \psi(\downarrow) \end{pmatrix}.$$

This applies to all Hadamardians, which can be naturally extended and transformed in the spin basis:

$$\frac{1}{\sqrt{2}} H. = \frac{1}{\sqrt{2}} (\sigma_x + \sigma_z), \qquad \frac{1}{\sqrt{2}} \dot{H} = \frac{1}{\sqrt{2}} (\sigma_x - \sigma_z)$$

$$\frac{1}{\sqrt{2}} .H = \frac{1}{\sqrt{2}} (\sigma_0 + i\sigma_y), \qquad \frac{1}{\sqrt{2}} \dot{H} = \frac{1}{\sqrt{2}} (\sigma_0 - i\sigma_y)$$

RECASTING LOGIC IN DIRAC NOTATION

TRANSLATING LOGIC INTO QUANTUM MECHANICS

The introduction of the Dirac bra-/ket-notation in matrix logic has brought together the physical and logical methods. We are moving more and more from the realm of speculation to the realm of computation. The matrix theory has not only confirmed all the results of classical logic, it has also opened up a wide range of new computational possibilities. With the matrix operator formulation came the new understanding of the fundamental link between logic and quantum physics. Matrix logic continues to attract growing attention due to its wealth of logical structures and promises to resolve the fundamental question of consciousness scientifically.

The matrix principle reveals that the operation of matrix multiplication lies at the foundation of logical thinking. Matrix logic consequently offers a realistic opportunity to unlock the mystery of the intelligence code, exploring the well-developed and powerful apparatus of matrix calculus. This project takes on particular significance because fundamental and beautiful connections are known to exist between the theory of matrices and quantum mechanics, and one is naturally motivated to seek the links between logic and quantum theory, where a different kind of intuition exists. In this section my purpose is to outline a relationship between the logical structure of quantum mechanics and the logical structure of thoughts. This is an important but brief section, as the use of the logical kets and bras, first conceived by the author and systematically formulated in 1980s, pervades this monograph and previous publications. There are two points which need to be emphasized here, which concern the differences between Dirac's notation and matrix logic. Dirac's

formalism offers a concise, consistent vector shorthand ideally suited to the needs of quantum mechanics. Dirac postulated the dual bra and ket spaces where the scalar product is linear in the postfactor but antilinear in the prefactor. The physics dictates that the change between the dual spaces requires not simply transposition but essentially the complex conjugation

$$c^*\langle\psi| \Leftrightarrow c|\psi\rangle.$$

This constraint is no longer relevant for logic, where the factors are real,

$$\langle p|q\rangle = \langle q|p\rangle,$$

and whatever information is held within a logical vector, the same information will be found in its transpose. For discussion of a logical problem, $\langle q|$ and $|q\rangle$ are equally informative, and neither is more important or fundamental than the other. The second point. To avoid confusion between logicians and physicists, the introduction of the Dirac notation in logic requires that we abandon the Shiefer stroke "|", the symbol a logician uses for the **NAND** operator, because Dirac's stroke has different meaning; it is, in fact, the unit operator:

$$\langle p|q\rangle \equiv \langle q|I|p\rangle.$$

A fundamental parallel can be drawn between differential and matrix operator logics and wave mechanics and matrix mechanics. We now illustrate the power of the new logical notation by considering two logical descriptions, analogous to the Schrödinger and the Heisenberg pictures. In the Heisenberg picture we have a dynamical variable represented by the quantum operator **A**, satisfying the Heisenberg equation of motion

$$i\hbar\frac{dA}{dt} = [A, H].$$

In the Schrödinger picture we employ the state vectors $|\psi\rangle$, satisfying the Schrödinger equation of motion

$$i\hbar\frac{d|\psi\rangle}{dt} = H|\psi\rangle,$$

where the Hamiltonian H is the same in both descriptions, since these are physically equivalent. In the Heisenberg picture a state vector is steady but dynamical variables evolve with time, while in the Schrödinger picture it is the state vector which evolves with time but the dynamical variables remain unchanged,

$$i\hbar\frac{d|\psi_H\rangle}{dt} = 0, \qquad i\hbar\frac{dA_S}{dt} = 0.$$

The states and operators of both pictures are linked by the unitary transformations

$$\psi_S = U\psi_H, \qquad U = e^{-\frac{i}{\hbar}Ht}, \qquad A_S = U^*A_HU = e^{-\frac{i}{\hbar}Ht}A_H e^{\frac{i}{\hbar}Ht},$$

and necessarily

$$\langle A_S\rangle = \langle A_H\rangle.$$

Bearing in mind that in quantum mechanics the formalism of the expectation value is the basis of all comparison between theoretical prediction and experiment, and that expectations should not depend on particular picture, we now take our case to logic. Since truth is not dependent on time, there is no explicit dependence on time in logic. To expand logical thinking in the two pictures, it is evident that time has to be excluded from the system and a dependence on truth variables introduced instead. A logical state in the Schrödinger picture is a unitary negation of a state in the Heisenberg picture

$$|\bar{q}> = \mathbf{NOT}|q>$$

where **NOT** is the logical evolution operator [Ref 91]. Indicating the different logical pictures by subscripts, we can write

$$|q_S> = \mathbf{NOT}|q_H>$$

and

$$\mathbf{L}_S = \theta \mathbf{L}_H = \mathbf{NOT} \cdot \mathbf{L}_H \cdot \mathbf{NOT}$$

where **L** is a logical operator.
Similar to

$$<\psi|U^*U|\psi> = <\psi|\psi>,$$

with a changeover of the logical pictures the scalar product is conserved:

$$<q_H|q_H> = <q_S|\,\mathbf{NOT} \cdot \mathbf{NOT}|q_S> = <q_S|q_S>$$

and the truth-value average remains unaltered:

$$<q_H \mid \mathbf{L}_H \mid q_H> = <q_H \mid \theta \mathbf{L}_S | q_H> = <q_H \mid \mathbf{NOT} \cdot \mathbf{L}_S \cdot \mathbf{NOT} | q_H>$$
$$= <q_S \mathbf{NOT}| \,\mathbf{NOT} \cdot \mathbf{L}_S \cdot \mathbf{NOT}|\mathbf{NOT} q_S>$$
$$= <q_S|\,\mathbf{L}_S|q_S>.$$

We observe that the Schrödinger and Heisenberg logical pictures are equivalent and, executed jointly, one undoes the unitary rotation of the other. Exploring this fact, one derives the fundamental contraposition inference rule:

$$< p|\,\mathbf{IMPLY}\,|q> = <\bar{p}\,|\,\mathbf{IF}\,|\,\bar{q}> = <\bar{q}\,|\mathbf{IMPLY}\,|\,\bar{p}>$$

where the implication operator is respectively in the two pictures:

$$\mathbf{L}_S = \mathbf{IMPLY} \quad \text{and} \quad \mathbf{L}_H = \mathbf{IF}.$$

Because a logical state either remains unaltered or negated:

$$|q> = \mathbf{YES}\,|q>, \qquad |\bar{q}> = \mathbf{NOT}|q>,$$

to take account of both options we introduce two fundamental operators:

$$\mathbf{H} = \mathbf{YES} - \mathbf{NOT}, \qquad \mathbf{M} = \mathbf{FALSE} - \mathbf{TRUE}.$$

We now have all the tools needed to formulate two logical descriptions in analogy to the Schrödinger and Heisenberg pictures. In one picture the logical state is steady and the operator is in motion, while in another the logical operator is steady and the state evolves. The pair of equations, defining the Schrödinger logical picture then:

$$-\frac{d|q>}{dq} = (\mathbf{FALSE} - \mathbf{TRUE})|q>, \qquad \frac{d\,\mathbf{IF}}{dq} = \mathbf{0}$$

In the Heisenberg logical picture the state is steady and the operator evolves:

$$\frac{d|q>}{dq} = 0, \qquad \frac{d<\mathbf{IMPLY}>}{dq} = <[\mathbf{IMPLY}, \mathbf{H}]>$$

where the operator **IMPLY** is unitarily transformed into the operator **IF**, and **M= FALSE-TRUE** is modified into **H=YES - NOT.**

To fully appreciate the significance of these results, we must make a conceptual and mathematical journey that is far from trivial. At this stage it perhaps may be clear that Dirac's formalism can be naturally transformed into a logical calculus and vice versa, and that the Schrödinger and Heisenberg pictures have fundamental relevance for the thought process. It should not go unnoticed that the Schrödinger and Heisenberg logical pictures are only equivalent to a degree: the inner product and the averages are conserved, but we have to modify **FALSE-TRUE** into **YES-NOT**, which are logically close but very different computationally. Nonetheless, later we will prove that these two operators both belong to the class of logical energy operators and that in the Boolean limit their averages coincide by modulo, making both pictures fully equivalent.

In the course of this study we will also determine that the quantum commutator is an optimal quantum-logical code and that the differentiation operator and the quantum brackets are new fundamental logical operations. Although quantum operations are involved in the thought processes, the logic which we commonly use for communication is primarily classical Boolean logic. We are far from learning to explicitly think quantum-mechanically. A traditional logician, to whom logic is a formal structure and not a dynamical process, may wish to exclude the logical derivatives from the equations. Since logical differentiation is a fundamental quantum-logical operation, this is not a wise move, but one which enables us to combine the two pictures into a logical relation which is purely structural:

$$<(\mathbf{FALSE}-\mathbf{TRUE})> = <[(\mathbf{YES}-\mathbf{NOT}),\mathbf{IMPLY}]>$$

although

$$\mathbf{FALSE}-\mathbf{TRUE} \neq [(\mathbf{YES}-\mathbf{NOT}),\mathbf{IMPLY}]$$

where

$$[(\mathbf{YES}-\mathbf{NOT}),\mathbf{IMPLY}] = \mathbf{NOT}\bullet\mathbf{AND}\bullet\mathbf{NOT}-\mathbf{AND}$$

but

$$\mathbf{FALSE}-\mathbf{TRUE} \neq \mathbf{NOT}\bullet\mathbf{AND}\bullet\mathbf{NOT}-\mathbf{AND}$$

However, in averages one regains the equality:

$$<(\mathbf{FALSE}-\mathbf{TRUE})> = <(\theta\mathbf{AND}-\mathbf{AND})>,$$

which is destined to play a fundamental role in the theory of the thought mechanism developed below.

Recasting logic in Dirac's notation and translating quantum mechanics into a logical calculus provide a new theoretical framework which in the end must lead to what has been defined as **experimental logic** [Ref 89]. Matrix algebra is a powerful technique for solving linear partial equations, control theory equations, signal processing problems, the minimax problem and many other problems in mathematics, physics, economics, etc. Apparently, any problem involving a set of independent multilevel computations of the

same type can be viewed as a matrix-matrix or matrix-vector multiplication problem. Many important computations such as convolution, Fourier transforms and the finite response filter are special instances of matrix-vector multiplications. The convolution problem can be represented as an inner product computation which is common to a number of computational routines, such as filtering, pattern matching, correlation, interpolation, polynomial multiplication and division.

Forging links between logic and matrix algebra, we develop a formalism with new deductive and inductive power and far-reaching implications for the ways logical inferences are achieved in logic. The matrix logic is not just another exotic calculus, but explores quantum algorithms not available classically. Although logic forms the core of the intelligent mind, our knowledge of high-level algorithms is still poor. The only algorithms we fully understand and manage confidently are the algorithms of classical computation, and these alone are in their present form insufficient to handle high-level cognitive functions such as abstraction, induction, intuition and creativity.

The study of intelligence has hitherto been carried out by developing and investigating different one-dimensional scalar logics. To understand high-level intelligence we requires not simply a modification of scalar logic but an entirely new dimensional generalization. Taking full step in maths and physics, matrix logic does precisely that, extending the power of computation to the domain of logical consciousness.

PARALLEL AND SERIAL UNIVERSES

FACTORIZING THE THOUGHT PROCESSES

George Boole hardly had it in mind but the involutive rule for logical addition

$$1 + 1 = 1$$

which he postulated hides within itself an unnoticed suprasymmetry, reflected in the degeneracy of algebraic logical operations with respect to Boolean addition and multiplication. Boolean logic algebra provides, albeit in an awkward way, for the equivalence of logical addition and product. Indeed,

nothing prevents us from writing the involutive equality differently, multiplying the right side by 1:

$$1 + 1 = 1 \bullet 1.$$

Then, since for the logical zero evidently

$$0 + 0 = 0 \bullet 0,$$

for any Boolean variable in general we have the suprasymmetry:

$$x + x = x \bullet x.$$

In terms of Boolean logical connectives this is written symbolically as:

$$x \vee x \equiv x \wedge x,$$

which reflects the fact of the cancellation of the off-diagonal logical elements.

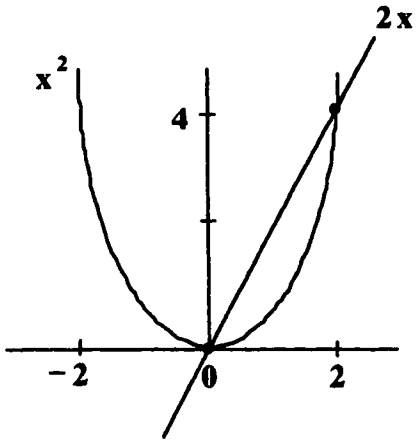
Suppose now that we abandon the unnatural Boolean rule for logical addition, and let the x behave as ordinary number variable. Then

$$x + x = 2x, \qquad x \bullet x = x^2,$$

and, if one wishes to retain suprasymmetry, one must have:

$$2x = x^2.$$

Now, instead of being satisfied at the points $x = 0$ and 1, the equality is satisfied for $x = 0$ and 2, the two points where the functions $2x$ and x^2 intersect:



Since that $2x$ is the derivative of x^2, now to satisfy suprasymmetry we must have a function which equals its own derivative:

$$f'(x) = f(x).$$

Since such a property is the characteristic feature of a wave represented by an exponent, the concept of a wave once again becomes critical to our analysis.

According to the reduction theorem of matrix logic any sufficiently complex logical thought is not factorable, as some trivial intellects might be. There are fundamental reasons which are responsible for this state of affairs. Abstract thinking relies heavily on the interaction of logical connective

operators. In field theory if the system of quantum particles $a, b, c,...$ is without interaction, then the total wavefunction is the product of individual wavefunctions:

$$\psi(a, b, c...) = \psi(a)\psi(b)\psi(c)...$$

But as soon as the interaction between the particles is switched on,

$$\psi(a, b, c...) \neq \psi(a)\psi(b)\psi(c)...$$

Whenever a wavefunction of a compound quantum system cannot be factorized into a product of the wavefunctions of its constituents, we say that the quantum system is entangled. Typically the effect of entanglement is associated with quantum-mechanical objects. Although it manifests itself most prominently in the quantum realm, there are no fundamental reasons to consider entanglement to be exclusively quantum-mechanical phenomenon, and the reduction theorem of matrix logic underscores the essential role of entanglement in logic [Ref 94]. It determines explicitly that any nontrivial logical thought is an entanglement, hence a knot, for which the cognitive matrix cannot be factorized into product matrices of elemental logical states:

$$\mathbf{L}(x_1, x_2,..., x_n, y_1, y_2,..., y_n) \neq \prod_{i=1}^{n} \mathbf{L}(x_i, y_i)$$

As with a general quantum process the entanglement among the logical states prevents, in general, the cognitive matrix from being serially factorized. This effect, considered in the context of computer science, can be recast in terms of serial and parallel logics. Factorizable logic is naturally accommodated by a serial transfer function, while distributive systems such as the brain or a supercomputer require parallel transfer functions.

PARALLEL BRAIN, SERIAL MIND

In almost every sense of the word, serial and parallel operations are thought of as incompatible opposites. It is also commonly assumed that the mind is based on operations that are sequential in nature, while the brain is a parallel distributed system, which introduces a major stumbling block when we approach the brain-mind as an integral whole. Are the serial and the parallel necessarily mutually exclusive? Because the brain is parallel system while the mind is serial one, the fundamental question arises of explaining this parallel-serial paradigm. My purpose now is to show that in matrix logic the automatic division into parallel and serial operations, to which we all have grown accustomed under the influence of computer science, can be replaced by parallel-serial duality, where parallel and serial are not competing and mutually exclusive modes of operations but inclusive components of the whole.

This approach is closely linked with the field of quantum physics. There was a time when we erroneously believed that waves and particles were mutually exclusive, but the wave-particle duality of quantum mechanics proved us wrong. At the dawn of modern physics, Albert Einstein had demonstrated

that waves can behave as particles, while in reverse Louis de Broglie associated a wave with every particle. Depending on the mode of observation an electron will display the properties either of a particle or of a wave, revealing in a quite spectacular way the limits of our intuition.

In an attempt to formulate an adequate scientific brain-mind theory we are inevitably drawn to the fundamental problem of quantum measurement, which introduces a measuring device and an observer as an integral part of the description of reality. Prior to measurement, the wavefunction of a quantum system is the coherent superposition of all possible states. The wavefunction of the observer is initially single, but splits into a great number of mutually orthogonal packets. The observer cannot decide which is the correct value of the system observable, and would have to be supplemented by a second observer to observe the first one; the same is true for the second observer who cannot decide, and so must be supplemented by a third; and so on. This catastrophe of infinite regression vividly exposes the paradox of the quantum theory of measurement. One cannot make predictions about the whole universe, because the universe must contain *all* the observers. Until the observer performs a measurement the world is in continual flux and no certainty about physical states is possible. To a second observer this system 'wavefunction+first observer' appears to be in similar fluctuation, even when the first observer carries out the measurement. The second observer is convinced that the first observer is in continual fluctuation, splitting into many copies of himself, even though he is making an observation. Only when the second observer performs, must the first observer again become singular.

Various proposals to escape from this infinite regress can be classified in computational terms as *serial*, which includes the Copenhagen interpretation, Bohm's hidden variable proposal etc., and *parallel*, due to Everett. According to Niels Bohr's original proposal the act of measurement selects a particular state, the eigenstate, out of the initial superposition. We cannot predict which state will be selected, but there is a probability distribution of possible outcomes. David Bohm's hidden variable approach reintroduces determinism: the indeterminable quantities are defined, but to conform to the predictions of quantum mechanics all such formalisms necessarily have a large-scale nonlocality built into them. For many years the search for hidden variables has been carried out, but none has been found. There are those who believe that the mind is the only hidden variable. A proposal was in fact made by Eugene Wigner that the infinite regress could be arrested by the intervention of the mind. This speculation was the first constructive appearance of mind in modern physics, accompanied by a mathematical description based on nonlinear departures from the Schrödinger equation when consciousness intervenes.

In a bold move, ignored for a long time by mainstream physicists, Everett proposed the many-world interpretation of quantum theory. This, to use our terminology, parallel model attempts to deny the collapse of the state vector, and takes the mathematical formalism of quantum mechanics as it was

originally presented. Many-world interpretation forces us to believe in the reality of all the simultaneous universes represented in quantum superposition. These universes cannot communicate physically with each other, because the respective state vectors are mutually orthogonal. Each time an observed quantum transition occurs, one universe splits into many parallel universes. Moreover, it is inherent in the many-world theory that every quantum transition, resulting from measurement-like interaction anywhere, constantly splits our local world into myriads of copies of itself. In the many-world formulation, the wavefunction does not collapse at the moment of a transition; rather, it splits into a large number of wavefunctions, each differing from the others in its energy. The observer in a particular universe is capable of measuring this energy, but another observer, in a parallel universe, would measure a different energy. Because it is purely a matter of chance which universe the observer is in, the particular result appears to be random. However, if the observation is repeated a sufficient number of times, then the result is predictable by the wave equation, since all universes are equally probable. The collapse of the state vector is avoided by postulating that within a single universe only initial and final states are relevant, the superposition having no reality unless a complete set of universes is taken into account.

THE SUPRASYMMETRIC TRANSFER FUNCTION

Hitherto serial and parallel logics have been considered to be separate and entirely different, just as in the past Schrödinger and Heisenberg quantum mechanics were thought to be different until their equivalence was established. Suppose we have a logical system residing in an eigenstate $|q\rangle$ and that we repeat energy measurement $\mathbf{n}$ times, each time obtaining the same value, corresponding to the eigenvalue of the **Lagrangian** $\mathbf{H}$. Acting serially on the eigenstate $|q\rangle$, the operator $\mathbf{H}$ expands it λ times, hence

$$\mathbf{H}\bullet\mathbf{H}\bullet\mathbf{H}\bullet \ldots\ldots\bullet\mathbf{H}\,|q\rangle = \lambda^{n}\,|q\rangle$$

In analogy to the many-world interpretation we can view this transition differently, inserting each $\mathbf{H}$ to its own parallel universe, and then consider the operator sum of such action:

$$(\mathbf{H}+\mathbf{H}+\mathbf{H}\ldots\ldots+\mathbf{H})|q\rangle.$$

The question which we now pose is the following. Can we envisage the equivalence of such serial and parallel actions? Is there an energy operator satisfying the equality

$$\mathbf{H}\bullet\mathbf{H}\bullet \ldots |q\rangle = (\mathbf{H}+\mathbf{H}+ \ldots)\,|q\rangle\ ?$$

The suprasymmetry of the **Lagrangian** entails

$$\mathbf{H}_1 \bullet \mathbf{H}_2 \bullet \ \ldots\ \bullet \mathbf{H}_n = \mathbf{H}_1 + \mathbf{H}_2 + \ \ldots\ + \mathbf{H}_{2^n}$$

which leads to

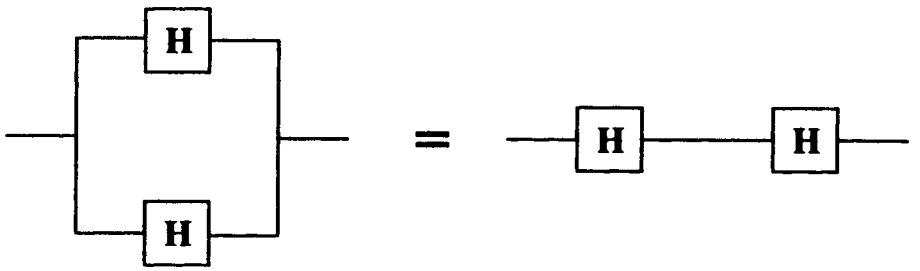
$$\mathbf{H}^n = 2^{n-1}\mathbf{H}$$

But for n = 2, we have $\mathbf{H}^2 = 2\mathbf{H}$, which is the suprasymmetry we are searching for:

$$\mathbf{H} \bullet \mathbf{H} = \mathbf{H} + \mathbf{H}.$$

A similar suprasymmetry can be established for the logical **Hamiltonian 1**, which is the spectral twin of **H**:

$$\mathbf{1}^2 = 2\mathbf{1}.$$



H + H **H • H**

Fig. 13
The equivalence of the parallel and serial transfer functions.

We have found the matrix logical functions for which the rate of change of the squared function equals the function itself:

$$\frac{d\mathbf{L}^2}{d\mathbf{L}} = 2\mathbf{L} = \mathbf{L}^2.$$

The operator derivative of a squared logical energy operator leaves it unchanged. The parallel mode is derivative of the serial, and equal serial. For a pair of identical elements the energy transfer function will not discriminate between the parallel and serial modes of operations. The parallel-serial duality in matrix logic stands in direct analogy to the wave-particle duality in quantum mechanics: a wave is clearly a parallel phenomenon, whereas a particle is naturally treated as serial one.

The seeds of suprasymmetry can already be found in Boolean logic. First of all, as opposed to the single distributive law of ordinary algebra, Boolean logic has two distributive laws, which invariantly swap the dual connectives **AND** and **OR**:

$$\mathbf{x} \wedge (\mathbf{y} \vee \mathbf{z}) = (\mathbf{x} \wedge \mathbf{y}) \vee (\mathbf{x} \wedge \mathbf{z})$$
$$\mathbf{x} \vee (\mathbf{y} \wedge \mathbf{z}) = (\mathbf{x} \vee \mathbf{y}) \wedge (\mathbf{x} \vee \mathbf{z})$$

This duality exchange is a suprasymmetry imposed in order to satisfy the laws of logical inference. The operations **AND** (logical product) and **OR** (logical addition) are changed over as we pass from one distributive law to another.

The scalar suprasymmetry $x^2 = 2x$ is not been stressed out, but is implicit. Introducing the operator derivative d/dH, we have a system with the equality

$$f'(H) = f(H)$$

However, the points 0 and 2 that solve the equation $2x = x^2$ are now found as the eigenvalues

$$H|q\rangle = \lambda|q\rangle, \qquad \lambda_{1,2} = 0, 2.$$

Beginning with the generalization of the Boolean postulate we have ended with an important new rule, according to which logical processes rely on a mechanism that does not distinguish between the parallel and the serial, casting new light on the distributive structure of the brain and the serial structure of the thought process. The brain-mind system is neither serial nor parallel, but both.

OPERATOR WAVES

Wave is a key physical concept next to the concept of a particle. It is central to physical theory, from ordinary water and sound waves to the electromagnetic field, De Broglie's waves and the abstract waves of probability epitomized by wavefunction. The wave-particle duality in quantum mechanics entails that elementary particles, such as electrons, have wave properties. Moreover, quantum mechanics was initially conceived as wave mechanics of a particle. A wavefunction is a wave of probability, not an actual wave, although some physicists were and are still open to the possibility that quantum descriptions may apply realistically. There is a whole field of physics in which macroscopic quantum effects occur. Superfluids and superconductors exhibit macroquantum properties. In a Bose-Einstein condensate, where each atom is in phase with the others, quantum waves extend across the condensate and can be observed. The microscopic can become macroscopic. However, strict quantum-mechanical phase coherence is required for the wavefunction to be assembled coherently into a single macroscopic 'wavefunction'. An attractive possibility in topologically closed classical+quantum systems, then, is a continual fusion-fission process: the fusion of microscopic phases into a macroscopic phase, and its subsequent fission into microscopic ones.

As we mentioned earlier, the notion of a wave is central to physics. Although logic is an unlikely matter to think in terms of waves, amplitudes, phases or energies, the feeling has persisted for a long time that some unknown wavelike quantity may be relevant for fundamental logical theory. The description of waves requires the exponent function, which is distinguished from other analytical functions by the fact that its derivative is the function itself:

$$\frac{d}{dx}e^x = e^x$$

The analysis of parallel and serial logics reveals that this distinguishing property of the exponent, $f'(x) = f(x)$, is not unique. Making use of the operator derivative, we find instances when

$$f'(\mathbf{L}) = f(\mathbf{L}).$$

This is the consequence of the serial-parallel duality concerning the logical synergy functions, having the property that the logical product equals the sum:

$$(\mathbf{YES} \pm \mathbf{NOT})(\mathbf{YES} \pm \mathbf{NOT}) = (\mathbf{YES} \pm \mathbf{NOT}) + (\mathbf{NOT} \pm \mathbf{YES}),$$

the proof of which is most transparent in a purely logical format. Write the operators as an algebraic sum of **YES** and **NOT**, and carry out the direct calculation:

$$(\mathbf{YES} \pm \mathbf{NOT})(\mathbf{YES} \pm \mathbf{NOT}) = \mathbf{YES} \pm \mathbf{NOT} \pm \mathbf{NOT} + \mathbf{YES}$$

$$= (\mathbf{YES} \pm \mathbf{NOT}) + (\mathbf{YES} \pm \mathbf{NOT})$$

which is reduced to either

$$\mathbf{1}^2 = \mathbf{21}$$

or

$$\mathbf{H}^2 = \mathbf{2H}.$$

Now consider the operator derivative

$$\frac{d}{d\mathbf{H}}\mathbf{H}^2 = \mathbf{2H},$$

but since $\mathbf{2H} = \mathbf{H}^2$ this can be re-written as

$$\frac{d}{d\mathbf{H}}\mathbf{H}^2 = \mathbf{H}^2$$

The squared logical synergy functions behave as wave exponents, preserved under operator differentiation. However, such invariance does not automatically entail the equivalence of parallel and serial actions. Consider

$$\frac{d}{d\vee} \ln \vee = \ln \vee$$

which behaves, of course in linear terms only, as the exponent under differentiation, but does not follow the rule of serial-parallel duality:

$$\ln \vee \bullet \ln \vee \neq \ln \vee + \ln \vee.$$

The fact that the synergy operators are suprasymmetric simplifies the solution of the eigenvalue problem:

$$\mathbf{H}|q\rangle = \lambda|q\rangle \Rightarrow \mathbf{HH}|q\rangle = \mathbf{H}\lambda|q\rangle \Rightarrow 2\mathbf{H}|q\rangle = \lambda\mathbf{H}|q\rangle \Rightarrow \lambda = 2.$$

LOGICAL WAVE OPERATORS

The logical operator waves, being invariant under differentiation, are important for treating the stream of consciousness as a topological current, directly linked to Matrix Statistics,

$$\mathbf{TAUTOLOGY} = e^{\pm\Phi}.$$

In relation to pure structural logic the operator waves stand as wave mechanics stands to matrix mechanics which are equivalent. This is not necessarily the case in matrix logic when we change over from **L** to **W(L) = exp(-L)**. New fundamental logical relations, unknown in static logic, become manifest. Of particular importance among these is a connection between logical wave operators and the conversion principle. To see that, we must understand that an operator wave generally comprises an integer *logical* part and fractional *physical* part:

operator wave = logical part + physical part

Here lies the germ of the idea. We can imagine a situation in which one or the other part is zero. Then if the noninteger part vanishes, the exponential gives a conversion from quantum field to logic. If the logical part vanishes, we have a reverse conversion. The two key theorems hold:

(1) The matrix operator wave of the identity ('I', **IS, YES**) has no logical part, providing a translation from logic to the physical realm:

$$e^{-IS} = \frac{I^2}{2!} - \frac{I^2}{3!} + \cdots$$

(2) The matrix operator waves of the annihilation and creation operators do not have a physical part, providing a translation from quantum physics to logic:

$$e^{a} = \rightarrow, \qquad e^{a^*} = \leftarrow.$$

The operator wave of 'I' is a purely physical infinite converging series, while the exponential waves of a and a^* are purely logical and finitely halt.

In wave logic the structural DeMorgan equality

$$\overline{<\bar{p}|\wedge|\bar{q}>} = <p|\vee|q>$$

is replaced by

$$\overline{<p|e^{-\wedge}|q>} = <p|e^{\vee^{-1}}|q>,$$

where one benefits from the fact that the operator wave of **AND** is the orthogonal transformation:

$$\theta_\wedge = e^{-\wedge} = W(\wedge),$$

while on the right hand side the operator wave requires the inverse.

We now compare the standard matrix logic of **TRUE** and **FALSE** with the corresponding logical wave operators which are the signed exponentiation of **TRUE** and **FALSE**:

$$W(L) = \pm \exp(-L)$$

Standard matrix logic obeys Matrix Statistics where **TRUE** and **FALSE** are idempotent

$$\mathbf{TRUE}^2 = \mathbf{TRUE}, \qquad \mathbf{FALSE}^2 = \mathbf{FALSE}.$$

The logical wave of **TRUE** and **FALSE** can be described as the difference of the idempotent **AND, NOR**, and the self-dual **ON, OFF**:

$$\exp(-\mathbf{FALSE}) = \mathbf{AND} - \mathbf{OFF}, \qquad \exp(-\mathbf{FALSE}^T) = \mathbf{AND} - \mathbf{ON},$$

$$\exp(-\mathbf{TRUE}) = \mathbf{NOR\text{ - }OFF}, \qquad \exp(-\mathbf{TRUE}^{T}) = \mathbf{NOR\text{ - }ON}.$$

Exponentiation is known to convert Lie algebra to noncommuting Lie groups. First of all, the wave operators alter the Matrix Statistics by mixing it multiplicatively with Fermi-Dirac:

$$\mathbf{W}(\perp)\mathbf{W}(\mathsf{T}) = -\,\mathsf{N} \bullet \mathbf{W}(\mathsf{T})\mathbf{W}(\perp)$$

as opposed to simply

$$\perp \bullet \mathsf{T} = \mathsf{N} \bullet \mathsf{T} \bullet \perp.$$

Secondly, the wave operators comprise the idempotent, $\mathbf{W}(\mathbf{L})^2 = \mathbf{W}(\mathbf{L})$, quadruplet

$$\mathbf{W}(\perp), \mathbf{W}(\mathsf{T}), \mathbf{W}(\dashv), \mathbf{W}(\vdash)$$

and the tripotent, $(\mathbf{W}(\mathbf{L})^3 = \mathbf{W}(\mathbf{L})$, quadruplet

$$-\mathbf{W}(\perp), -\mathbf{W}(\mathsf{T}), -\mathbf{W}(\dashv), -\mathbf{W}(\vdash),$$

which in fact is the global $e^{i\pi}$ phase shift of the idempotent wave operators:

$$\mathbf{W}(\mathbf{L})e^{i\pi} = -\mathbf{W}(\mathbf{L}).$$

Since

$$\mathbf{W}(\mathbf{L}) = e^{-\mathbf{L}} = \mathbf{I} - \mathbf{L}$$

and

$$\ln \mathbf{L} = \mathbf{L} - \mathbf{I}$$

the tripotent quadruplet of wave operators are the logical logarithms:

$$\ln \perp = -\mathbf{W}(\perp), \quad \ln \mathsf{T} = -\mathbf{W}(\mathsf{T}), \quad \ln \dashv = -\mathbf{W}(\dashv), \quad \ln \vdash = -\mathbf{W}(\vdash),$$

with the fundamental property for the integer section of the series:

$$\ln^3 \mathbf{L} = \ln \mathbf{L}.$$

Many logical operators can be treated as operator waves, approximated by the linear section of the power series expansion. Of particular interest for us are the logical waves generated by quantum-mechanical spin operators:

$$e^{-\sigma_x} = \mathbf{H}, \quad e^{-\sigma_z} = \mathbf{H}_{diag}$$

where **H** is a suprasymmetric logical **Lagrangian**, diagonalized concomitantly with the diagonalization of the spin operator.

An important form of the logical operator waves are the matrix powers of matrices, among which the most fundamental and unique are the matrix powers of implications, providing for internal, purely logical, and autonomous self-conversion to a fixed point, fully equivalent to quantum field operator conversion:

$$\rightarrow^{\rightarrow} = e^{\rightarrow \ln \rightarrow} = e^{a} = \rightarrow.$$

Implication, which is at the heart of the matrix principle of cognitive logic, is a **unique** fundamental logical membrane on which a quantum operator wave **finitely** halts, with all nonlinear terms of the power series expansion confined to the **vacuum. IF IF** times is **IF**. The self-potency is the **exact** power, but there are also beautiful approximations for **IF** and its inverse in terms of the canonical pair of logical operators:

$$\vee^{\wedge} \approx \mathbf{IF}, \qquad \wedge^{\vee} \approx \mathbf{IF}^{-1},$$

where one uses only linear logical terms of the expansion, and changes to the inverse by simply swapping the base and the power.
It is useful to re-express this result in the language of a quantum system defined in an infinite spatial volume, where the corresponding Hilbert space is divided into mutually orthogonal sectors, with each sector preserved by any local operation. For example, an electric charge has an infinite range electric field. Therefore no local action can create or destroy a charge, for to destroy a charge we must also destroy the electric field lines extending to infinity, and no local procedure can accomplish this task. A topological charge that participates in cognitive phenomena is associated with the implication waves in 5-dimensional spacetime. The rotated charge acquires a phase no matter what its distance from the origin of matrix space. The number of revolutions is countered by the off-diagonal element in $\rightarrow^n$ and no local operation can destroy the phase. This situation is similar to the Aharonov-Bohm effect. If an electron is transported around a perfectly shielded magnetic solenoid, its wavefunction acquires a phase which is a topological property; it depends only on how many times the electron encircles the solenoid, and is unchanged when the path is smoothly deformed. A circling electron acquires a certain phase no matter what its distance from the solenoid. If we consider two charges, widely separated and well isolated, then any process that changes either charge would have to act coherently in the whole system. The topological charges are robust and immune to local disturbances.

THE EXOTIC STATISTICS

A study of the topological cognitive states produced from the logical vacuum indicates that in the compactified space containing nonorientable manifolds, fundamental logic obeys Matrix Statistics. A changeover of **TRUE** and **FALSE** produces neither a bosonic **+I** nor a fermionic **-I** factor but another square root of the matrix identity, **N**. The fact that Bose-Einstein and Fermi-Dirac are not adequate for the description of the logical mind has far-reaching implications for logic. But the new statistics can be explored in quantum field theory as well. Classical solitons can be quantized as bosons

$$q(f_1)q(f_2) = e^{2\pi i}q(f_2)q(f_1)$$

or as fermions

$$q(f_1)q(f_2) = e^{\pi i}q(f_2)q(f_1)$$

An interchange of two nonoverlapping solitons of odd degree produces a phase factor **-1** of the corresponding states. This is a consequence of the way the gauge group acts in Fock space. Starting from a quantization of ordinary chiral fermions in an external gauge field with the gauge group **SU(3)** and looking at the states which are produced from the vacuum by the action of nontrivial topological transformations, in 4 dimensions there is an option to quantize solitons such that they do not obey either canonical commutation

relations or anticommutation relations. Observing that the integral logical charge of the Matrix Statistics factor is the quark charge

$$\mu(\Phi) = \frac{1}{3}$$

the conversion from logic to the quantum field gives the exotic quark form of Matrix Statistics:

$$q(f_1)q(f_2) = e^{2\pi i/3}q(f_2)q(f_1)$$

where $e^{2\pi i/3}$ is a third root of unity and there is a nonlocal term which is calculated as an integral of a topological density. The quark root is a nontrivial fractional phase, which is indistinguishable in all its observable effects from an Aharonov-Bohm phase arising from a fundamental gauge field, and its consequences are spectacular. Cancellation of the factor of two

$$q(f_1)q(f_2) = e^{\pi i/3}q(f_2)q(f_1)$$

gives three odd multiples of iqon: 1/6, 3/6 and 5/6, which is a nontrivial prediction of the theory.

The mixtures of the bosonic, fermionic and matrix factors can give rise to a family of exotic statistics. We have seen already that the logical wave operators mix Matrix Statistics with Fermi-Dirac:

$$\mathbf{W}(\perp)\mathbf{W}(\mathsf{T}) = -\,\mathbf{N} \bullet \mathbf{W}(\mathsf{T})\mathbf{W}(\perp)$$

The diagonalization of Φ = **NOT** leads to yet another square root of identity

$$\mathbf{N}_{\text{diag}} = \sigma = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which is a unique persymmetric matrix, explicitly mixing the bosonic and fermionic factors and bringing all statistics under one umbrella. In a sense Bose-Einstein and Fermi-Dirac statistics can meaningfully be treated as the eigenstate limits of Matrix Statistics, where the eigenvalue **+1** of the diagonalized **NOT** corresponds to bosons and the eigenvalue **-1** to fermions. This naturally invites another name for Matrix Statistics which, in the above-stipulated sense, can be called Bose-Dirac statistics.

These enticing results take our analysis to the deep areas of physics, from the fundamental level of unified field theory all the way up to the complex level of consciousness. Matrix Statistics is destined to play a decisive role in the experimental verification of the topological theory of the brain. Specifically, it must answer the key question: **how many '*logical particles*' or nibbles are allowed in a cognitive state**? A quantum statistics determines the number of particles that can occupy a quantum state. For Bose-Einstein statistics there is no limit on the accumulation of particles, but only a single fermion can impregnate a given quantum state. Consequently, the spin-1/2 particles obey Fermi-Dirac and those with integer spin Bose-Einstein statistics. Personal experience tells us that the number of mental elements (concepts) which consciousness can process simultaneously is quite small. What is important is that according to matrix logic the number of nibbles which can be accepted by a cognitive state is greater than one (which excludes fermions)

but finite (which excludes bosons). Analysis of nonorientable cognitive states, produced by polarization of logical vacuua, suggests that cognitive matrix logic either operates outside the Bose-Einstein and Fermi-Dirac statistics or that it is supersymmetric in the sense that it somehow lets both the fermionic determinant and the regular bosonic sum coexist.

It is difficult to predict where the exploration of Matrix Statistics will eventually bring us. Besides its relevance for consciousness studies, it has repercussions for purely physical problems, opening up new ways of looking at fundamental interactions and clarifying the reasons for the existence of the three basic families of elementary particles. M(atrix) theory is most natural and promising way towards a superunification which cannot be accomplished without taking account of the logical consciousness.

MATRIX BRANES

BRAINS AND BRANES

It has been observed that the underlying problems in the theory of the brain, in quantum field theory and in cosmology are strikingly similar. The descriptions of thinking brains, elementary particles, black holes and strings unexpectedly give rise to similar questions, most importantly to the question of dimensionality. The studies of various string dualities indicate that all five known string theories in **10** dimensions are closely related to each other. They represent five corners of the moduli space for the vacuum of the one and same matrix theory, which in yet another corner of the moduli space is most conveniently viewed as a theory in **11** dimensions. Quantum particles can be conceived as strings, or more generally as **n**-dimensional membranes or simply **n**-branes. A particle, instead of being pointlike, is in reality a minute bubble, a (mem)brane closed in on itself. Supersymmetry severely restricts the possible dimensions of a brane. In the spacetime of **11** dimensions a brane has only two spatial dimensions and looks like a sheet. If one of the **11** dimensions is curled into a circle, we can wrap the brane around it, pasting the edges together to form a tube. If the radius of the brain is sufficiently contracted, the rolled-up brane ends up looking like a string in **10** dimensions. M(atrix) theory is best thought of as the discretized worldcone quantization, which is a compactification on a lightlike circle of radius R= **n** /**P**, where **P** is the momentum quantized in integer multiples of 1/R. Accordingly, the worldcone is boosted, and thereby is Lorentz-contracted, so that a transversing **n**-brane will fit into the longitudinal space, causing no infinities. Choosing an **n** too large can introduce a needlessly large number of degrees of freedom, whereas

it is desirable that there are neither too many nor too few of them. Matrix logic is revealing a link between the microscopic and the macroscopic. On a very small scale close to the Planck length of 10^{-33} cm, the universe looks the same as it does at large scales. The laws for large and small are essentially and remarkably the same.

LOGICAL VOLUME

In this section we introduce new abstract primitives of cognitive logic, namely the matrix membranes or **L**-branes, and then show that the dimensional structure of logical branes corresponds to the fundamental structure of superstring theory. A logical (mem)brane is a cognitively extended object with matrix degrees of freedom on which a thought can end [Ref 79]. The notion of a logical brane closely parallels the notion of the worldvolume, which in relativity theory is **4**-dimensional:

$$dV = \prod_{i=1}^{4} dx_i = dx\,dy\,dz\,dt,$$

and **11**-dimensional in superstring theory.

Definition. A logical (mem)brane or volume is defined by the matrix product

$$V_n = L_1 \bullet L_2 \bullet L_3 \bullet \ldots \bullet L_n$$

where $V_n \in \Omega$ and $L_1, L_2, L_3, \ldots, L_n$ is the ordered sequence of the matrix-logical coordinates.

According to matrix logic the cognitive coordinates are represented by logical matrices. The product of these matrices determines a matrix volume (mindvolume) which we call a logical (mem)brane or simply a **brane**, which is in itself a matrix. In the Fig 14 we give an example of a **2**-dimensional logical membrane which is visualized as a 'surface'. Since any logical matrix can be extended as a product of other matrices, including itself, if idempotent, a resulting **L**-brane 'forgets' its dimensional 'history' and in this sense can be of an arbitrary dimension. Basically the dimensionality of a logical membrane is unknown, or, more correctly, is 'known' only to the membrane itself. For example, the (mem)branes

TRUE = AND•OR
TRUE = TRUE •TRUE
TRUE = AND•AND•OR
TRUE = TRUE •TRUE •TRUE•TRUE

are all volume-equivalent, even though their logical contents and dimensions are different. We see that branes with identical volumes can be generated by different matrix-logical products; hence, a matrix 'line', a 'surface' and a volume are indistinguishable, which has nontrivial consequences.

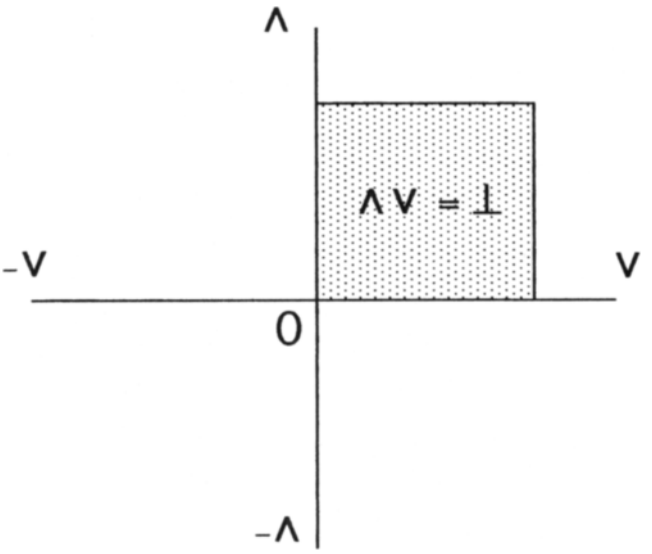


Fig. 14
The logical brane **TRUE**, formed, clockwise as a product of the matrix coordinates **AND** and **OR**.

The fundamental feature of the L-branes is that their coordinates are promoted to matrices, the idea which inspired m(atrix) theory [Ref 88]. Cognitive space is noncommutative, with the membranes exhibiting properties that can be explained only by quantum principles. Although a logical brane is an extended object it is not a pure abstraction but physically realized through the vacuum of quantum theory. Due to the uncertainty relation, the vacuum is not a void but actually teeming with activity, with virtual particles and antiparticles constantly appearing out of nowhere and disappearing back into the vacuum. The transient virtual L-branes may carry the negative (logical) energy which has historically been anathema to physicists, but is fundamental for the realization of consciousness.

IRREDUCIBLE LOGICAL BRANES

The dimensions of a logical brane determine the number of available degrees of freedom. When scalar logical functions are promoted to matrix coordinates we have **17** basic logical degrees of freedom: **16** Boolean dimensions plus **TIME**, which are described by the following standard set of matrices [Ref 88]:

NONE, YES, NOT, AND, OR, IMPLY, IF, NAND, NOR,

$$\text{ON, OFF, TRUE, FALSE, } \widetilde{\text{TRUE}}, \ \widetilde{\text{FALSE}}\text{, ONE, TIME.}$$

It might be clear that due to the inclusion of TIME a situation may arise when a logical volume is negative.

We are concerned with the question: what is the irreducible number of matrix logical dimensions? Given an n-dimensional system, we are usually interested in determining its minimal irreducible basis. In relativistic physics such an irreducible basis is 4-dimensional spacetime, and the seven extra dimensions of string theory must be compactified to get the actual 4-dimensional world. In logic, in order to cut 17 matrix dimensions of the mindvolume down to the irreducible and functionally complete basis, we can explore the fact that it is possible to express some logical operators via the others. For example, we know that

$$x = y$$

is equivalent to

$$(x \rightarrow y) \text{ AND } (y \leftarrow x).$$

Instead of

$$x \rightarrow y$$

one can write equivalently

$$\bar{x} \text{ OR } y.$$

Instead of

$$x \text{ NOR } y, \ x \text{ TRUE } y, \ x \text{ BUT NOT } y$$

we derive respectively

$$\bar{x} \text{ AND } \bar{y}, \ x, \ x \text{ AND } \bar{y}.$$

In the end the propositions of nonnegative Boolean logic can be expressed in the functionally complete triplet of connectives

$$\{\text{NOT, AND, OR}\}.$$

Making use of the DeMorgan equalities this major basis set can be, as is well known, cut down to the minor basis sets

$$\{\text{NOT, AND}\} \text{ or } \{\text{NOT, OR}\},$$

or even more extravagantly one comes down to the minimal basis sets {NOR} and {NAND}, relying on the equalities:

$$\bar{x} = x \text{ NOR } x,$$
$$x \text{ AND } y = \bar{x} \text{ NOR } \bar{y},$$

or

$$\bar{x} = x \text{ NAND } x,$$
$$x \text{ OR } y = \bar{x} \text{ NAND } \bar{y}.$$

Scalar and matrix degrees of freedom are fundamentally different. In order to eliminate the scalar features entirely, we retain only the non-Abelian dimensions and exclude those coordinates which follow the quadrature difference rule for scalars:

$$A^2 - B^2 = (A + B)(A - B),$$

which is another way of saying that we exclude the commuting coordinates:

$$[A, B] = 0.$$

In our scheme the **17** logical degrees of freedom are represented by matrices, and we use the commutation criteria in the dimensional reduction of logical branes. The purpose is to obtain a critical dimensional limit of the mutually noncommuting coordinates:

$$[L_i, L_j] \neq 0, \quad \forall\, i, j.$$

This means that if two logical coordinate matrices commute we can throw one of them out, since knowledge of one will give information on the other. Clearly, we can at once eliminate **NONE** and **YES**, which commute with any logical matrix. There are four more matrices which have to be thrown out in order to satisfy the condition of mutual noncommutation. This concerns the following commuting products

[NOT, ONE] = 0, [AND, NOR] = 0, [IMPLY, OFF] = 0, [IF, ON] = 0.

After excluding all coordinates which commute, we end up with the phase space of **11** matrix logical dimensions, which is the fundamental irreducible basis space, comprising the following coordinates:

{NOT, AND, OR, IF, NAND, OFF, TRUE, FALSE, $\widetilde{\text{TRUE}}$, $\widetilde{\text{FALSE}}$, TIME}

The number of noncommuting logical dimensions coincides with spacetime dimensions in m(atrix) theory, indicating a link between matrix logic and strings. Superstrings are consistent in **11**-dimensional spacetime with supersymmetry restricting the number of possible dimensions. In the spacetime of **11** dimensions floats a supermembrane which looks like a sheet, and if one of the dimensions is curled into a circle, we can wrap the membrane, gluing the edges together to form a tube. For a circle with a sufficiently small radius, the rolled-up membrane ends up looking like a string in **10** dimensions, yielding the Type IIA superstring.

There are many competing predictions of how the real world works. Since there are many different ways of compactifying **11** dimensions down to four, the central problem is to determine the right way. Kaluza-Klein theories contend that we are unaware of the extra dimensions because they are compact, extending only across distances far smaller than the size of an atom. In the Randall-Sandrum theory the additional dimensions are essentially infinite in extent, and the **3**-brane on which we live is just part of a **5**-space. In matrix logic the **11**-dimensional **L**-brane, which we obtained by disposing of the redundant commuting coordinates, can be contracted further, and most importantly the end result of such reduction is a **4**-dimensional irreducible logical basis, just as a **4**-spacetime is an irreducible physical basis. To prove this, we rely on a new feature of matrix logic in which logical connective operators interact, in contrast to traditional calculus where expressions like

AND OR or **OFF NAND**

are utterly meaningless. In contrast in matrix logic they are 'ordinary' examples of the matrix product interactions, yielding the logical branes:

AND•OR = TRUE and **OFF•NAND = NOR.**

This should make clear our next step of reducing the **11**-dimensional logical brane to four basic dimensions. The criteria which we use to eliminate a particular logical dimension is whether the corresponding coordinate can be derived as the matrix product of other coordinates. As we have shown in previous studies, the irreducible **multiplicative** matrix logical basis consists of the three matrices: {**NOT, AND, OR**}. Then, taking the time coordinate into account, the **11** logical degrees of freedom of the original **L**-brane will be cut down to four:

$$\{\mathbf{NOT, AND, OR, TIME}\}$$

The matrix-logical coordinates **NOT, AND** and **OR** emerge as the counterparts of the space coordinates x, y and z, and **TIME** as the counterpart of the time coordinate t. The coordinates **NOT, AND** and **OR** are symmetric while the logical coordinate **TIME** is antisymmetric. The logical branes with time are of particular significance. The time dimension possesses quartric identity and the fifth power of the logical time volume is the time coordinate itself:

$$\blacktriangle^5 = \blacktriangle .$$

Whenever a logical brane involves logical time, we can introduce **5**-fold symmetry into the brane. In this manner the fundamental **4**-dimensional brane becomes equivalent to the octal **8**-brane since the time volume $\blacktriangle\blacktriangle\blacktriangle\blacktriangle\blacktriangle$ is equivalent to a single coordinate $\blacktriangle$:

$$N \wedge \vee \blacktriangle = N \wedge \vee \blacktriangle\blacktriangle\blacktriangle\blacktriangle\blacktriangle$$

The properties of the logical **4**-brane are determined by the two following theorems, which show explicitly that cognitive logic is quantized in the 'units' of time and can be constructed purely axiomatically as an analog of a third-quantized theory with singularity.

Time quantization theorem

$$[\,\mathbf{NOT, AND}\,] = \blacktriangle, \quad [\,\mathbf{NOT, OR}\,] = \blacktriangle, \quad [\,\mathbf{OR, AND}\,] = \blacktriangle$$

Proof by inspection.

Triple anticommutator theorem

$$\{\mathbf{NOT}, \{\,\mathbf{AND}, \{\,\mathbf{OR}, \blacktriangle\}\}\} \equiv \mathbf{0}$$

More generally, denoting the **NOT, AND** and **OR** by $\mathbf{L}_i$, i = 1, 2, 3, for any cyclic shift of i, j and k

$$\{\mathbf{L}_i, \{\,\mathbf{L}_j, \{\,\mathbf{L}_k, \blacktriangle\}\}\} \equiv \mathbf{0}.$$

Proof

If a single anticommutator anticommutes, the triple anticommutator is necessarily empty.

$$\{\mathbf{AND}, \blacktriangle\} = \blacktriangle$$

$$\{\,\mathbf{OR}, \blacktriangle\} = \blacktriangle$$

$$\{\mathbf{NOT}, \blacktriangle\} = \mathbf{0}$$

CONSCIOUSNESS AS A UNILATERAL MEMBRANE

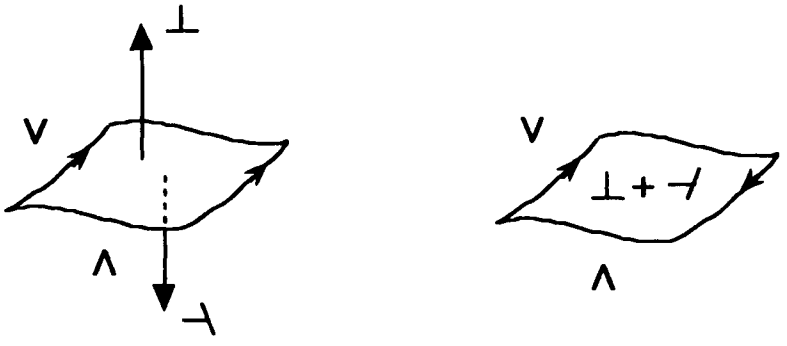
The logical volume of a brane is an additive quantity. If the logical volume of the brane L_1 is V_1 and that of the brane L_2 is V_2, then

$$V(L_1+L_2) = V(L_1) + V(L_2).$$

This is easily seen by considering a matrix membrane with a certain logical volume, and then presenting the matrix as the sum of two matrices. The force holding membranes together is associated with virtual interactions of the logical vacuum. Matrix logical bracket closure

$$\langle p| \text{LOGICAL BRANE} | q\rangle,$$

independently of the resulting value, cancels out a logical volume. We distinguish two kinds of membranes, those contractible to a scalar point and those which are not. Topologically the first kind corresponds to a convex space with zero genus, the second to a space punctured with holes. While ordinary logic is fully identified by an open space, cognitive logic refers to itself, which implies the formation of loops. The logical degrees of freedom are associated with matrix dimensions in cognitive phase space, and both in a formal and a fundamental sense consciousness can be described as a nonorientable logical membrane, effectively and selectively isolating itself from the environment. The logical brane is another constructive way of describing the interacting operators in the factorized terms of fundamental reduction in matrix logic. Logical branes can intersect, get wrapped and entangled, giving rise to (topo)logical knots of varying complexity. The underlying algebra of logical branes is superalgebra. For a knot to carry a logical charge the bilateral matrix membranes must be first converted to a nonorientable topology, not just in an abstract mathematical sense but in the actual physics of the brain.



Bilateral membrane Unilateral membrane

Fig. 15
An example of bilateral and unilateral L-branes.

For a bilateral logical membrane the two sides can be identified by treating the logical volume as the matrix vector product with an orientation. In the example we consider above, the logical surface of one side then is $\perp = \wedge\vee$, and that of the other $\dashv = \wedge\vee$. However, for the unilateral membrane, which, if cut, retains a (topo)logical charge, the orientations cannot be specified, the membrane is nonorientable, and its volume then identified by the matrix sum $\perp + \dashv$, in this instance equals the sum of the matrix coordinates of the membrane:

$$\perp + \dashv = \wedge + \vee$$

The conversion from orientable to nonorientable topology, leading to the creation of (topo)logical charges, has important implications in the context of the topological theory of the brain, where cognitive elements are non-Hermitian and the effect of self-awareness is attributed to a violation of parity symmetry. When an orientable logical membrane is differentiated, it closes on itself, and singularity appears in the logical phase space. Consequently, adding new thoughts or mems to a system will puncture it with holes, preventing the contraction of a logical membrane to a point. Manipulating (topo)logical charges, the thinking process can dynamically change the genus of the system, locally changing its underlying spacetime topology.

Below we will be concerned with the problem of the embedding of a logical brane in the Hilbert space and with the reverse problem of the insertion of a quantum operator into a logical brane. These problems are closely linked with the question of connecting the logical quantum numbers with the quantum numbers, which, in turn, forces us to confront another challenge, that of connecting **continuum** and **discretum**. We must understand how a discrete eigenvalue can be embedded in a continuous spectrum, the cognitive continuum, and how a logical membrane can be confined to a particular subspace of the Hilbert space, without creating the havocof the breakdown of the laws of quantum mechanics and without offending the covariant relations in general. We are at ease connecting the continuum and discretum statistically, but run into problems whenever the connection must be done explicitly and locally. To connect a continuum to discrete states we have to reconcile the reversibility of unitary quantum evolution and the irreversibility of the inductive thought process. Although the brain at bottom is quantum-mechanical, it is not possible for the thought process to run backward, the way a quantum particle can reverse the direction of time in quantum field theory. The identification of the time operator for the description of irreversible processes becomes essential. But there are other obstacles. Even if the thought process could conceivably run backwards, it must be unitary, hence, Hermitian, and therefore cannot be observed cognitively. Either a thought can reverse itself only on the unconscious level, which is near Hermitian, or through a back door, by creating a cognitive time loop, where the past is recovered not by unitary regression but by 'compactification' of the time

dimension of a logical brane, with apparently the same result but achieved by different means.
Treating logical branes as factorized terms of the fundamental product of matrix logic, we determine that the dimension of a brane is $2n-1$, $n = 1, 2,...,$ where n is the number of atomic products in a fundamental reduction:

Fundamental product	Matrix dimension	Spins
<||>	1	4
<||><||>	3	8
<||><||><||>	5	12
<||><||><||><||>	7	16
<||><||><||><||><||>	9	20
<||><||><||><||><||><||>	11	24

The dimension of the branes are odd and each new atomic product adds two dimensions, a pair, to a brane. Coherently, the number of spins needed to form a logical brane is even, with a nibble of spins being added to each new level. If the spins are the electrons of an atom, a 3-dimensional matrix brane can be realized by the oxygen atom ${}_8O^{16}$, an 11-dimensional one by calcium ${}_{20}Ca^{40}$. Because the covariant logical inferences involve the octet of spins, the 3-branes are important for experimental verification of matrix logic.

THE CAT AND THE LIAR

Those who play with cats must be expected to be scratched

Miguel de Cervantes

THE INTELLECTUAL CATASTROPHES

Science is a relentless assault on human intuition. Common sense has failed us on many occasions in the past, and will most certainly fail us in the future. How often have the ideas and theories we cherish, the feelings and beliefs we hold so dear had to be abandoned and as we have been forced to make a U-turn because the experimental facts reveal that were entertaining a

wrong idea or premiss. In order not to be trapped in controversies such as whether, and if so, how, human thought can be represented by physical processes, let us consider two fundamental intellectual catastrophes, one occurring in logic and the other in quantum physics, which can help to untangle the paradox of the brain.

We enter the world with an exaggerated belief in truth, naïvely taking for granted that things can be divided into true or false, and that it is within the power of the human mind to distinguish the true from the false. But as soon as the faculty of logical thinking is developed we come across the Liar paradox, known for centuries. If history is to be believed it was first discovered by a Creatan thinker in ancient Greece, proclaiming

"ALL CRETANS ARE LIARS".

Everybody lies, said the Liar. If everybody lies, the statement of the Liar is false. In other words, the Liar, a Creatan, is lying. But if he is lying, then the Creatans are not liars. Then the Liar tells the truth, then all Creatans are liars, and the vicious circle begins to revolve all over again, never ending.

The Liar paradox has a long and distinguished curriculum vitae. Such is the power of this intellectual catastrophe in undermining the fundamental axiom of the mind that it still reverberates to the present day. There has been great deal of discussion of the Liar in the literature. Numerous papers and books have been written about the paradox [Ref 48]. A countless number of mathematicians, logicians and philosophers, fools and geniuses, have thought about it with amazement or indignation, claiming from time to time that a solution has been found. Russell's paradox in the set theory and Gödel's incompleteness theorems are technical mathematical versions of the Liar, and in fact have been inspired by it. Gödel paraphrased the paradox with the statement "This sentence cannot be proved true". If the sentence is correct, then it cannot be proved to be correct. Patrick Grim was first to analyze the Liar in terms of fuzzy set and chaos theory [Ref 27]. We have made an attempt to describe the paradox in terms of negative logical values, treating it as a dynamical oscillation process in time where self-referential deduction periodically enforces two mutually excluding and incompatible conclusions [Ref 88, 91].

Counterintuitive as it is, we have no choice but to conclude that the Liar must be both true and false! The paradox forces consciousness into an alternating loop of reasoning which does not halt, oscillating indefinitely between the true and the false. Consciousness fluctuates. Finite reasoners such as ourselves are generally smart enough to break out of such a vicious circle rather than to continue in it forever. This, however, is not a solution but a defeat. The paradox forcefully tells us that our intuition is fundamentally at fault. **Different answers will be given to the same question in different instants of time.** Binary intelligence fails to accommodate the Liar. In a major contradiction to the basic axioms of classical logic it is

possible to deduce from the same initial data a proposition that is both true and false. But even if one abandons the Law of Excluded Middle and pursues the intuitionistic approach, the question of finding a fundamental explanation of the paradox remains unanswered.

The events take an unexpected turn when it is realized that the mystery of the Liar can be explained by another mystery from the seemingly unrelated field of quantum mechanics, the Schrödinger cat paradox. The latter concerns the coherent superposition of macroscopically distinguishable states, known as cat states. Progress in science is often based on connecting different fields previously considered to be unrelated. Such unification is particularly relevant in brain science, presenting physics with not only formidable technical difficulties, but also a major conceptual challenge. While naturally a physical system, the brain is at the same time essentially an information-logical system. Therefore any meaningful study of the brain cannot be based on physical considerations alone, be they classical or quantum, but necessarily has to include the logical cognitive degrees of freedom, fundamentally entwined with its physical mechanism.

In quantum theory unitary time development can result in macroscopic quantum superposition or Schrödinger's cats, which must then somehow be explained away. The cats have been of considerable interest, but the quantum superposition of macroscopically distinguishable states is very difficult to obtain. We can push up the classical-quantum border slightly to form for example mesoscopic superposition of atoms but in macroscopic reality we never observe Schrödinger cats. Schrödinger cats are made of a very large number of molecules. The interactions with the macroframe causes quantum coherence to decay into a statistical mixture very rapidly, which is used to explain why one cannot obtain a macroscopic coherent superposition of an alive and a dead cat. The difficulties with the Cat paradox are so significant that they have prompted proposals to modify quantum theory, despite all its unquestionable experimental success. Wigner suggested that the failure to obtain macroscopic quantum interference is due to a nonlinear term unaccounted for in the Schrödinger equation, which cancels out the coherence between macroscopic states. Mesoscopic coherent interference effects between distinct states of an individual atom have been monitored experimentally but an effective control over the coherent superposition of macroscopic states remains as elusive as ever. From Bohm's hidden variables approach to Gell-Mann-Hartle's consistent histories many unsuccessful attempts have been made to make the cats disappear, and one begins to wonder whether physics is on the right track. Instead of trying to get rid of the embarrassment of Schrödinger's helpless cat, wouldn't it be more logical, without taking the unfortunate cat too literally, to look, instead of Hilbert space, for other spaces where cat states can actually exist? What if thought is a cat state? A Schrödinger's cat in every head? For seventy years theorists and experimentalists have been searching for Schrödinger's cats in the **outer** world of quantum mechanics, missing the fact

the cats are a permanent fundamental feature of the **inner** world of cognitive logic.

The principal thesis of this section is to show that the Liar paradox is a macroscopic coherent superposition realized by the brain, indirectly reflecting the quantum nature of the elements of which it is made up. The Schrödinger Cat paradox not only explains the Liar paradox, with both paradoxes seeming to be one and the same thing, but a more general notion of truth is needed, one which allows for partial inconsistency and self-reference. This notion comes from topology. The logical rules of inference are the classical limit of probability rules in which conditional probabilities are reduced to definite Boolean true or false [Ref 89]. The existence of the coherent superposition of incompatible events, persisting up to the point at which a measurement is made, is the major conceptual difficulty of quantum theory. It is commonly believed that this is an exclusively quantum phenomenon without a classical macroscopic analogue. But, as the Liar shows, classical logic itself is beset with conceptual difficulties of superposition, the interpretation of which remains controversial. Because both the Liar and the Cat have an important bearing on the theory of consciousness, it would be wrong to dismiss these issues as merely philosophical. The Cat is an unavoidable consequence of the principle of linear superposition which is central to quantum mechanics. The wavefunction of a quantum system is a coherent superposition of all possible states, including the orthogonal states of the system, which can be mutually exclusive, one the negation of the other. For example, if spin-up and spin-down are the two possible states of a spin-1/2 particle, the particle will also exist in a coherent superposition of both states $|up\rangle \pm |down\rangle$. Projecting such superposition to the macroscopic level leads to absurd situations. In Schrödinger's cruel gedanken experiment a poor cat is in a box together with a cyanide capsule which may release its content depending on the state of a quantum detector measuring the radioactive decay of some atoms. The laws of classical logic break down in the quantum realm, because both alternatives the cat alive and the cat dead must be present in the state of quantum superposition. For the outside observer, the cat is in a linear superposition of both alternatives, which must coexist and interfere with each other. The absence of quantum superposition at the classical level has been a serious problem in quantum mechanics: quantum interference at the microscopic level implies a superposition between distinguishable macroscopic states. Quantum measurement not only detects the pre-existing properties of a measured system, it in part creates in self-referential fashion the properties it reveals. In the macroworld, when observing a nice little cat we can determine that the cat is either perfectly alive and playful with probability p(alive) or unfortunately has played out all her games and dead with probability p(dead). Probability theory demands that

$$p(\text{alive}) + p(\text{dead}) = 1.$$

However, the Schrödinger cat, subjected to the strange demands of quantum theory behaves in quite a weird manner, existing in a bizarre coherent superposition of both dead and alive. Laplace, who viewed probability as an improvement on common sense would be disappointed to see the rules of classical probability break down. The complete sample space must include p(alive+dead), which would give meaningless value, no longer confined to the unit interval:

$$p(\text{alive}) + p(\text{dead}) + p(\text{alive+dead}) > 1.$$

The Cat cannot be accommodated by classical probability, which cannot exceed unity. Quantum theorists get around the problem by making use of a wavefunction which does not stand for actual physical properties but is merely a tool for calculating probability. Likewise, the treatment of the Liar in terms of classical probability leads to a violation of familiar probability laws. Suppose we characterise the Liar by the conditional probability $p(A|B)$ where A and B are the two intertwined true and false outcomes of the paradox. Then the Liar's conditional

$$p(A|B) = \frac{p(A \wedge B)}{p(B)}, \qquad p(B) \neq 0,$$

is not defined for $p(B) = 0$, yielding the uncertainty

$$\frac{0}{0}.$$

Infinite reality and finite theory differ. One can be sceptical and doubt the possibility of macrophysical Cats but the Liar is clearly an actual macroscopic coherent superposition. We need just one small but bold logical step to connect the two paradoxes into a single one. To achieve this goal we have to turn for help to topology, which provides the adequate theoretical framework. Consider a bilateral manifold and some system which can reside on either side of the manifold, with probability p(up) on one side and with the probability p(down) on the opposite side, hence

$$p(u) + p(d) = 1.$$

Suppose now that our system is the Liar (or the Cat) in true state (alive) when found on a given side of the manifold, and in the false state (dead) when found on the opposite side. Without informing the Liar or the Cat, let us twist the edges of the manifold and glue them into a Möbius unilateral topology. Since the manifold has 'lost' sides and become nonorientable, the question whether the Liar is truthful or dishonest, likewise whether the Cat is dead or alive, cannot be answered classically. We have changed nothing in the Liar and in the Cat states. But we have a strange and very real manifold in which the Cat and the Liar are definitely on both 'sides' at the same time, because these now form a convex. We arrive at absurd situations, uninterpretable in terms of classical probability:

$$p(\text{true}) + p(\text{false}) = 1 + 1 = 2.$$

or

$$p(\text{alive}) + p(\text{dead}) = 1 + 1 = 2.$$

But a nonorientable Möbius strip is not absurd and does actually exist in the macroscopic world. A new reality, quantum in essence but macrophysical in form, emerges on it. This reality requires for its description neither classical probabilities nor complex probability amplitudes but improbability instead.

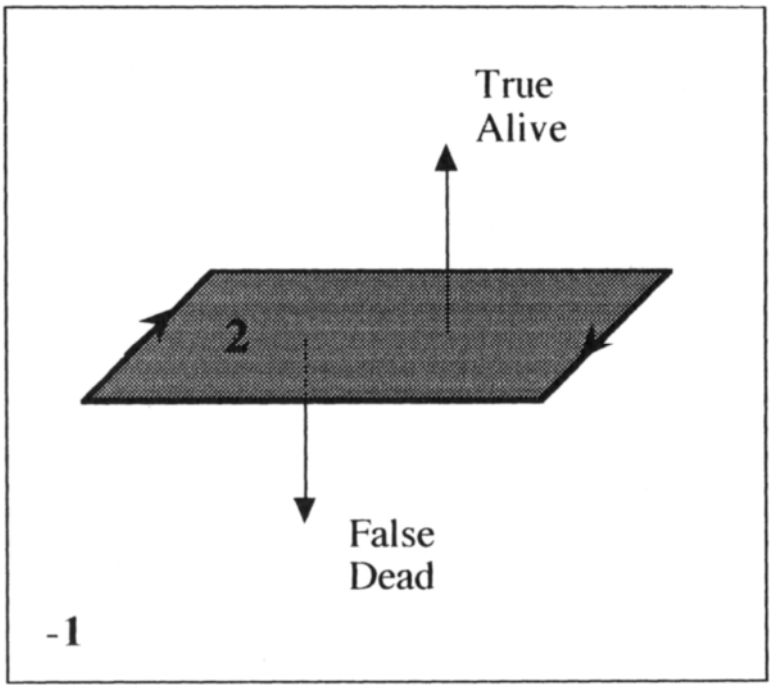


Fig. 16
The Liar or the Cat residing on a unilateral Möbius strip. For orientable topology the probability of false 0 is complement of the true 1. For nonorientable topology -1 is a complement of the improbability 2.

The Möbius strip, that darling of mathematicians, has been known for century and a half. What we didn't know is that we can use it to describe a macroscopic Schrödinger cat state and the Liar, actually realized as a cognitive thought. A topological approach to consciousness indicates that a very different form of actualization occurs in a self-measuring system. The process of

conversion from quantum truth to classical truth is tampered with because the observer becomes an integral part of the system and the distinction between the observing and the observed vanishes. In technical terms this entails that consciousness no longer needs to convert quantum amplitudes into certainties, but instead, swaps improbabilities and probabilities. Outside the framework of wavefunction, classical (topo)logical states can get coherently mixed as if they were quantum superpositions. Since wavefunction is not needed to account for such superposition, a fruitful way forward would be to look for other physical principles that can. Such a new principle we find in topology, which provides new clues and ultimately an access to the intractable physics of consciousness, without relegating the mind to mechanical interactions. The affinity of the Cats and the Liar suggests that Schrödinger cat states exist in the thinking brain, and one paradox can be used to explain the other.

These results also provide a new and unconventional framework for the study of Schrödinger cat states. Quantum truth is very different from classical truth. Indeterministically it describes often mutually contradictory possibilities. Quantum systems are spread out theoretically over all spacetime. A particle can be both here and there, it is both a particle and a wave. Different quantum truths all coexist simultaneously in wavefunction until the act of measurement selects one of them to be a classical truth. Quantum measurement connects the realm of quantum possibilities into the real of classical certainty. In reverse manner to the decoherence of a quantum system through interaction with the macroworld, a macrosystem can *emit* coherences, thus creating a cat state. The road between classical and quantum is two-way, with topology providing a framework both for the design of the microphysical Cat and the macrophysical Liar.

THIS IS ABSOLUTELY TRUE±FALSE

Who does not enjoy Shakespeare's drama and poetry? Luckily the great writer lived in more simpler times than ours. In modern culture Shakespeare is replaced by Schrödinger. "To be **OR** not to be" gives way to the absurdity of "to be **AND** not to be". The Liar is the fundamental example of the **true±false** states of logical consciousness. In the context of the two dual theories of the brain, the geometrical and the topological, one can explain the Liar as a Schrödinger cat which entails the microphysical quantum basis of consciousness. But no less meaningfully one can explain the Liar as a cognitive nonorientable Möbius which entails the macroscopic topological foundation of consciousness. The fact that there exist coherent superpositions not necessarily represented by the wavefunction has an important bearing on cognitive logic. Möbius states are macroscopic coherent superpositions **true±false**. We have used the Schrödinger cat to explain the Liar paradox. But topology solves both. In the final analysis the new effects, neither classical

nor quantum, must exist in the topological brain. Topology is critical for the theory of consciousness. Under the usual assumptions of classical logic true and false are separable. Under the usual assumptions of quantum theory quantum information cannot be cloned. An object cannot be observed in two places at once. However, if not all natural processes are governed by unitary evolution, the laws of quantum mechanics will need revision, and consciousness provides a compelling reason for a modification of quantum theory. In topological consciousness a single percept can be cloned into two, signifying the emergence of a cognitive Möbius.

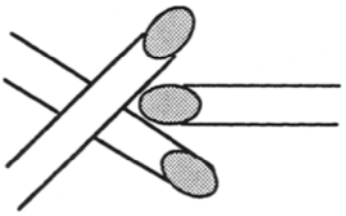


Fig. 17
Cognitive two-in-one 'two-slit experiment'. A finger will be cognized as two fingers, if placed between the crossed middle and index fingers.

QUANTUM MATHEMATICS

Truth is stranger than fiction.

George Byron

We equate truth with mathematics. The incredible, almost mystical power of mathematical deduction has puzzled many thinkers. In Maxwell's field equations, Einstein's mass-energy relation, Schrödinger's equation of motion, Wigner's irreducible group representations and many other instances physics celebrates the magic of mathematical consciousness. Whether mathematical truth is in the outer world or the inner world of abstractions, once the axioms are correctly guessed, the mathematical structures built around them obtain a kind of life of their own. Symbols can be trusted, and we rely on the ability of formalisms to describe the known effects and to predict the unknown. When a mathematical description of some phenomenon is written down, we expect it to tell us things we did not envisage initially.

Physics is digital. A powerful numerology is built into physical systems. A proton weighs **1836** times as much as an electron; there are **6** different flavours of quarks, the second orbit of an atom is saturated with precisely **8** electrons, the charge of a quark can be in absolute value only **1/3** or **2/3** of the charge of an electron. This list can go on and on. The interaction of

fundamental particles such as protons, neutrons, quarks and leptons can be quantified digitally in terms of a certain set of quantum numbers, changing to another number set. The quantum world is almost like a computer which, receiving an input set of data, processes them to give us an output number. At the fundamental level, the world, which we perceive as classical and continuous, is in fact discrete and essentially numerical. It is possible, if only in theory, to assign to each and every particle a distinct set of quantum numbers, and this is what quantum physicists actually do to identify a particle, and in doing so reduce the whole complex physical theory of the universe to a digital computation. There will be, however, a catch if objects and states exist which refuse such straightforward digitizing.

In searching for absolutes, mathematics provides us with a powerful abstract means of acquiring knowledge about the real world. There is a sense of mystery about this strange ability of mathematics to tackle and reveal the laws of physics, sometimes without any actual physical tests. Even more so because serious difficulties exist in the foundations of mathematics, since Cantor gave his famous diagonal proof concerning the properties of infinite sets. Classical intuition often breaks down when we enter the domain of the infinite. In a sense mathematics is macrophysical science. The true results of mathematics, as opposed to those of physics, cannot be ambiguous. It is a common belief that mathematical statement necessarily ought to be definite, and whenever an undecidable result occurs we should discard it as invalid and inconsistent.

Is there a new lesson for us to learn? Open any standard textbook on mathematics, and find the following innocent-looking series, known to mathematicians since 1703:

$$1 - 1 + 1 - 1 + 1 - 1 + \ldots$$

This infinite series is a sequence of units alternating in sign. The simplicity of the series, however, is misleading. If we place brackets the following way

$$(1 - 1) + (1 - 1) + (1 - 1) + \ldots = 0$$

the sum of the series will be nil. But if we place brackets differently, the sum is unity:

$$1 + (- 1 + 1) + (-1 + 1) + \ldots = 1.$$

The textbook will tell us that the series does not converge, but oscillates back and forth between **0** and **1**, never reaching a limit. A question which naturally presents itself is how it is possible that one and the same sequence of numbers can yield a different result, simply because one chooses a different way of placing the brackets. How can the result of mathematics depend on one's mental choice? How it is possible that the result of a precise mathematical computation depends on how one decides to look at it? Is mathematics physical or mental? Is it deterministic or nonclassical? What would happen if the strict provisions of two-valued classical logic were not built into our reasoning?

Consumed by the overriding concern for the macrophysical nature of mathematical results, we lost track of the possibility of a quantum mathematics, a mathematics where mutually excluding possibilities may exist in perfect

harmony in the domain of the infinite. Indeed, if we abandon the finite classical mode of reasoning, we can see the series as converging to a quantum limit of coherent superposition, like the Schrödinger cat. From the classical standpoint the series is in a nonconverging endless process of oscillation, but in quantum mathematics, classical in form but due to the infinity factor quantum in result, the series converges to a different kind of dual number which is defined by at least two ordinary numbers. Instead of saying that the series does not converge, we can say that it does converge but to a superposed (dual) quantum number:

$$1 + (-1 + 1) + (-1 + 1) + \ldots \Rightarrow s.$$

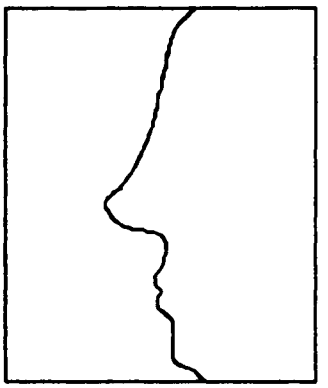
In this conceptual switch we treat a number not just as a value but also as a process. The terminology suggests that superposed numbers are essentially quantum-mechanical. Although the quantum-mechanical level seems to be suitable, the superposed numbers exist in the macroworld as one easily obtains coherent superposition macroscopically on a Möbius strip without wavefunction. The series does not converge in the orientable world, but can converge in a nonorientable topology.

An adequate mathematics is needed to gain an understanding of the outer world of physics or the inner world of the mind. The exact sciences are based on the commonly reaffirmed premiss that physics obeys certain mathematical rules, and does not simply conform to mathematics because we have been able to make increasing use of it. The theory of the brain is no exception. The study of consciousness is becoming less and less the subject of philosophy and more and more the subject of exact science. The problem of consciousness can be only grasped with appropriate math. There was a time when the whole of mathematics was just the few first natural numbers, until today in French the number *eighty* is denoted as *quatre-vingts*, the 4 times 20. But then the notion of negative numbers evolved, leading later to the notion of zero number. Many centuries passed before the concept of a complex number, then the quaternion, then the octonion and the matrix emerged. This prompts the question whether consciousness has sufficiently evolved mathematically to understand itself or do we still lack a critical mathematical insight?

Is there an actual reality corresponding to quantum mathematics? What kind of reality can correspond to numbers which combine mutually excluding values? If one had time to carry out infinite tests, one could not exclude the possibility that consciousness would converge to a quantum limit of coherent superposition of the two classical states. Topology indicates that quantum mathematics is relevant to the real world. For example, there seems to be no essential difference between odd and even numbers, but if numbers are counted as actual physical particles, bosons or fermions, the difference between the odd and even becomes fundamental. Is there a supersymmetric theory which is in a position to treat both sort of numbers in one unified framework? On a Möbius strip there is no problem with quantum mathematics. We exist on 3-brane, but there is no guarantee that it is not part of some hyperdimensional topology we unaware of.

Mental brackets

Superposed numbers exist not only in quantum mechanics but also can be found in the macrophysical world of consciousness. Consider an examples from psychology indicating the reality of superposed numbers. Suppose we want to answer a question: whether the line in the following figure outlines a face or a section of a vase:



Seeing is believing. Depending on how we place attention's mental *brackets*, we may see the face or the vase. At the same time we are well aware that a single line runs through the picture. We have one gestalt but with two different meanings, attending to one or to the other, one at a time. Although at lower cognitive levels we can separate the two percepts, at higher levels it can be difficult or even impossible. Our cherished true **or** false is not enough when consciousness oscillates between orthogonal states, producing interference patterns out of phase with itself. Describing cognitive states by superposed numbers, we can combine mutually excluding alternatives into superposition.

LOGICAL ROTORS

TOPOLOGICAL PHASE AND ATOMIC ORBITS

We suggested that topology is essential in enabling the brain to harness cognitive effects. What are the links of this idea to quantum physics? The topological phase is acquired in logical differentiation and can be quantified as a multiple of the fermionic half-twists, which determines the (topo)logical potential:

$$\infty(k) = \oint M dq = 2\pi(n + \frac{1}{2}) = k\pi$$

$$n = 0, 1, 2, \ldots, \qquad k = 2n+1,$$

where n is the winding number, specifying the number of times the closed curve runs round in an anticlockwise sense, and M is logical momentum, satisfying the commutation relation $[q, M] = 1$. Clearly, when n runs the bosonic numbers $n = 0, 1, 2, \ldots$ then $(n + 1/2)$ runs the fermionic numbers $1/2, 3/2, 5/2, \ldots$. The topological potential is an odd multiple $k\pi = (2n+1)\pi$ of the elemental topological phase π and is $\hbar^{-1}$ times the Bohr energy of the quantum oscillator:

$$\oint p dx = 2\pi\hbar(n + \frac{1}{2}), \qquad n = 0, 1, 2, \ldots$$

where $\hbar$ is the Planck constant divided by 2π, and the position and momentum operators satisfy the commutation relation:

$$[x, p] = i\hbar .$$

The topological potential, multiplied by the factor $\hbar$, gives the Bohr quantum energy which opens up a possibility of formulating the quantum mechanics of the atom as a topological theory. Atomic levels are topological rotors, which form nonorientable atomic orbits, where the ground level $n = 0$ is a basic Möbius level without a knot.

A wider implication of this scheme is that the physics of the atom can be treated as a dynamical logic in a fundamental sense, where the quantization stems from the underlying topological properties of matter. In this framework the idea of the two alternative dual treatments of the brain, geometrical (quantum-physical) and (topo)logical (cognitive), gains important new ground. Symbolically speaking, the physical brain is the logical brain, multiplied by the action quantum $\hbar$:

$$\textbf{PHYSICAL BRAIN} = \hbar \cdot \textbf{LOGICAL BRAIN},$$

hence the fundamental constant $\hbar$ can be defined through the ratio

$$\hbar = \frac{\textbf{PHYSICS}}{\textbf{LOGIC}}$$

In quantum mechanics, when we consider the quantization of energy the whole emphasis is shifted to the quantum action $\hbar$, the prefactor, whereas the postfactor $(n + \frac{1}{2})$ figures only in the background. In the topological formulation it is the topological potential $2\pi(n + \frac{1}{2})$ which emerges as the central element and $\hbar$ becomes merely a transition operator.

Topological potential reflects different levels of complexity. The knots corresponding to different rotors are characterized by the same genus, but have different topological phases, determining the amount of logical work required to twist the bilateral topology into a nonorientable rotor. Creating the (topo)logical phase on a logical string is analogous to winding up a clock's spring, which, if released, will rotate back. Accumulation and dissipation of (topo)logical phases are conveniently described by rotors of different signs.

We determine that the topological phases of the logical rotors of **TRUE** and **FALSE** are opposite in sign:

$$\text{rot TRUE} = 1, \qquad \text{rot FALSE} = -1.$$

Theorem

The rotors of the symmetric dual operators **AND** and **OR** are stable against unitary rotation:

$$\text{rot}\,\theta\wedge = \text{rot}\,\wedge,$$
$$\text{rot}\,\theta\vee = \text{rot}\,\vee.$$

Proof

$$\text{rot}\,\theta\wedge = \text{rot}\uparrow = -\bar{q}-(-\bar{p}) = \bar{p}-\bar{q} = q-p = \text{rot}\,\wedge,$$
$$\text{rot}\,\theta\vee = \text{rot}\downarrow = -q-(-p) = p-q = \text{rot}\,\vee.$$

□

Operation **rot** increases the rank of the tensor, and manipulating topological current, one can run different dynamical computations on a logical loop.

Theorem

The rotors of **AND** and **OR** sum up to zero:

$$\text{rot AND} + \text{rot OR} = 0,$$

hence:

$$\text{rot AND} = -\,\text{rot OR}$$

Proof

$$\text{rot}\wedge(p,q) = \frac{\partial\wedge}{\partial p} - \frac{\partial\wedge}{\partial q} = q - p,$$
$$\text{rot}\vee(p,q) = \frac{\partial\vee}{\partial p} - \frac{\partial\vee}{\partial q} = \bar{q} - \bar{p} = p - q.$$

□

The **rot AND** and **rot OR** compute the comparison function, forward and backward, respectively:

$$\text{rot}\wedge(p,q) = \langle p|\blacktriangle|q\rangle$$
$$\text{rot}\vee(p,q) = \langle p|\blacktriangledown|q\rangle$$

where

$$\langle p|\blacktriangle|q\rangle = -\langle p|\blacktriangledown|q\rangle.$$

Because free (topo)logical charges do not exist but are an emergent effect, the corresponding divergences must be zero. Logical differentiation results in superposed states which are information singularities. The important element of this scheme is that consciousness gains access to altering quantum evolution. To determine that critical feature, we express the information singularities as follows:

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} -e^{-2\pi i(n+\frac{1}{2})} \\ e^{-2\pi i(n+\frac{1}{2})} \end{pmatrix}$$

Each run of a logical rotor inserts or deletes the action quantum from the loop, while both components must remain strictly coherent. The twist of the

lower component clockwise is equivalent to a twist of the upper component anticlockwise. Setting the topological energy of the rotor T equal to

$$T = 2\pi(n + \tfrac{1}{2}),$$

we can write the corresponding operator wave as

$$\hat{C}(T) = \cos T - i \sin T$$

which is equivalent to

$$\hat{C}(n) = e^{-2\pi i(n + \frac{1}{2})} = e^{-iT}$$

Making use of the relation between topological potential and quantum energies:

$$H = \hbar T,$$

$\hat{C}(n)$ becomes

$$\hat{C}(n) = e^{-\frac{i}{\hbar}H}$$

which is nothing else but the quantum evolution operator:

$$\hat{C}(n) = U(H) = e^{-iT}$$

We are given an important reason to believe that logic is directly linked to quantum evolution. *Free will* is a logical rotor which must have access to underlying fundamental processes in the brain.

SELF-MEASUREMENT AND BACKREACTION

Logical rotors are self-measuring systems. Because a measurement is always defined with respect to a system of reference, the observables, be they physical or logical are always, relative observables. In the laboratory we never measure the absolute position of a particle, but the distance between the particle and some reference object. The reference system, which can be a macroscopic object or simply another heavier particle, is always there, even if one does not refer to it explicitly. Naturally, a relative measurement affects both the measured particle and the reference system. It is in this respect that logical measurement differs considerably both from its classical and from its quantum-mechanical counterparts. New features are present in the closed topology of self-referential measurement, in which the backreaction cannot be disregarded. Effective logical differentiation is a covariant differentiation; the reference system is not fixed but itself is in motion, inducing change in the basis. Logical differentiation in the presence of the quantum-mechanical backreaction gives rise to an effective topological potential. The induced topological effect is interpreted by our brain as a cognitive thought, thus providing a fundamental link between topology, quantum measurement and logical consciousness.

Awareness is a topological effect which can result in external action like speech or motion of the body, or internal action like the logical momentum which applies to the system itself. Suppose cognitive interiorization relies on

measurement of a spin σ in a direction defined by a reference particle R, itself a quantum particle. The total Hamiltonian is given by

$$H = H_R + H_\sigma + H_{R\sigma}$$

where H_R stands for the Hamiltonian of the reference particle, H_σ for the Hamiltonian of the spin, and $H_{R\sigma}$ is the interaction Hamiltonian. While in physical measurement $H_{R\sigma}$ is put in to account for the interaction between the measuring and measured systems, in a self-measuring system it arises naturally, inducing a topological effect whenever the spin is measured. For a self-measurement the interaction Hamiltonian dominates all the other terms, which then can be disregarded [Ref 78]. The appearance of the topological phase can be obtained by transforming to a quantum reference frame, where the spin observable becomes directly measurable and the backreaction felt by the reference particle corresponds to the topological potential. The potential describes the backreaction on the reference frame, which takes the form of a fictitious charge at the origin $r = 0$, with a flux in the z direction. In the absence of measurement the σ_x component of the spin is a constant of motion. Since the topological potential is equivalent to a pure gauge transformation, it causes no observable effect on the reference particle. During the measurement, the self-interaction rotates the spin towards the principal axis alignment σ_z which in turn leads to observable effects. The total phase accumulated in a cyclic motion of the reference particle is the path-independent topological phase equal to the topological phase of a logical rotor:

$$\infty(k) = \oint M dq = k\pi.$$

The consequences of self-measurement and backreaction on the reference particle which takes the form of a topological potential are more complicated for a fermionic spin which flips sign under a 2π rotation. But the accumulated topological phase can be registered by means of interference, which automatically cancels the ambiguity and constructs single-valued solutions. This purely topological effect depends crucially on the particular choice of vacuum.

In string theory similar effects are determined to one-loop order by calculating the one-loop renormalization of the gauge couplings in the presence of a background gauge field. The threshold correction is then deduced by comparing the coefficients of the gauge field strength terms in the bare and one-loop effective Lagrangians. Such a renormalization calculation is essentially analogous to standard field-theoretic one-loop vacuum polarization, and to the calculation of the running of gauge couplings, except that in string theory the full calculation involves evaluating the 2-dimensional worldsheet correlation function $< F^a_{\mu\nu} F_a^{\mu\nu} >$ on the torus, while in matrix logic calculation is carried out on the Möbius loop. The procedure in string theory generalizes the vacuum polarization calculation in such a way that the full backreactions are automatically incorporated. Similarly, the calculation on the Möbius

incorporates the nontrivial double-valuedness of the fermionic eigenfunctions and thereby preserves logical invariance, a counterpart of worldsheet conformal invariance.

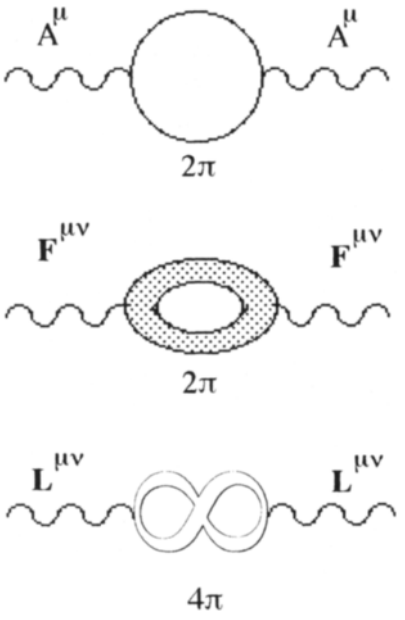


Fig. 18
In field theory calculation of the running of gauge couplings involves evaluating the one-loop vacuum polarization shown at the top. In string theory this polarization generalizes to become the torus diagram shown in the middle. In the diagram at the bottom vacuum polarizations give rise to unilateral topology where the 4π identity of a logical rotor cancels out negative values.

QUANTUM KNOTS

In this section we introduce the important new concept of quantum knots. A quantum knot is an achiral knot which cannot be continuously deformed into its mirror image, just as one quantum energy level cannot continuously evolve into another but is quantized. The basic example of a quantum knot is a trefoil which contains a defect that cannot be smoothed out.

The quantum knots we will describe by commutators, and for the sake of generality also by anticommutators.

The matrix-logical vacuum is a complex structure governed by symmetry, and must be further analyzed. Complete additive polarization of the vacuum **0** gives rise to an algebra of **80** logical (2×2) matrices which sum to zero:

$$\sum_{i=1}^{80} \mathbf{L}_i = \mathbf{0}$$

Among **80** matrices there are **26** symmetric,

$$\mathbf{X}_{ij} = \mathbf{X}_{ji},$$

representing **13** matrix-logical coordinates $\mathbf{X}_i^{sym}$ (i = 1, 2,... 13):

NOT, IS, AND, OR, OR^{-1}, NAND, NAND^{-1}, NOR, 1, H, $^{\bullet}$H, H., σ.

The weights of these coordinates are spread among the **9** values,

$$\mathbf{w}(\mathbf{X}_i) = -4, -3, -2, -1, 0, 1, 2, 3, 4$$

and the traces among the **5** different values,

$$\mathbf{Tr}\ \mathbf{X}_i = -2, -1, 0, 1, 2.$$

The choice of symmetric coordinate matrices reflects the fact that we consider the Noether Hermitian charges.

Next we introduce the *quantum* matrix coordinates Q_i, or the topological knots, which can be obtained either as the bosonic commutator

$$Q_i = [\mathbf{L}_i, \mathbf{L}_k]$$

or as the fermionic anticommutator

$$Q_i = \{\mathbf{L}_j, \mathbf{L}_n\}$$

where

$$Q_i \in \Omega,$$

$$\mathbf{L}_i \in \Omega,\ \mathbf{L}_k \in \Omega,\ \mathbf{L}_j \in \Omega,\ \mathbf{L}_n \in \Omega.$$

The quantum coordinates Q_i, represented by traceless matrices,

$$\mathbf{Tr}\ Q_i = \mathbf{Tr}\ [X_i, X_k] = \mathbf{Tr}\ \{X_j, X_n\} = 0, \quad \forall\ i, k, j, n,$$

whose weights, like the traces of symmetric matrix coordinates, are spread among the **5** values

$$\mathbf{w}(Q_i) = -2, -1, 0, 1, 2.$$

while

$$\mathbf{Rt}\ 0 = Q_i.$$

A quantum coordinate Q_i serves as a template, antisymmetric $[\mathbf{L}_i, \mathbf{L}_k]$ or symmetric $\{\mathbf{L}_j, \mathbf{L}_n\}$, for calculating a bosonic or β-topological charge by tracing over the squared quantum coordinate

$$\mathbf{Tr}(Q_i)^2 = \mathbf{Tr}[\mathbf{L}_i, \mathbf{L}_k]^2 = \mathbf{Tr}\{\mathbf{L}_j, \mathbf{L}_n\}^2$$

In this manner we identify the four β-charges

$$\beta = \{-2, 0, 2, 4\}$$

Numerically this is expanded by the factor of **2**, the standard set of logical quantum numbers of matrix logic:

$$\lambda = \{-1, 0, 1, 2\}$$

$$\beta = 2\lambda.$$

Each of the **13** symmetric coordinates X_i can be mapped to a corresponding quantum knot, antisymmetric and symmetric, except for the spin quantum coordinate σ which has only the antisymmetric template,

$$\sigma, N, \blacktriangle, M, \tilde{M}, \backslash, /, {}^{\circ}\!\rightarrow, \rightarrow_{\circ}, {}^{\circ}\!\leftarrow, \leftarrow_{\circ}, \dot{H}, H.$$

The number of bosonic templates significantly exceeds the number of fermionic ones:

Quantum coordinate Q_i	Number of bosonic templates $Q_i = [L_i, L_k]$	Number of fermionic templates $Q_i = \{L_j, L_n\}$
σ	180	0
$\blacktriangle$	136	40
M	80	40
$\tilde{M}$	80	40
\	136	40
/	136	40
${}^{\circ}\!\rightarrow$	104	32
$\rightarrow_{\circ}$	104	32
${}^{\circ}\!\leftarrow$	104	32
$\leftarrow_{\circ}$	104	32
$\dot{H}$	80	16
H.	80	16

The intersection of the set of symmetric coordinates $\{X_i\}$ and the set of quantum coordinates $\{Q_i\}$ is not empty

$$\{X_i\} \cap \{Q_i\} \neq \varnothing$$

and is populated by **8** fundamental traceless matrices:

$$\pm\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \pm\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \pm\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \quad \pm\begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix},$$

which up to a numerical factor are self-inverses:

$$Q_i Q_i \sim \mathbf{I}.$$

Because all Q_i are traceless, only traceless X_i can inhabit the intersection subset.

TEMPLATE SUPERSYMMETRIZATION

Supersymmetry requires that every bosonic contribution is balanced by a corresponding fermionic contribution. Logical supersymmetry can be obtained by the isomery, condensing a pair of identical nibbles, just as two electrons are condensed into the Cooper pair in the superconductivity effect. A topological current is a collective excitation whose carriers cannot be isolated into monocarriers. Although the quantum coordinate σ is distinguished by having only an antisymmetric template and a monospin code has no symmetric realization, the condensation of two spins,

$$\sigma + \sigma = 2\sigma$$

leads to supersymmetrization where bosonic contributions can be balanced by exactly the same amount of fermionic contributions. The supersymmetric code is given in Table.1.

The quantum spin coordinate σ is the z-component of spin, and with the aid of the commutator the double spin it is naturally connected with the x and y components

$$2i\sigma = [\sigma_x, \sigma_y].$$

In string theory supersymmetry is a symmetry whose charge transforms as a spinor, which relates the masses and couplings of fermions and bosons. In logic supersymmetry is also a spinor symmetry which balances the number of bosonic (commutator) and fermionic (anticommutator) contributions.

The spin commutator template $\sigma = [\mathbf{L}_i, \mathbf{L}_k]$ is unique and the most effective template for the generation of the bosonic topological charge

$$\beta = \mathrm{Tr}\, \sigma^2 = \mathrm{Tr}[\mathbf{L}_i, \mathbf{L}_k]^2$$

and the fermionic topological charge

$$\phi = \mathrm{Tr}\, \sigma = \mathrm{Tr}[\mathbf{L}_i, [\mathbf{L}_j, \mathbf{L}_k]]$$

where the triple commutator $\sigma = [\mathbf{L}_i, [\mathbf{L}_j, \mathbf{L}_k]]$ has **8160** different matrix-logical realizations, the highest number in the logical code.

THE ANTISYMMETRIC TEMPLATES $2\sigma = [L_i, L_k]$		THE SYMMETRIC TEMPLATES $2\sigma = \{L_j, L_n\}$	
1	-1 -1 -1 -1 -1 1 -1 -1	1	-1 -1 -1 -1 -1 0 0 1
2	-1 -1 -1 -1 0 1 -1 0	2	-1 -1 0 -1 -1 0 0 1
3	-1 -1 -1 -1 1 1 -1 1	3	-1 -1 1 -1 -1 0 0 1
4	-1 -1 1 -1 -1 -1 -1 -1	4	-1 0 -1 -1 -1 0 0 1
5	-1 -1 1 -1 0 -1 -1 0	5	-1 0 0 -1 -1 0 0 1
6	-1 -1 1 -1 1 -1 -1 1	6	-1 0 0 1 -1 -1 -1 -1
7	-1 1 -1 -1 -1 1 1 -1	7	-1 0 0 1 -1 -1 0 -1
8	-1 1 -1 -1 0 1 1 0	8	-1 0 0 1 -1 -1 1 -1
9	-1 1 -1 -1 1 1 1 1	9	-1 0 0 1 -1 0 -1 -1
10	-1 1 1 -1 -1 -1 1 -1	10	-1 0 0 1 -1 0 0 -1
11	-1 1 1 -1 0 -1 1 0	11	-1 0 0 1 -1 0 1 -1
12	-1 1 1 -1 1 -1 1 1	12	-1 0 0 1 -1 1 -1 -1
13	0 -1 -1 0 -1 1 -1 -1	13	-1 0 0 1 -1 1 0 -1
14	0 -1 -1 0 0 1 -1 0	14	-1 0 0 1 -1 1 1 -1
15	0 -1 -1 0 1 1 -1 1	15	-1 0 1 -1 -1 0 0 1
16	0 -1 1 0 -1 -1 -1 -1	16	-1 1 -1 -1 -1 0 0 1
17	0 -1 1 0 0 -1 -1 0	17	-1 1 0 -1 -1 0 0 1
18	0 -1 1 0 1 -1 -1 1	18	-1 1 1 -1 -1 0 0 1
19	0 1 -1 0 -1 1 1 -1	19	1 -1 -1 1 1 0 0 -1
20	0 1 -1 0 0 1 1 0	20	1 -1 0 1 1 0 0 -1
21	0 1 -1 0 1 1 1 1	21	1 -1 1 1 1 0 0 -1
22	0 1 1 0 -1 -1 1 -1	22	1 0 -1 1 1 0 0 -1
23	0 1 1 0 0 -1 1 0	23	1 0 0 -1 1 -1 -1 1
24	0 1 1 0 1 -1 1 1	24	1 0 0 -1 1 -1 0 1
25	1 -1 -1 1 -1 1 -1 -1	25	1 0 0 -1 1 -1 1 1
26	1 -1 -1 1 0 1 -1 0	26	1 0 0 -1 1 0 -1 1
27	1 -1 -1 1 1 1 -1 1	27	1 0 0 -1 1 0 0 1
28	1 -1 1 1 -1 -1 -1 -1	28	1 0 0 -1 1 0 1 1
29	1 -1 1 1 0 -1 -1 0	29	1 0 0 -1 1 1 -1 1
30	1 -1 1 1 1 -1 -1 1	30	1 0 0 -1 1 1 0 1
31	1 1 -1 1 -1 1 1 -1	31	1 0 0 -1 1 1 1 1
32	1 1 -1 1 0 1 1 0	32	1 0 0 1 1 0 0 -1
33	1 1 -1 1 1 1 1 1	33	1 0 1 1 1 0 0 -1
34	1 1 1 1 -1 -1 1 -1	34	1 1 -1 1 1 0 0 -1
35	1 1 1 1 0 -1 1 0	35	1 1 0 1 1 0 0 -1
36	1 1 1 1 1 -1 1 1	36	1 1 1 1 1 0 0 -1

Table 1

Supersymmetric realization of the double spin 2σ.

Other supersymmetric coordinates are the duplicated logical momentum operator and its transpose

$$2M \text{ and } 2\tilde{M}$$

and the symmetric Hadamardians representing the unbroken topological phases

$$2\dot{H} \text{ and } 2H..$$

Thus, altogether there are five supersymmetric matrix coordinates:

$$\{2\sigma, \pm 2M, \pm 2\tilde{M}, \pm 2\dot{H}, \pm 2H.\}.$$

Among these, the double spin 2σ is most effective code; the number of realizations for other supersymmetric coordinates is three times less than for the double spin.

INFORMATION SOLITONS

A topological theory of the brain has contact to the spin in quantum field theory. As we indicated earlier a supersymmetric topological current is similar to a superconductive flux which is associated with nonclassical states involving macroscopic numbers of paired fermions, tunnelling between the minima of the effective potential. In 4-dimensional spacetime there are solitonic field configurations which behave like fermions under rotations. A nonlocal term is an integral of a five-dimensional topological density. An interchange of two nonoverlapping solitons of odd degree produces a phase factor -1 of the corresponding states in Fock space. In 4 dimensions classical solitons can be quantized only as fermions or bosons, but in higher dimensions there is an option of quantizing solitons such that they do not obey either canonical commutation or anticommutation relation.

The fact that the logical rotors can be quantized as solitons which are either Bose-Einstein or Fermi-Dirac motivates us to seek the answer to the physical mechanism of logic in a higher dimensional framework. The brain may be Bose-Einstein or Fermi-Dirac, one at a time, but the mind, obeying Matrix Statistics, is neither or simultaneously both! As soon as a logical rotor is formed, the (topo)logical current will propagate without resistance, reduced to zero. To obtain a solitonic solution to logical rotors, we start with the states which are produced from the logical vacuum by the action of (topo)logically nontrivial gauge transformations. With the aid of logical derivatives the solitons are classified by a winding number in the compactified space with a fixed metric and orientation which can be positive or negative.

According to Kaluza-Klein a single theory in five dimensions - one of time plus four of space - could combine electromagnetism and gravitation if one of the space dimensions were compactified, meaning that the size of the universe in that dimension is small compared to the smallest distances probed by experiments. Originally, Kaluza-Klein theories did not regard the extra dimensions as being physical; these arose as imaginative ways to unify gravity and electromagnetism. In a topological framework the fifth dimension was associated with information [Ref 79]. Today, a consistent quantum theory of gravity is embedded in a matrix theory which requires **11** dimensions. In superstring theories, particles are vibrations of the basic string, and there is an infinite sequence of such vibrations.

Cognitive systems that actually occur in the world must be described as solutions to the dynamical logical equations when logical theory is recast into matrix dimensions. Matrix theory plays an essential role in helping untangle

the underlying physics of the thinking brain in a topological framework. In a nonlinear approach the logical knots appear as stable solutions to the truth development equations resulting in superbits [Ref 91, 92, 93]. Knotlike configurations also appear as stable solitons in certain relativistic quantum field theories. In a surprising turn of events, the problem of the cognitive brain parallels the problem of black holes in cosmological studies. In both instances we are dealing with 4-dimensional singularities or 4-dimensional Klein bottles. If black holes are described as a collection of D-branes with attached strings, the entropy computed from the classical solution agrees with the black hole entropy given by the number of states of strings and D-branes, although a general proof of this result in Minkowski spacetime is still lacking. Spinning massless strings of finite length in 4-dimensional space behave like particles with nonzero rest mass and intrinsic spin. A string formed at a phase transition very early in the history of the universe might provide the density perturbations to start the condensation process. Knotted strings have solitonic behaviour in the sense that localized string perturbations propagate without losing coherence. In the case of the collision of two string packets the coherence is lost, but after collision both packets acquire their initial shape and propagate freely with the same velocity.
The extended objects, such as p-branes and D-branes, are nonperturbative solutions of the low energy effective action. They have been instrumental in the analysis of dualities in string theory and in providing insights into string dynamics in various dimensions. At present, it is accepted that the five string theories are manifestations of various phases of one underlying matrix theory. The low energy effective action of the M-theory can be identified with that of D=11 supergravity, and there is mounting evidence that M-theory encompasses and unifies string theories and string dynamics. The low energy effective action of this theory contains the antisymmetric tensor, besides the graviton, in its bosonic sector. Therefore, the membrane that couples to the three index antisymmetric tensor field is expected to be the natural extended fundamental object in eleven dimensions, with 5-brane as its solitonic counterpart; consequently, a lot of attention has been focused on the study of the branes in D=11 theory and their implications.

An important question is whether the fundamental supermembrane can provide the degrees of freedom of M-theory and whether the quantum theory is a consistent one. If the quantum theory is inconsistent, the answer to the above question is negative. This issue of consistency is very intimately connected to the behaviour of the U(n) matrix model with large n. In the recent past the proposal of the M(atrix) model has led to very interesting developments. The m(atrix) theory reveals various salient features of the M-theory. According to the basic postulate of the m(atrix) theory, the dynamics of the eleven-dimensional M-theory finds its description in the many-body quantum mechanics of n D0-branes of the type IIA theory in the limit $n \Rightarrow \infty$. Through dualities the compactified m(atrix) theory has close connections with super Yang-Mills theories. In this context, the attempt must

be made to unravel how much matrix logic 'knows' about spacetime, and what is a consistent way to encode the logical matrix into spacetime. A solution to this problem is based on the ideas of topology, which concerns the global properties of objects that remain unchanged when we deform them locally. The brain stores and manipulates information in a global form that is resistant to local perturbation, and has the topological character of a soliton. If we continuously deform the logical knot without cuts, the soliton remains invariant and transports the same information. In seeking physical implementations of cognitive logic, we are led to contemplate a realization of cognitive solitons in which information is encoded in a form that can be measured and manipulated through topological interactions that are immune to local disturbances. One of the discoveries of recent decades has been that intelligent logic must be non-Abelian. Non-Abelian effects are known to arise in systems with non-Abelian gauge fields. There are some fundamental non-Abelian gauge fields, but unfortunately not very many, and none of these seem to be suitable for the brain, where non-Abelian effects arise as collective topological phenomena in two-dimensional systems. The topological properties of the quantum-theoretic brain are insensitive to microscopic details. The cognitive content is stable even as we deform the Hamiltonians of the theory. If the deformation is sufficiently small, we can study its effects using perturbation theory. Two types of nonperturbative effects are possible. The ground state of the theory is a topological condensate with a long-range interaction between charged solitons which cannot be separated. In a gauge theory, this would correspond to electric confinement. Alternatively, a condensate of Noether charges might appear in the ground state. Then the topological excitations would be confined, and the long-range effects would be destroyed. In a gauge theory, this will correspond to magnetic confinement. There is a phase boundary, beyond which either confinement will be encountered. The identification of this boundary will lead to verification of the theory that behaves as specified. A hope is that we can find gauge phenomena emerging as collective effects in systems with only short-range interactions.

LOGICAL MEMBRANE ACTION

The study of string dualities has revealed that five consistent pertrubative superstring theories in **10** dimensions are closely linked to each other. They represent five corners of the moduli space for vacuua of the M-theory, most conveniently viewed as a theory in **11** dimensions. M-theory relies crucially on the idea of supersymmetry, an ultimate model aimed at

joining fermions and bosons into a general description. Supersymmetry requires that for each boson having integer spin 0, 1, 2, ... there is a particle with the same mass but half-integer spin 1/2, 3/2, 5/2, and vice versa. Although no such superpartners have yet been found, theorists have retained belief in supersymmetry, primarily because it provides a framework in which the known fundamental forces may be united.

The most general superalgebra in **11**-dimensional spacetime is spanned by a Majorana spinor supercharge Q_α (α = 1, ... , 32), the **11**-momentum P_m, a **2**-form central charge Z_{mn} and a **5**-form central charge $Y_{m_1 \ldots m_5}$. The only nonvanishing anticommutation relation is the anticommutator

$$\{Q_\alpha, Q_\beta\} = (C\Gamma^m)_{\alpha\beta} P_m + \frac{1}{2}(C\Gamma^{mn})_{\alpha\beta} Z_{mn} + \frac{1}{5!}(C\Gamma^{m_1 \ldots m_5})_{\alpha\beta} Y_{m_1 \ldots m_5}$$

where Γ^m are constant Dirac matrices, $\Gamma^{m_1 \ldots m_p}$ (p = 2,5) are their antisymmetrized products, and C is the real charge conjugation matrix. The structure of this superalgebra reflects important aspects of M-theory.

M(atrix) theory can also be thought of as discretized light-cone quantization, with compactification on a lightlike circle. Accordingly, the longitudinal momentum is quantized and transverse **d** coordinates are compactified on a **d**-dimensional torus. M-theory is a limit of maximally supersymmetric quantum mechanics of *N*x*N* matrices describing D0-branes. The action of the matrix model is expressed by a *N*x*N* matrix, and the dimensionally reduced Hamiltonian is given by

$$\frac{1}{2g} \mathrm{Tr}\left(\sum ([X_a, X_b])^2 - \sum [\theta^\alpha \Gamma^a_{\alpha\beta} [\theta^\beta, X_a]] \right)$$

where the first term $([X_a, X_b])^2$ accounts for the bosonic contribution and the double commutator for the fermionic contribution.
The superalgebra corresponding to the supersymmetry transformations contains the **2**-form central charge

$$Z_{ab} = \frac{i}{2} \mathrm{Tr}[X_a, X_b]$$

and the energy of the membrane state is proportional to the square of the membrane charge

$$E \sim -\frac{1}{4}(\mathrm{Tr}[X_a, X_b])^2$$

The dimensional reduction from **D=11** to **D=10** produces a string out of a membrane. To construct a superstring stress energy tensor we need to relate the matrix variables in the Hamiltonian to the superstring variables, under the reduction to **D=10**. The commutator $[X_a, X_b]$ in the strongly coupled superstring theory corresponds to the membrane topological charge in matrix theory. M-theory is essentially the theory of a single object, but one which takes on various forms in different dual formulations of the theory.

The branes convolute to space with a varying number of internal holes, violating the law of classical topology. It is conjectured that strings ultimately choose the spacetime with the lowest energy and inhabit it, giving rise to the

elementary particles and forces as we know them. The role played by the fermionic and bosonic parts of the Hamiltonian in matrix theory is crucial. The fermionic part is given by the double commutator

$$\Gamma^{a}_{\alpha\beta}\,[\theta^{\alpha},[\,\theta^{\beta},X_{a}\,]]$$

Integration over fermionic variables θ^{α} yields the determinant Pfaff, which identifies the map for the bosonic potential

$$\mathrm{Tr}[\,X_{a},X_{b}\,]$$

This map

$$W^{c}=\gamma^{c}_{kk}[\,X_{k},X_{k}\,]$$

defines the new variable W^{c} in terms of the membrane topological charge. For logical membranes the quantum knot's commutator optimally encodes topological charges

$$[\,X_{a},X_{b}\,]\rightarrow\sigma$$

The commutator tends to σ, indicating that bosonic and fermionic contributions to the supersymmetry of the **11**-dimensional supermembrane, even though appearing on essentially different grounds, in the end lead to one optimal energy solution. Membrane undulations are thought to give rise to particles and asymmetric degeneracy, probably due to the fact that the thinking brain exists in the world of particles and not of antiparticles. These facts suggest that the realization of supersymmetry as translations in superspace is not the ideal way to think about it. It appears that the entire world sheet supersymmetry of superstring theory is contained in just the bosonic part of the m(atrix) theory Hamiltonian. The fermionic part of the Hamiltonian (action) is exactly the one which gives us the picture-changing transformation we need.

Applying this method consistently to matrix logic emphasizes the connection between the fundamental coding octave in matrix logic and the Duff's superstring octonionic membrane. A local symmetry of a brane action and spacetime supersymmetry can be converted into the global supersymmetry of the worldvolume. The intelligence code is a self-dual matrix code whose normed algebras may exist in the form of real scalars, complex numbers, quaternions and octonions, so generally the codewords are multiples of **4**, with the associativity law breaking down for the octonionic regime.

In m(atrix) theory we reduce the Lagrangian to **D=0**, that is to a zero-dimensional *N*x*N* matrix model by choosing the gauge 0 and dropping the kinetic term in the action. The thus reduced Lagrangian, then, is given by

$$\frac{1}{2g}\mathrm{Tr}\Big(\sum([\,X_{a},X_{b}\,])^{2}-\sum([\theta^{\alpha}\Gamma^{a}_{\alpha\beta}\,[\theta^{\beta},X_{a}]]\Big)$$

where the squared commutator refers the bosonic contribution and the double commutator to the fermionic contribution to the overall action.

For a logical membrane we eliminate the factor 1/g from the equation, by setting it to unity, and retracing the action we get

$$\frac{1}{2}(\sum [L_a, L_b]^2 - \sum [L_c[L_d, L_e]])$$

where the logical coordinates L_a, L_b,... can be chosen from the general code of logical matrices L_i (i = 1,..., 80). For the sake of simplicity we consider a single action and drop the summation signs:

$$\frac{1}{2}([L_a, L_b]^2 - [L_c[L_d, L_e]])$$

The aim of these transformations is two-fold. Expressing the Lagrangian in terms of topological charges only, namely the bosonic topological charge

$$[L_a, L_b]^2$$

and the fermionic topological charge

$$[L_c[L_d, L_e]],$$

we seek to establish whether the sum of (topo)logical charges can diagonalize the synergy operators **H** or **1**, and if so to determine the conditions under which this occurs. The simulation of the random walk through the excited states of the logical vacuum has determined that the logical action expressed by the diagonalized logical operators $\mathbf{H}_{diag}$ and $\mathbf{1}_{diag}$ is indeed the optimal (quanternion/octonion) quantum code

$$\frac{\mathbf{H}_{diag}}{2} = \frac{1}{2}([L_a, L_b]^2 - [L_c[L_d, L_e]])$$

or explicitly

$$\frac{\mathbf{H}_{diag}}{2} = \frac{1}{2}\left(\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} - \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\right) = \frac{1}{2}\begin{pmatrix}0 & 0\\ 0 & 2\end{pmatrix}.$$

Logical energy is the **AND** sum total of the topological bosonic and fermionic energies. The identity bosonic charge **I** is a logical volume, obtained by squaring the matrix logical coordinates $\sigma = \mathbf{N}_{diag}$ and **N**. It is the optimal charge with **180+136 = 316** realizations, while

$$\sigma = [L_c, [L_d, L_e]]$$

is the optimal fermionic logical charge with **8160** realizations. Left to its own devices a logical brane will choose to occupy a minimax state with maximal logical energy and minimal Noether energy. The logical energy is treated as topological energy associated with a knot in a set of information field lines. Numerically, the overall logical charge of a logical action $\mu(L)$ equals the quark charge, independently of whether the logical system is rotated into principal axis alignment:

$$\mu(L) = \int_0^1 dq \frac{\mathbf{H}_{diag}}{2} = \int_0^1 \mathbf{AND}\, dq = \frac{1}{3}$$

or not:

$$\mu(L) = \int_0^1 \mathbf{H}\, dq = \int_0^1 (\mathbf{I} - \mathbf{N}) dq = \frac{1}{3}$$

Until now the determination of the logical energy action was based one way or the other on the ideas of unitary quantum evolution and the Schrödinger equation of motion. We now establish a more fundamental line of deduction whereby the logical membrane action emerges as the optimal solution for the topological charges contributing to the total energy of the octonionic logical branes.

FROM BLACK HOLES TO GRAY BRAINS

Do the symmetries of the logical mind and the fundamental symmetries in physics follow from the some general basic laws? If consciousness is a consequence of a natural law, and not a random artifact, the potential for the emergence of intelligent systems must have existed already in the initial singularity of the Big Bang. The mind creates the patterns of thoughts, none of which are precisely the same. A neuroscientist assumes that these patterns have inherited their structure from the way the cortical neurons interact with each other by means of bioelectrical spike potentials and neurotransmitters. Further down the road we descend to the level of supramolecules and molecules. The molecular reactions, in turn, are determined by the composition of the electronic shells of the atoms. From here on we must deal with the symmetries of quantum theory, which in turn should be derived from unitary symmetries, and ultimately from supersymmetry. There are the **U(1)** symmetry of the Maxwell equations, where "**1**" stands for the fact that there is only one photon, and the **SU(2)** which rotates proton and neutron or electron and neutrino in the electroweak theory. The strong interactions can be represented by the symmetry group **SU(3)** which rotates three particles that make up the strongly interacting particles, the baryons. In the grand unified theory the group **SU(5)** can shuffle five particles: the electron, neutrino and three quarks. A search continues for an unknown group **?(7)** which might account for the graviton and the gravitino, two fundamental particles required for a complete field theory. A theory which gives a simple origin of the symmetries found in particle physics is the product

$$U(1) \times SU(2) \times SU(3) \times SU(5) \times ?(7).$$

There is the notion of topological defects, infinite or closed cosmic strings, conjectured to have been formed at the time of the cataclysmic event of the Big Bang. Although the theory of the Big Bang is grotesque and technically unverifiable, either due to finiteness of time or due to the quantal response, it is useful and stimulating. Matrix logic suggests that the fundamental quantum particles can be treated as topological knots, distinguished by the different number of crossings and helicity. If only we could better understand knots, we could unravel both the secrets of innate matter and living and 'thinking' matter. At the beginning of time the universe was a perfectly symmetrical condensate of Noether and topological energies, with the electromagnetic, weak and strong forces and gravity all tied up in a single singularity, the prime superknot. When the singularity exploded and the knot began to cool the gravity broke off from the **?(7)** symmetry. In the resulting **SU(5)** group the strong force was united with the electroweak interactions but then **SU(3)** symmetry broke into **SU(2)** and the knot-free **U(1)**. After 10^{-3} seconds, at temperatures of about 10^{14} **°K**, quarks began to condense into neutrons and protons, which soon - after 3 minutes - condensed into stable nuclei. 300 thousand years down the road, electrons started to gather around nuclei and atoms began to form, the universe became transparent to light and outer space became black. Five billion years transpired before the first galaxies appeared. Ten billion years later the solar system was born, and several billion years after that the remnants of topological energy were sufficiently condensed to give rise to the **5**-fold symmetry of life and ultimately consciousness.

There is a huge discrepancy between the time scales which were required for the creation of the physical universe and that of the mental *universe*. It took only seconds and minutes for the nuclei of atoms to form but many billion of years for the '*atoms*' of the mind to emerge. Why does the symmetry of matter and forces break down so rapidly, allowing fast development of the universe, while the symmetries of life and mind's are so exceedingly slow to manifest themselves? There is reason to believe that the explanation goes with its roots to the question of entropy and the related subject of (bio)logical information. The Big Bang was a sequence of phase transitions, leading to the increase of entropy in the expanding universe. Matter and the various effects associated with it are consequences of phase transitions. It is known that phase transitions, such as crystallization, magnetization or more simply freezing or boiling, are not smooth but abrupt events which are initiated by microscopic defects in the atomic structure that grow rapidly. The defect can be thought of as a quantum error or a fault which acts as a seed around which the transition effect expands exponentially. In a biological system, producing free energy, the entropy process is slowed down, arrested and even reversed in the operations of the intelligent brain. Logic and consciousness enter into the picture, finally after many billion years finding a dynamical support living system in which to manifest themselves.

LOGICAL BANG
BLOWING UP THE FAREY SEQUENCE

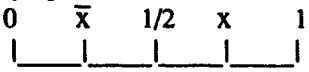
The groups of unitary symmetry are central to the formalism of quantum theory, starting with the Pauli **SU(2)** group for spin, Heisenberg's isospin, Wigner's irreducible representations, and ending with Gell-Mann's quarks, Yang-Mills's fields, strings and **11**-dimensional supergravity. Matrix logic offers a different language. which also has ramifications for cosmology. If information evolution accompanies the unitary evolution of the Big Bang, we must consider a concomitant Logical Bang. As we determined earlier the set of integral logical charges is a finite increasing sequence of irreducible fractions, between 0 and 1 [Ref 94]:

$$0, \frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{5}{6}, 1 \in [0, 1]$$

It is possible to determine logical charges through unitary symmetry groups, such as Gell-Mann's **SU(3)** or the Okubo group, but a different way of deduction exists in matrix logic.

Enter a Logical Bang on the unit interval, where the integer logical charges 0 and 1 are Boolean values:

Divide the interval into two halves, the universe and anti-universe, populating them with the complementary 'particles' x and $\bar{x}$, such that $\bar{x} + x = 1$:



The fractional logical charge 1/2 is now available. How can we further expand the unit interval in a fundamental way, so that the complete set of logical charges becomes manifest?

Definition. A Farey sequence F_n of order **n** is a finite increasing sequence of irreducible fractions, between 0 and 1 inclusive, whose denominators do not exceed the natural number **n**. If a/b and c/d are any two adjacent terms of the sequence, then the determinant of a matrix erected from these terms is -1:

$$\det\begin{pmatrix} a & c \\ b & d \end{pmatrix} = -1.$$

The classical definition of the sequence of C. Haros (1802) and J. Farey (1816) did not employ matrices, as our definition does, but it is easy to see that both are equivalent. The number of terms in a Farey sequence will be called the dimension of the sequence. Until F_7 these follow the prime number series, with collapse occurring at F_7:

$$2, 3, 5, 7, 11, 13.$$

Expressing the Boolean boundaries 0 and 1 as 'fractions' 0/1 and 1/1 respectively, we blow up the logical interval in steps as follows:

F_1 $\frac{0}{1}, \frac{1}{1}$

F_2 $\frac{0}{1}, \frac{1}{2}, \frac{1}{1}$

F_3 $\frac{0}{1}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{1}{1}$

— —

F_4 $\frac{0}{1}, \frac{1}{4}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{1}{1}$

F_5 $\frac{0}{1}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{1}{1}$

— —

F_6 $\frac{0}{1}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{1}{3}, \frac{2}{5}, \frac{1}{2}, \frac{3}{5}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{1}{1}$

— —

F_7 $\frac{0}{1}, \frac{1}{7}, \frac{1}{6}, \frac{1}{5}, \frac{1}{4}, \frac{2}{7}, \frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{1}{2}, \frac{4}{7}, \frac{3}{5}, \frac{2}{3}, \frac{5}{7}, \frac{3}{4}, \frac{4}{5}, \frac{5}{6}, \frac{6}{7}, \frac{1}{1}$

— —— —— —

The process accelerates in a nonlinear fashion. Quark charges are created in F_3 but the universe must inflate to the F_6 for the iqons, the elementary logical charges, to emerge. Significantly, the Farey sequence growth, starting from F_3, carries within itself the seeds of its own destruction. The crunch of certain sequences, excluding F_4 which is not contractible, may occur because we can expel from a sequence certain terms (underlined) without breaking down the condition det = -1 in the contracted universe. The F_3 contracts, because we can exclude the quarks charges from it, returning to F_2. This is not possible for the F_4, which is stable, but F_5 and F_6 live without quarks, while in the F_7 six particles, in appropriate order, can be eliminated, without offending the unit determinant condition, and changing the dimension of the sequence from 21 to 15.

THE THEORY OF EVERYTHING

The Theory of Everything attempts to connect quantum theory and relativity. It is fair to say that a theory seeking a status of a theory of *everything* must also be able to account for intelligence, that definite fact of the universe. The Theory of Everything, as it stands, apparently does not

include consciousness. It is a sterile theory of spacetime, particles and fields, devoid of any content. This is a major flaw which requires remedy.

Before the Big Bang matter was a singularity of infinite density. In a topological framework infinite density can be put in correspondence to a knot with the infinite number of crossings. In a loop, made of a many times twisted string, after some point with each twist of the string its length increases, while the linear space it occupies shortens, approaching zero. The unknotting of the infinite knot should be accompanied by a growing supply of energy. The source of the amiss energy responsible for the observed accelerating expansion in cosmology is unclear, and puts contemporary physical theory to a crucial test. Topological energy very well could be responsible for a mysterious force causing the accelerating expansion of the universe. Surprisingly enough, the Big Bang in cosmology, the Logical Bang of consciousness and Matrix Theory, all go with their roots to the field of topology, providing for the emergence of fermions, quarks, neutrinos, and the 5-dimensional space of the mind.

The grand unified theories predict that the early universe underwent successive phase transitions which were accompanied by the formation of topological defects, such as cosmological strings and string loops. A cosmic string under certain conditions exhibit a supercurrent which alters the string's spacetime. There is a nonzero momentum along the string in the direction of the current flow, and this modifies the properties of the cosmic string and loops. Charged loops have sufficient angular momentum to stabilize centrifugally. The inclusion of momentum when constructing the energy-momentum tensor leads to exotic spacetime properties. The Thatcher-Morgan tensor contains components

$$T_{tz} = T_{zt} = -M\,\delta(x)\,\delta(y)$$

which describe the momentum in the string along the z-direction. Adapting Cartesian coordinates (t, x, y, z) with metric signature diag(+,-,-,-), in natural units $\hbar = c = 1$, we get

$$T_{\mu\upsilon} = \delta(x)\delta(y)\begin{bmatrix} E & 0 & 0 & -M \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -M & 0 & 0 & -S \end{bmatrix}$$

where E, M, S are energy, momentum and tension respectively. Due to nonzero momentum spacetime acquires mixed metric components, which give rise to oscillatory behaviour for g_{tt}, g_{zz} and g_{tz}. This oscillation of the metric components form multiple horizons at various distances from the string loop. In the string spacetime particles are deflected in the z-direction as they approach the string, effectively isolating the defect from the outside universe.

The principles of relativity have provided a conceptual framework for modern physics and the understanding of the universe in fundamental terms. According to these principles the laws of physics should not depend on particular systems of reference. Relativity theory, however, was born in a bosinic era, prior to the discovery of fermions and quarks, the fundamental particles with fractional spin and charges. As we indicated early, the world

looks very different for a bosonic observer and for a fermionic observer. Each observer may formulate a different theory of physics and one is led astray in trying apply both theories simultaneously. Quarks present an even greater challenge. Imagine how the world would look to someone residing on a quark. Plato's prisoner, caged in a cave, having to reconstruct the world through shadows, would be a trivial example in comparison to an observer on a quark. Bizarre things would manifest themselves. The world would split into incomplete parts and the sides of objects would disappear, just as they 'disappear' on a Möbius strip, and 'materialize' when we cut the strip. Apart from becoming quantized, the observer residing on a quark would discover that one-third or two-third of his charges have mysteriously vanished. Due to reasons that are very different from those of Bohr's complementarity principle, the reality of the observer is fundamentally incomplete. The only way the observer could compensate, if at all, for the incompleteness of his world is through abstract thinking and consciousness. Consciousness thus becomes essential for the Theory of Everything. Since the concept of spatial localization is inapplicable to the thought process, which is independent of the way we measure distances, the origin of logic is topology, providing the framework in which the physics of consciousness can be properly understood. Einstein's equivalence principle does not distinguish between gravity force and accelerated motion. The concept of topological equivalence is applicable everywhere, from cosmology to strings, and it is intriguing to speculate that the equivalence principle could be part of a wider principle of topological equivalence.

INFORMATION COLLAPSE

Physics studies the phenomenon of gravitational collapse leading to the formation of spacetime singularities or **black holes**. A black hole is 'black' because the gravity force there is so intense that nothing, not even light, can escape from it. An event horizon is formed around a black hole, and whatever crosses it never returns. If you drop a coin into a black hole, you'll never know whether it lands heads or tail. The objects that are sucked into the hole are literally squeezed out of existence. Just as energy vanishes from volumes of space without a trace, at the black hole singularity the laws of physics break down completely, and even space and time disappear. In the bizarre world behind the Schwarzschild radius, time stops. Information falling into a black hole is lost and does not reappear when the black hole subsequently evaporates. This is inconsistent with the idea of unitary quantum-mechanical evolution and apparently either requires a significant modification of quantum mechanics or a significant breakdown of the usual understanding of locality. The essence of the argument is very simple: because of the highly distorted causal structure of the black hole spacetime, the emitted radiation is actually on the same time

slice as the collapsing body that disappeared behind the event horizon. When the information that was initially encoded in the collapsing body is eventually emitted, then that information must be in two places at once. In other words, the quantum information must be cloned, which is known to be impossible under the usual assumptions of quantum theory [Ref 108]. This suggests that not all physical processes are governed by unitary time evolution, and therefore the laws of quantum theory need revision.
A black holes possesses entropy, which measures the disorder of a system by accounting for the number of quantum states available. The quantum theory of a black hole suggests that black holes are in fact not entirely black but radiate energy. In the classical theory of black holes, at infinity it is always possible in principle to observe the collapsing black hole and there would be no loss of information. But in quantum theory, the number of photons emitted before the event horizon forms is finite, and loss of information is unavoidable.

Surprising similarities found between the problems of cosmology and the problem of the conscious brain lead us to consider singularities of a different kind, not necessarily connected with gravity, namely information collapse, occurring in systems with nonorientable topology [Ref 79]. The information loss could occur not only for macroscopic black holes, but on a microscopic scale as well, because of virtual black holes that would appear and disappear in the vacuum state. The idea basically boils down to the need to consider vacuum **grey holes** which have direct repercussions for consciousness. Gravitational collapse leads to the formation of a black hole, while information collapse leads to the formation of a grey hole [Ref 79]. Nonorientable topologies which before were considered merely as mathematical curiosities, enter physics in fundamental way as a basis of the mind. Consciousness is a quantum information grey hole. Information falling into a black hole is lost and does not reappear when the black hole evaporates. Information entered into a grey hole is locked-in within the **5**-sphere with the unit matrix radius, analogous to the Schwarzschild radius of the black hole.

It is useful to compare these ideas with the scheme of the Black Box. Consciousness is a non-Hermitian Black Box which differs from the Black Box of information theory or from the S-matrix method in quantum physics in that once something is inside the cognitive Box, there is no means of observing it. This is not a matter of having crude instruments, it is a law of nature. While an ordinary Black Box is a system whose internal organization we *choose* not to know, a cognitive Black Box is a new fundamental source of uncertainty. We simply *cannot* gain knowledge of it through laboratory measurement, and this has nothing to do with Heisenberg's uncertainty or Bohr's complementarity. The Black Box of consciousness is an information singularity, a grey hole which, as opposed to a black hole, retains partial contact with the outside universe: information falling into a grey hole can be retrieved. After perceptual information crosses over to the cognizable realm, it can be 'recovered'. The nonorientable topology of a grey hole is realized through vacuum loops whose logical **information horizon** is analogous to

the **event horizon** of a black hole. The realization of grey holes by virtual vacuum loops leads to a potentially serious conflict with the law of physics: mass conservation. When a vacuum loop swallows a nibble or a pair of actual particles which then vanish, the mass of the total system should decrease. This bizarre situation of the cognitive brain sucking particles into a vacuum would undermine mass conservation, unless we allow a grey hole to create ('radiate') information and through this process to restore conservation. This explanation of the creative mind has a strong affinity to the Hawking theory according to which quantum effects will cause Black Holes to emit particles, evaporating it in the process. **Black holes radiate energy; grey holes create information.**

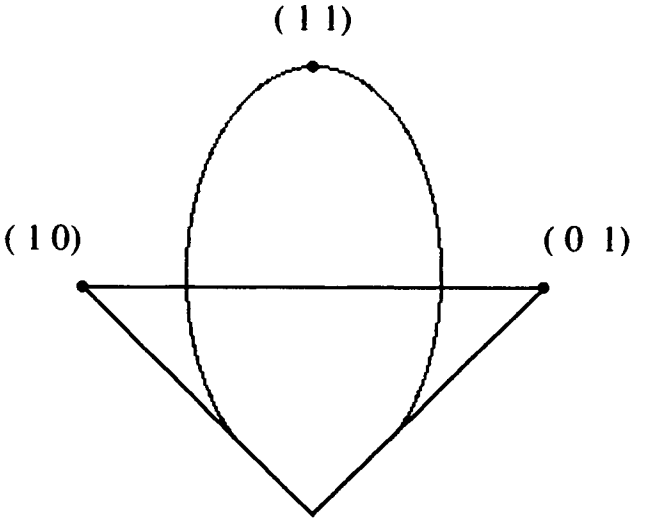
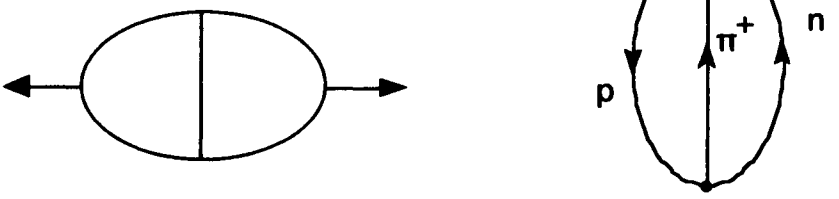


Fig. 19
Formation of a grey hole. The 'logical cone' bends inward towards the singularity (1 1), which is similar to the worldcone of a black hole where the worldlines tip so that light falls straight into the black hole's centre.

The strange forces of the vacuum can be created and observed in the two different ways: top-down and down-up. Otto von Guericke demonstrated the unsuspected strength of a vacuum with two teams of horses making a futile attempt to separate two metal hemispheres bound by nothing more than the absence of air in the sealed space between them. In the Feynman vacuum diagram no worldlines lead up to the interaction and none lead away from it. The vacuum loop literally comes out of nowhere and vanishes without a trace. Although we normally construe a vacuum as an empty space, vacuum diagrams

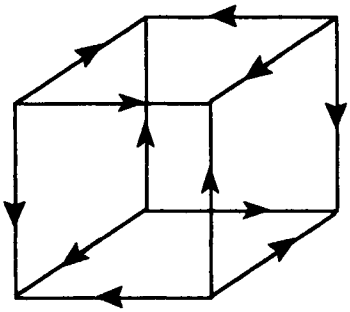
contradict this view, indicating that in the real world there is no such thing as an entirely empty space. It is an idealization. When a pair of virtual particles emerges from the void, a vacuum loop is created **bottom-up**. But if the actual particles are properly calibrated and brought together to form a loop, they fall into a grey hole, creating a vacuum singularity **top-down**. If it were not for the law of mass conservation, a vacuum loop could suck-in pair after pair, creating a strange effect of 'dematerialization'. The void is not an illusory fiction but real. Through the exchange of virtual particles, it produces actual forces, the Casimir effect, experimentally measured by M. Sparnaay (1958) and by Steve Lamoreaux (1997).



Guericke 1654 Feynman 1949

Fig. 20
Harvesting the unsuspected forces of a vacuum.

Minkowski geometry gives us an effective tool to visualize the interactions in **4**-dimensional spacetime. But it is not easy to visualize a singularity associated with a nonorientable hyperdimensional cognitive thought. Gravitational collapse leads to the black hole singularity. But gravity is not the only means to obtain a singularity in actual space. The information collapse is entirely due to topology. In **3-D** a grey hole may look as follows, 'cube' completely isolated from the outside universe:



Nature favours minimal surfaces because a minimal area stores minimal energy and is therefore physically stable. The connected sum of six Klein bottles, each bottle in itself a sum of two projective planes, gives us **12** projective planes altogether. The cube turns into a nonorientable singularity, which we can analyze but cannot actually visualize. Choosing a nucleon to realize a projective plane, the area-minimizing forces give the sphere of a nucleus of the carbon atom, the elementary building block of life. A suitable approximation model of a grey hole is given by the chemically nonactive electron shells of noble gases: it is not possible to 'communicate' with the electrons chemically. The world must be full of virtual grey holes, the vacuum singularities which momentarily decouple from the outside universe. However, the lifetime of vacuum loops is too short to create a cognitive effect. As rapidly as they appear, grey holes dissolve, leaving no traces of their existence. To harvest the special properties of the vacuum, the lifetime of grey holes must be extended beyond the duration dictated by the uncertainty principle. When a pair of particles is lost to a vacuum grey hole, they do not reappear when the grey hole cancels out. To retrieve the lost information, it is necessary to recreate the grey hole in its entirety, which indicates that no explanation for consciousness can be conceived outside the second quantization, which apparently is also not enough so that a third-quantized theory is needed. A pair of particles falls into a vacuum loop and a pair comes out. Equivalently, one can think of a virtual pair as a single grey hole moving on a closed loop. An electron-positron pair can be created from the vacuum by applying strong electric field. In black holes the dense gravitational field converts virtual particles into real ones, inducing particle production from the vacuum. In a reverse manner, the dense information field of the brain can trap pairs of real particles, vanishing without a trace. Although grey holes are off-shell and need not satisfy any field equations, they contribute to the topological energy and must produce observable effects. We suggest that consciousness is one such effect, where grey holes continually arise as quantum fluctuations. The origin of the forces which come into play in consciousness has remained a mystery. The newly acquired topological understanding of grey holes provides us with a fresh and potentially fruitful way to think about the problem of the thinking brain. The Einstein vacuum equations describe a massless field of helicity 3/2. The elementary topological charge is associated with the Möbius-3/2, subjected to a closed cut. When a matrix brane is differentiated and closes on itself, the nonorientable topologies of gravitational and information collapse are indistinguishable. We arrive at a topological equivalence which puts the grey holes in the brain and the black hole in cosmology in a dual relation. Due to deflection of particles a cosmic string is effectively isolated from its environment. Due to information collapse a logical string effectively decouples from the outside universe. It seems that one general topological law manifests itself both on a cosmological scale and on the microscale. The ramifications of this hypothesis for the theory of the brain are significant. It may also help solve one of the biggest problems of string theory: how to crunch **11**

dimensions down to 4. Curling spacetime in a black hole gravitationally or a cognitive space into a grey hole informationally both result in decoupled systems that are equivalent topologically. For example, the Dirichlet brane can be interpreted as a black hole or a black brane. An open string may be regarded as the closed string, part of which is hidden behind the black branes. This leads to an interpretation of black holes as intersecting black branes wrapped around the curled dimensions. It turns out that the mass of a black brane vanishes as a hole it wraps around shrinks. This feature miraculously affects the spacetime itself, allowing one spacetime with a certain number of internal holes to change to another with a different number of holes, giving rise to quantum topology. If black holes and the matrix branes are thus related, the thinking brain becomes an integral part of the Theory of Everything, where topology plays the key role. A fundamental issue pertaining to the mind's ability to influence the brain has puzzled scientists for a long time. What distinguishes grey holes from black holes is that for a grey hole the event horizon can be punctured, letting classical information out and quantum information in. Finally the theory of a grey hole must take into account the effects of quantum information. Consciousness is sealed off from the outside universe, but not entirely. In strong resemblance to the zero-oscillation of a physical vacuum, the ground state of a grey hole possesses non-zero energy. Consciousness cannot be cancelled. There is no logical operation or cancelling command we can give to ourselves which could terminate the cognitive process. In the nonperturbed limit there always will be a nonvanishing term in the reduced implication:

$$\lim_{a^* \Rightarrow 0} \mathbf{IF} = 1.$$

THE INCREDIBLE KNOT OF CONSCIOUSNESS

INTRODUCING GAUGE SYMMETRY INTO LOGIC

We explain consciousness as a topological effect and treat coherent superposition as quantum knots, the impurities welded into non-Abelian gauge theory. According to this approach, accumulating the topological defects or charges, the brain develops cognitive function. The consequences of this scheme are as beautiful as they are deep. Topology is revealing the meaning of

consciousness conceptually and eventually, in a not too distant future, will also do so on the experimental level. However, formidable obstacles must be overcome. Physicists, chemists and molecular biologists are beginning to realize that important features of physical and biological phenomena can be understood by adopting the knot-theoretic point of view [Ref 79, 82, 87, 40]. There is a fundamental relation between the knot invariants and gauge symmetry. The third quantization and general relativity have interesting contacts with knot theory [Ref 3]. A theory has global symmetry if it is unchanged when we make the same rotation at every point in space and time. However, if the theory remains the same when we make a different rotation at each point in space and time, it possesses a more complex local or **gauge symmetry**. With the development of matrix logic, gauge symmetries, so fundamental to modern physics, crossed over to logic and brain science. It is a step forward with spectacular repercussions for the fundamental theory of consciousness. Gauge symmetry underlines human intelligence. The intelligence code remains invariant with free alteration of local logic, because the inference rules, the codons of tautology and contradiction, are gauge symmetries [Ref 89]. As universal cognitive constants the inference rules are unchanged under arbitrary interpretation of the constituent logical variables. A local modification of logic has no impact on global theory, which is invariantly valid, in spite of a local change or error. With the proposed new role of gauge symmetry in the brain the topological concept of the brain gains particular significance. As in all topological problems, the particular size or shape of a cognitive knot is irrelevant. The topological properties of a knot remain invariant when we locally deform the knot. All that matter are the number and direction of crossings, over and under, and their arrangement in relation to other crossings. In seeking a physical implementation for consciousness, we are led to contemplate topological interactions.

The problem of distinguishing between different tautologies in the thought process can be reduced to the problem of finding the invariants that tell one knot from another [Ref. 39, 50, 87]. Some invariants do better than others. The common way is to turn a knot into an algebraic expression, a polynomial which serves as a label for the knot. Unlike the knots themselves, the polynomials can be told apart just by looking. If the polynomials for two knots are different, then the knots are different. Unfortunately, if they are the same, the knots may or may not be the same. The gauge symmetry provides the invariants of a particular power, capable of distinguishing quantum knots where other invariants failed. The basic element of a quantum field theory is the vertex, in which several particles merge into a new particle which then may disintegrate into other particles. The basic elements of cognitive thought are knots, whose transformation must preserve the nonorientable topology which is at the basis of self-reference. Quantum algebra is a unitary Lie algebra with an antisymmetric product that satisfies Jacobi identity. Dealing with quantum knots it is preferable to take a unitary approach and consider the group of unitary transformations in the neighbourhood of unity

$$\mathbf{U} = 1 + i\mathbf{A}$$

where **U** is unitary and **A** is Hermitian.
However, for consciousness the laws of unitary evolution break down. The implication thought is nonunitary,

$$\rightarrow = \cos \mathbf{a} + \sin \mathbf{a} = 1 + \mathbf{a}, \qquad \leftarrow = \cos \mathbf{a}^* + \sin \mathbf{a}^* = 1 + \mathbf{a}^*,$$

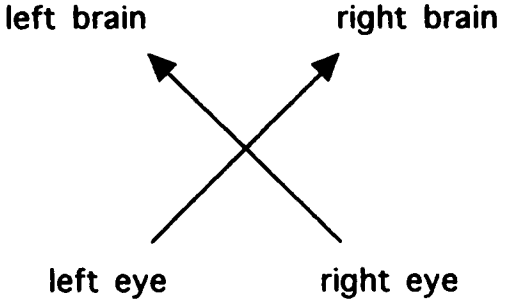
and the creation and annihilation operators $\mathbf{a}^*$ and $\mathbf{a}$ are non-Hermitian. The fundamental self-terminating series signify the emergence of the topological defects. There exists such a neighborhood $O(1)$ of the unit operator and such a neighborhood $O(0)$ of the nil operator in which the 1-1 correspondence between $\mathbf{a}$ and $\mathbf{a}^*$ from $O(0)$ and $\rightarrow$ and $\leftarrow$ from $O(1)$ is finitely guarantied. To be of any practical use the thoughts must be finite, and a marvellous sequence of cancellations occurs, eliminating all potentially infinite terms and yielding a finite answer. A topological wave abruptly halts and the unique implication knot is 'frozen' in a local subspace of matrix space. The formation of a logical knot is very much like a phase transition which is an abrupt event that begins with the formation of miscroscopic defects in the atomic structure, acting as seeds around which the global effect expands rapidly.

In previous studies we found that the thought process essentially requires nesting (a code within a code) and begins to operate after a certain high level of hierarchy is achieved [Ref 91]. Nesting is an important mechanism of the intelligence code, which makes arbitrarily long inferences possible. It also important when one considers the topological knots of consciousness. When a logical knot is inspected at higher resolution, one discovers that it is not really a single knot, but an array of a simpler knots. If we examine a simple knot in the array, we again find that the simple knot is in fact an array of still simpler knots. And so on, very much like a renormalization process in quantum field theory.

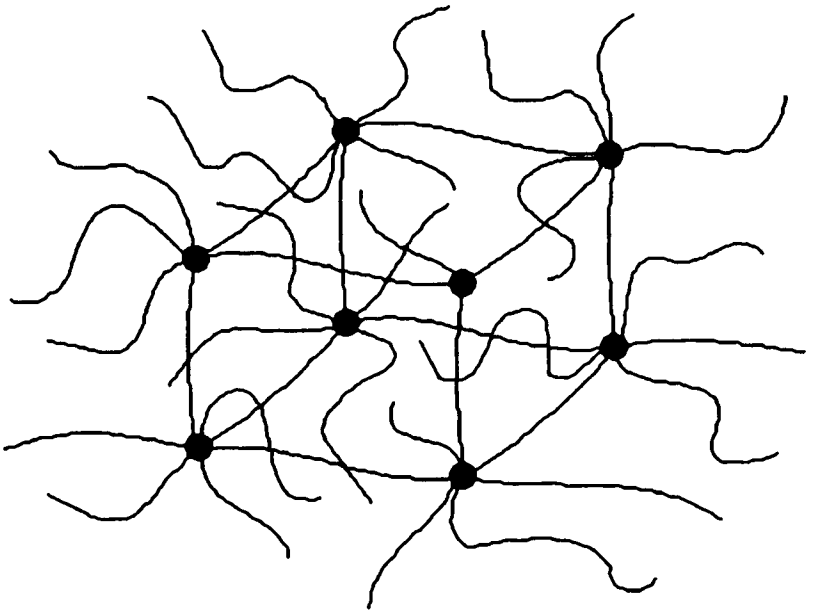
The cognitive matrix is realized by array of topological knots of varying complexity. What do we know at present about the knots of consciousness? Can we understand and untie the complex logical knots? In context of the two dual theories of the brain which we proposed earlier, one considers two kinds of charges, Noether and topological charges, analogous to electric and magnetic charges. Cognitive knots carry topological charges, which we can label as the elements of a finite group; then the dual Noether charges will be labelled as irreducible representations of the group. The duality between these two kinds of charges, which is a matter of choice in ordinary physics, is fundamental for the physics of the brain. A modification of either charge introduces changes into its dual. We can freely exchange the charges, controlling the cognitive knots by topological operations or equally well by the dual Noether operations. These operations are not flawless, but if we perform them repeatedly, we can design a cognitive knot with an intended value of topological charge. Abstract logical operations are pure topological operations. Knots have nontrivial interactions of their own, even in the absence of any Noether charges. Topology sheds new light on cognitive phenomena but also has raised new

questions, many of them unanswered. How can we preserve the Poincaré invariance of a cognitive knot when the physics of the brain changes locally? Are the logical knots macrophysical, microphysical or semiclassical? There are many possible physical realizations of knots, and one must be open to different possibilities. Just as there are many possible geometries, but only the Euclidian and Riemannian ones are actually physically realized, theoretically we can consider infinitely many knots, but in actual physics, including the physics of the brain, the number of actual basic knots is finite, just as in the periodic table the number of chemical elements is finite, becoming unstable as the weight of nuclei increases. The determination of logical knots and their place in the knot table is similar to the task of grouping chemical elements according to regularities in the periodic table. Each chemical element has a certain position in the table, and from this it is possible to predict how it will react with other elements, what kind of compounds it will form, and what sort of physical properties it will have.

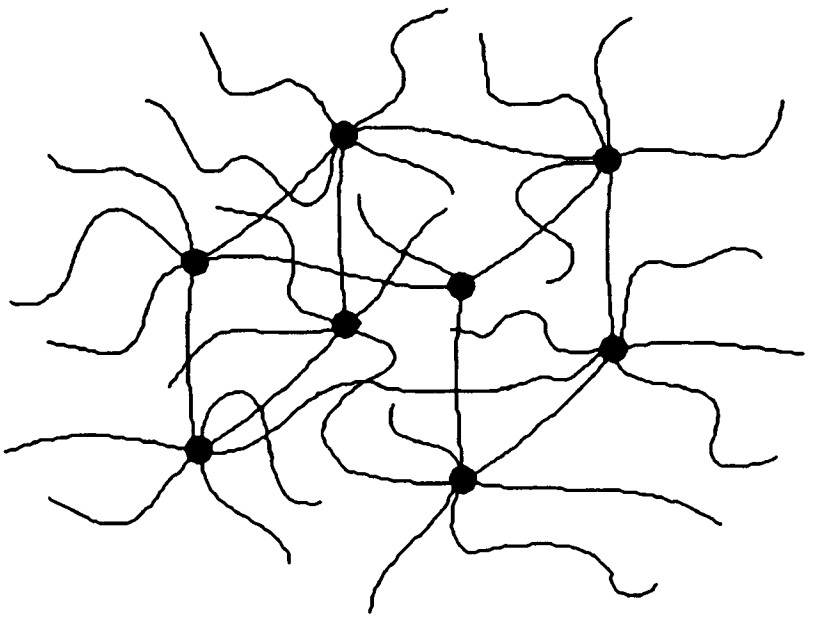
A similar scheme applies to cognitive knots. A potential for knots exists on different levels: macroscopic, macromolecular, quantum or strings, and these levels may be connected by dualities. Let us begin with macrobiology, where the knot theory can be applied in fundamental way. There is a strange and poorly understood twist of the neuronal connections in the brain. Perceptual inputs cross over, most prominently in visual perception, where the information from the left eye goes predominantly to the right brain, and from the right eye to the left brain:



The twist of neuronal connections in the middle brain has never been satisfactory explained, but gains much significance if viewed in the topological framework. Nature seems to have prepared the brain very early in evolution for the emergence of the logical knots needed for intelligence and consciousness. The existence of the left and right brains together with the twist of neural connections provides a neurobiological basis for realization of nonorientable topology in the brain, in itself a complex knot. An example of the conversion from orientable to nonorientable topology, due to a twist of neural links, is shown below:

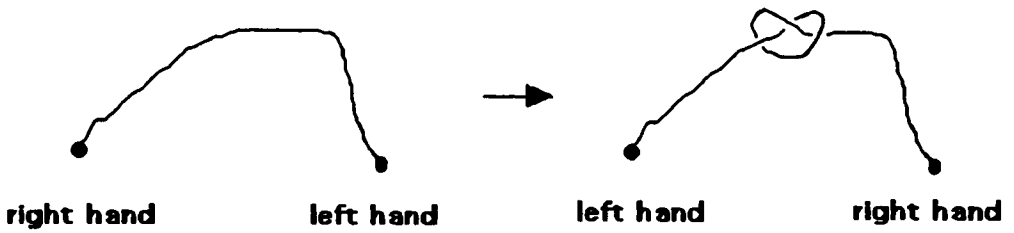


(a) neural realization of orientable topology



(b) neural twist, producing nonorientable topology.

The existence of similar, although not identical, left and right brains, which might be seen as an unnecessary redundancy, in fact provides a basis for the creation of topological charges, reaching its utmost complexity in the topological charge of consciousness. To see more clearly how a neural twisting can tie knots, subsequently transferred onto the high-level cognitive loops in the brain, make a simple topological experiment. Cross your arms Napoleon-style and take a rope, each hand holding the given end of the rope. When the arms are uncrossed, the knot will appear on the rope, signifying the transfer of a 'charge' from your arms to the rope:



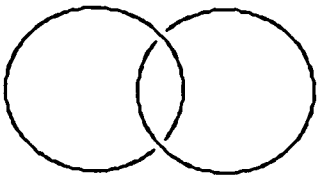
The analysis of topological knots in perception provides many useful ideas, contributing meaningfully to the ultimate goal of identifying the 'incredible' knot of consciousness. According to the proposed topological theory of the brain the transfer of perceptual macrological knots to micrological coherent superposition is the underlying mechanism of the interiorization of the outside world. Nonorientable consciousness is a natural end-result of biological evolution that began by excluding right-handed molecules from life processes. Asymmetrically selecting left-handed molecules for life, nature began to create the conditions for the emergence in the brain of nonorientable topology. This brings us to macromolecular knots.

The traditional neurological model of the brain gives prominence to neurotransmitters, the chemicals which are released into the synapses under the impact of action potentials, mediating in this way the interactions between the neurons. This is in essence a geometrical model. The topological model begins from a different premiss, which takes us to deeper levels. Where geometry thinks of structures as rigid, in topology distances are irrelevant. If a certain knot can be bent, twisted, stretched, squeezed or otherwise deformed into another knot, without being cut, punctured or untied, then the two are equivalent. Topology provides a new way of understanding how one macromolecular structure changes to another in life's basic processes of replication and recombination. Knots are the most natural language for life. The electron microscope picture of DNA clearly show knots, with turns and crossings. In this unusual joining of topology and biology we gain new information about enzymes that is otherwise impossible to obtain. A very tiny fraction of DNA, only 5%, is actually used for biological synthesis. What Nature does with the remaining 95%? The genetic role of DNA is well-known

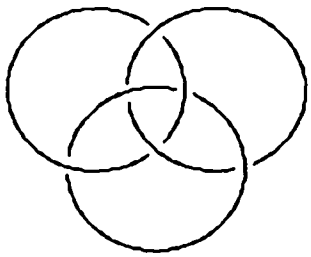
but it is also a suitable structure for the accumulation and storage of topological charges. Although one should not start to think about the macromolecules of DNA as 'thinking' molecules, genetic studies do intersect with cognitive studies. When the ends of a double helix DNA are connected, forming a loop, and then the chemical bonds connecting the two strands of the molecule are dissolved, separating the strands, the resulting structure is a macromolecular knot, carrying a topological charge [Ref 87]. This shows that the ***CUT*** and ***ZIP*** operations, which we introduced earlier, are not just abstract mathematical but actual biophysical operations, possibly the fundamental operations of the information dynamics of the living cell. In biochemistry we are not concerned with the exact number of twists of living molecules, but this number may be responsible for the accumulation of a particular topological phase, and the knotted structures may indicate the role of DNA in the realization of cognitive logic. This also may explain why neurons do not replicate, as the genetic use of DNA will be blocked in molecules with closed Möbius-type topology. The supercoiling of DNA can store topological energy which is later released, like the energy in a twisted rubber band. The action potential activates enzymes. When the enzyme breaks the strands, a knot is formed in which the number of crossings equals the number of half-twists, excluding the basic twist-**1/2**, producing an 'elemental' Möbius which under a closed cut gives a circle.

Besides neuronal and molecular knots we must also look at deeper levels. In fact we can study knot transformations, ignoring the physical nature of objects to which these transformations apply. Knots can be defined axiomatically, be they neurons, proteins, nucleons, electrons, quarks or strings. Although a general algebraist can ignore the physical nature of logical knots, one hopes that eventually it will be possible to formulate meaningful and testable predictions concerning the actual physical knots of consciousness. In the same manner as group theory is instrumental in the theory of elementary particles, with an irreducible representation of a group identifying and predicting a particle, topological charges are the key to understanding cognitive 'particles'. The topological charges are saved and persist in the form of knots, from simple neuronal knots in perception to the high-order conceptual knots such as those tied up in logical paradoxes. This warrants a more detailed study of knots, which provide a fertile ground and new effective tools for the study of consciousness.

The development of the theory of knots began over a hundred years ago, and physicists were the first to see its potential. Kelvin believed that atoms are knotted vortices in the ether and would be better understood if we could understand knots better. If quantum particles are knots, knots is the stuff the matter is made of. An atom is a composite knot, which can be stable or may decay into knots of lesser complexity. Here are some ideas. Think of a meson, made of two quarks, as two entangled knots, and of a nucleon, made of three quarks, as Barromeo knot, the latter symbolizing the cohesion of the three branches of an Italian family: if one is cut, the other two separate:



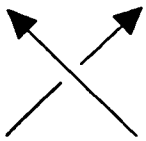
meson



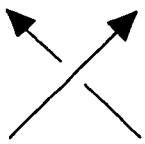
nucleon

The nucleon is strong as the Barromeo family, and the constituent quarks do not separate.

A topological knot is a closed loop or systems of entangled loops smoothly embedded in 3-dimensional Euclidian space R_3. Apparently three dimensions, as it turns out, are the only dimensions in which knots stay knotted. A knot can be pictured by a diagram which is a projection of a knot into a plane, where the crossings at transverse points can be left-handed or right-handed:

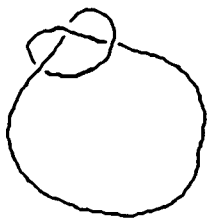


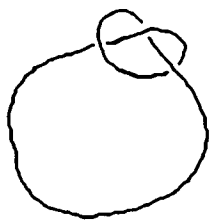
Left crossing



Right crossing

A crossing is worthless or nugatory if there is a circle in the projection plane meeting the diagram transversely at that crossing, but not meeting the diagram at any other point. Nugatory crossings can obviously be removed, so that they cannot occur in a diagram of a minimal number of crossings. The diagram is irreducible if none of its crossings is nugatory. In three dimensions the trivial knot, or **unknot**, is a circle with no crossings. In four dimensions the trivial **unknot** is a hypersphere. The simplest possible knot, the **trefoil**, has three crossings; it exists in the left and right isomeries, drawn next:





There is one knot with four crossings, the **figure-eight** knot, and just two with five. But **165** distinct knots have **10** crossings, and the total through **13** crossings is **12965** knots.

Different knots are equivalent if there is a homeomorphism of R_3 to itself, taking one knot to the other. A single knot can be represented by infinitely many diagrams, but only a finite number of diagrams will have a minimal number of crossings. Deciding whether two knots are the same or not turns out to be a puzzle. The first cataloguing of knots were done by Peter Tait and C. N. Little. Tait invented a scheme for encoding knot diagrams, enumerating all possible diagrams up to a given crossing number and then grouping together those diagrams that represent the same knot type. He considered all knots up to **7** crossings and grouped them together by knot type, obtaining the minimal diagrams. Daunted by the combinatorial explosion for larger crossing numbers, Tait stopped at seven crossings. Grouping the diagrams by knot type rather than enumerating all possible diagrams remains to this day the most difficult part of knot tabulation. For a large crossing number, the sheer number of possible combinations is so huge that the enumeration remains difficult in practice, even with the support of a computer.

There are alternating and nonalternating knots. Initially it was wrongly believed that nonalternating knots are not possible. These do not exist with fewer than **8** crossings, and the first proof of the existence of a nonalternating knot did not appear until 1930. Although nonalternating knots do not appear until **8** crossings and do not predominate until **13** crossings, the share of knots which are alternating tends exponentially to zero with an increasing number of crossings. After a quarter-century of laborious handwork, Tait, Kirkman, and Little created a table of alternating knots through **11** crossings and nonalternating knots through **10** crossings. In the absence of a rigorous theory, they could not know whether their tables were correct, and some errors came to light later, but the table of alternating knots through **10** crossings has stood the test of time. The era of rigorous knot theory began in the early part of the 20th century with the advent of topology. In 1949, Schubert proved that every knot can be uniquely decomposed as a connected sum of **prime knots**. In close analogy to numbers theory, a knot is prime if it cannot properly be decomposed as a connected sum. In the light of this result, it is only necessary to tabulate prime knots; the composite knots are then easily constructed by taking connected sums. Knots can be enumerated starting with a set of irreducible projections and then inserting crossings in a systematic way.

In the early days of topology it was found that a trefoil is chiral knot: the right-handed trefoil and left-handed trefoil are distinct and cannot be manipulated into each other. Those knots, like the figure-eight knot, which remain equivalent to their mirror images, are endowed with achiral symmetry. Achiral knots are important to chemists, who are often concerned with the right- or left-handedness of molecules. Tait identified all achiral alternating knots up to **10** crossings. He did not consider nonalternating knots, but, as it

happens, there are no achiral nonalternating knots with fewer than **12** crossings. In analogy to the Mendeleev table in chemistry, very complex but quite rigorous laws define the table of knots. For alternating knots there are no achiral knots with odd crossing numbers, but an achiral nonalternating knot with **15** crossings is known to exist.

Another aspect of refining the notion of equivalence for knots is to consider the orientation, or direction, of the knot curve. A knot is invertible if there is an ambient isotopy carrying the knot onto itself, but with its direction reversed. The trefoil and other prime knots are invertible, and it is not immediately clear that there exists a knot which is not invertible. This question was not resolved until 1964, when an infinite family of noninvertible knots was discovered, beginning at **9** crossings. The situation with a low number of crossings is not typical. However, only a tiny proportion of the knots up to **16** crossings are invertible.

As already stated, we are interested in prime knots and must therefore, exclude composite knots. It is obviously of fundamental importance to be able to show that the prime knots really are prime. In general, deciding whether a given knot is prime is a nontrivial problem. Fortunately, in the case of alternating knots, there is a theorem by Menasco which states that a reduced alternating diagram represents a prime knot if and only if the diagram is itself prime. A diagram is prime if there is no circle in the plane cutting the diagram transversely in two points, with crossings lying on both sides. Informally, an alternating knot is prime if and only if it looks as if it is prime.

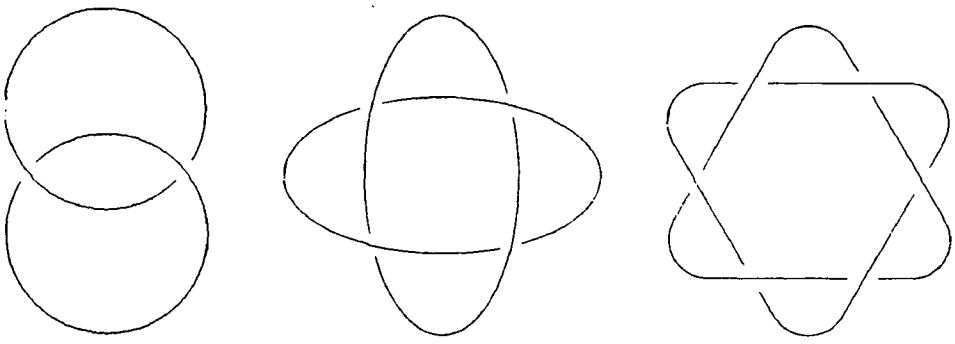
The classification of knots through **11** crossings marks the end of the era of manual calculation, as all subsequent calculations have been carried out by computer. In an outstanding *tour de force* Hoste, Thistlethwaite and Weeks have tabulated all knots through **16** crossings, and have begun the **17**-crossing list. With the aid of the computer they have extended the table to include all prime knots with **16** or fewer crossings. With almost two million knots now in the table, they have provided a great deal of data. Of the first 1701936 knots amazingly all but **32** are hyperbolic. In principle, there does exist an algorithm for determining whether or not two knots are equivalent: knots are equivalent if and only if their complements are homeomorphic. If a knot complement admits a complete Riemannian metric of constant Gaussian curvature -1, in other words the knot is hyperbolic, then such a metric is unique. Thus, two hyperbolic knots are equivalent if and only if their complements are isometric. A complete knot invariant is used to remove all duplicates from the list, which previously resolutely refused to be distinguished. A fundamental property of hyperbolic knots that they cannot be composite. The nonhyperbolic knots are so few in number that they are easily dealt with separately.

An important class of knots are the torus knots, which are prime knots. A torus knot is a closed curve sitting on a standardly embedded torus which wraps the torus **n** times meridianally and **m** times longitudinally. The integers **n** and **m** are necessarily coprimes; otherwise, we have a torus link. The torus

knots are all chiral and invertible: if we rotate through half a turn in the projection plane or flip right and left and top and bottom, the knot is mapped onto itself with reversed orientation. If a knot is placed inside a solid torus, and the torus is itself is knotted in R^3, then the knot is a satellite knot. The satellite knots are prime knots, which follows from a simple geometric argument.

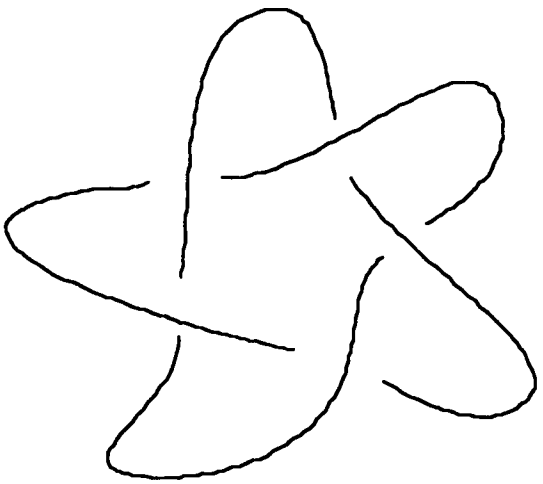
To connect the theory of knots with cognitive logic, we consider composite knots made of alternating, chiral and invertible elements. Logical thought can be represented as convex and nonconvex knots. Different knots correspond to thoughts of different complexity. Thoughts are nonlocal and can be read out from any point on the knot. Thinking is a natural topological transformation, with knots being inserted, substituted and deleted in a systematic way. To maintain the nonorientability of the cognitive knot certain conditions must be satisfied. Convex knots, carrying topological charges, begin with three crossings. There are insertions which will preserve the crossing number and those which will not. Crossing number **n**, which is the number of times the knot's curve self-intersects, is put in correspondence with the potency of logical operators. Consequently, in (2×2) matrix logic we study 5 different kinds of knots with **n** = 2, 4, originating from orientable manifold loops and with **n** = 3, 5, 7, originating from nonorientable loops. Since we are concerned with prime knots up to 7 crossings only, nonalternating knots, which start from 8 crossings, will not appear in (2×2) matrix logic. Although we still have to learn much more about knots, the alternating knots of matrix logic can be well understood, in particular since the advent of the Vaughan Jones polynomial in 1984, which resolves a number of conjectures concerning alternating knots. The alternating knots are obtained by the close cut of closed manifolds with twists. The convexity or nonconvexity of knots is dictated by the odd or even number of twists **n**, which not only determines the complexity of the original topology but also whether it is orientable or nonorientable. The cut of the orientable manifolds produces knots with 'split personality', which is the entanglement of two loops with a varying number of crossings, equal to **n**. Next we drawn three nonconvex knots, each a pair of entangled circles, for **n** = 2, 4 and 6:

n = 2 **n** = 4 **n** = 6



For an even number of half-twists **n**, the close cut will divide a closed loop into an alternating knot comprising two 'inseparable' circles, which suggests a link between knots and quantum entanglement that could have a topological origin. If we increase **n**, then the entanglement increases, and we can entangle even more by adding another half-twist, producing a highly entangled knot.

In contrast, the knots resulting from loops subjected, prior to close cut, to the odd number of half-twists **n** = 3, 5, 7 are convex knots. These are made of single self-intersecting lines and are of great significance for cognitive logic. The odd number of half-twists yields the nonorientable Möbius-**n**/2 topology. The close cut of Möbius-1/2 produces a circle or **unknot**, with a length twice the length of the original nonorientable loop. The close cut of the Möbius-3/2 produces a trefoil knot. The alternating knot with five crossings, shown below, comes from the Möbius-5/2:



Each two half-twists increase the value of fermionic spin, running the series 1/2, 3/2, 5/2, 7/2. The basic crossing number is **n**=3, which gives an elementary topological charge, associated with the trefoil knot. By iteration from elementary charges and joining many prime knots together we can design cognitive knots with a high degree of complexity. Since matrix logic does not require nonalternating knots, we consider prime knots up to **7** crossings only but should we stop at seven crossings?

BEYOND SEVEN CROSSINGS

Thoughts are quantum knots. The forces which come into play in consciousness, and which have stayed a mystery for a long time, appear to be topological in nature. Topology thus gains the status of the fundamental science

of the mind. The thinking brain exerts control over its intrinsic topology. These results should be taken as a starting point for exploration rather than the end of the voyage. From simple knots a complex dynamical knot of consciousness emerges. Although in (2×2) matrix logic we do not consider nonalternating knots, it is tempting to speculate on the possibility of higher forms of intelligence beyond the seven crossings. A general (topo)logical operation consists of five fundamental steps: **twist, glue, cut, move, zip.** Logical differentiation is responsible for the first two steps, logical momentum for the fourth, and logical integration for the fifth. What the physical basis is of the ***CUT*** operation we do not know, unless one chooses DNA molecules as the carriers of cognitive knots; then ***CUT*** is the well-understood genetic process of the separation of the two strands of the information macromolecule. This is an unlikely solution to consciousness, and there is not one shred of physical evidence to support direct DNA involvement in the thought process. It would be strange if Nature decided to implement the two fundamental life functions, reproduction and consciousness, in the same substrate. But the DNA model has a great heuristic value, showing the possibility of the biological realization of knots of high complexity. Ultimately we must decide whether the realization of knots in the brain is evolutionary or fundamental. At the fundamental level we find the prime knots, with relatively low complexity. Particles are prime knots, atoms are composite ones. Still, until inanimate gives way to living, the knots remain relatively simple. Biology introduces higher levels of knot complexity, which reach its ultimate degree in the thinking brain. When the complex knot of consciousness is formed from the array of simple knots, it is practically impossible to untie it, without totally breaking the system.

Topology is a science of the continuous, but gives rise to discrete charges. The logical derivative, converting logical membranes into a nonorientable loops, sustains a perturbation that connects the continuum to quantized states. Because we have linked the nonorientable Möbius-**n**/2 and the fermionic spin-**n**/2, the number of crossings in a knot is a quantum number. For a given type of logical knots, convex and nonconvex, the number of crossings in a knot can increase or decrease only in multiples of 2, a fact with consequences for the understanding of logical energy, where one energy level is inexplicably absent. Contracting the topology for a microscopic particle gives the idea of treating atoms as knots distinguished by different number of crossings. We can then ask the following questions. What is the biggest number of crossings? Are there physical reasons for a topology to become unstable after sufficiently many twists? Knot theory can explain the observed exceptional stability of the doubly even atoms as knot entanglement. If an electron's orbit is a knot, than it would not emit unless the ***CUT*** operation were applied. Understanding of atomic periodicity in topological terms might turn out to be useful for predicting new fundamental effects which quantum mechanics cannot identify.

Just as we can encode some number as the number of half-twists or equivalently as the number of crossings in a knot, we can in principle read out this number from a given topology. If knots are actual spins, we will then read bosonic and fermionic numbers, encoded in the 'winding' number. This joining of topology and quantum mechanics provides a framework for designing the quantum numbers one needs. When Matrix Statistics reduces to the eigenstates, the particles, which we call bosons and fermions, appear. Hence, one can determine whether a particle is a boson or fermion by counting the number of crossings in a knot. The topology of alternating knots enables us to store and retrieve quantum information in a systematic way and to perform logical inferences predictably. But we cannot effectively control topological charges and keep them within the range of allowed values if the knots are nonalternating. Could superior forms of intelligence exist beyond the seven crossings? The line between genius and insanity is thin. It is intriguing to speculate that cognitive pathology can be attributed to violations of the topology of the brain, when it extends to the 'forbidden region' of nonalternating knots. Eventually technology will allow neuroscientists to test the topology of the brain in laboratory.

THOUGHT TELLS THE BRAIN HOW TO SPIN, SPIN TELLS THE BRAIN HOW TO THINK

The natural laws formulated mathematically in quantum theory no longer deal with the elementary particles themselves but with our knowledge of them.

Werner Heisenberg

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NUMBERS AND OPERATORS

The whole numbers has the dear god made.
All else is man's work.

Leopold Kronecker

QUANTUM THINKING

We are driven by the desire to put thoughts into exact mathematical physical equations, certain that if such a project is successful it will open new vistas of science and mix the fundamentals of general field theory and the elements of logic in a way quite unlike anything previously encountered in physics. The problem of finding adequate mathematics for the intelligent brain is made harder by the fact that physics makes use of two fundamentally different descriptions: the classical description which relies on numbers and the quantum description which relies on operators. In the mechanics of Newton such physical quantities as position, energy and momentum are observables which are fully determined by real numbers, remaining invariant in different systems of reference. Classical probability is an example of such an invariant real number. Calculated in one reference system or spacetime (x_1,y_1,z_1,t_1) it must remain unchanged in another reference system or spacetime (x_2,y_2,z_2,t_2):

$$p(x_1,y_1,z_1,t_1) = p(x_2,y_2,z_2,t_2).$$

In quantum mechanics the actual probabilities are replaced by complex probability amplitudes or the wavefunction $\psi(x, y, z, t)$, and the descriptions in terms of number functions are converted into the descriptions in terms of operators. It is not that classical physics does not make use of operators and nonscalars like vectors and tensors, nor that in quantum physics we do not have ordinary number functions; we do, but this usage has no bearing on and does not reflect the underlying principles. A changeover from number to operator description in quantum theory is not simply a changeover between two equivalent descriptions but a transition to a higher level of abstraction, a transition from classical to quantum thinking. The conceptual reform caused by the operator description in quantum theory is similar to the change brought about by the transition from realistic classicism to abstract painting, where the message is not offered explicitly but has to be derived. The equations of classical physics *represent* values, while the equations of quantum physics *encode* them. What is important is that the complex amplitudes of the wavefunction serve as an *operator* replacement of classical probability. Although this is implicit in the formalism of the 2nd quantization, where the wavefunction becomes an operator, it is often poorly understood that a changeover from real to complex probabilities is an organic part of the conversion from classical to operator description.

Prior to the discovery and development of matrix logic, the classical function description had been a standard way of thinking in logic too, from Boolean logic to Birkhof-von Neumann quantum lattices. Matrix logic, however, revealed, quite unexpectedly, that the fundamental structure of logical theory is very similar to quantum descriptions. Even though the quantum hypothesis of the brain has failed in many important respects, and in spite of great expectations has not been able to offer nontrivial predictions, it has brought into focus the importance of operators for the theory of logic and consciousness. Whatever the prospects and future of quantum mechanics in brain science might be, the principles of quantum thinking are of paramount significance for the understanding the cognitive brain scientifically.

Before we can meanigfully study the logical brain in the operator framework, it will be instructive to consider and compare some fundamental examples of classical description, which is deterministic, and quantum description, which is not. The classical principle of causality implies that if the numbers defining the initial state of a system are known, integrating the Hamiltonian equations of motion

$$\dot{q} = \frac{\partial H}{\partial p}, \qquad \dot{p} = -\frac{\partial H}{\partial q}$$

we can determine a state of the system at any moment in the future. Similar deterministic and number-oriented thinking applies when we employ nonscalar vector quantities, as in the Maxwell's vacuum equations for the electric and magnetic fields **E** and **B**,

$$\text{rot}\,\mathbf{B} = \frac{\partial \mathbf{E}}{\partial t}, \qquad \text{rot}\,\mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}.$$

Scalar thinking is not necessarily one-dimensional, since it can also apply to **n**-dimensional spaces, like Minkowski **4**-spacetime, or to the Einstein energy-momentum tensor equation in general relativity

$$G_{ij} = -\chi T_{ij}.$$

Scalar, vector or tensor functions in classical descriptions tell the whole story explicitly. It is not the complexity of a quantity, but what this quantity stands for that is significant. If the position or energy of a classical system is described by some function, then a particular value of such a function identifies a physical observable directly and completely. While in the classical world we need nothing else for the decision process, in the quantum world strange things happen. Even though the results of quantum measurements, like classical ones, are also expressed as real numbers, now these numbers are not available directly but have to be derived as eigenvalues of corresponding operators. The precision and strict determinism of Newtonian mechanics and classical fields give way to the uncertainties of quantum mechanics. The concepts of the position and velocity of a particle become approximate concepts and no amount of effort nor sophisticated devices will let us escape back to the convenience of the classical world. This is not a matter of having crude instruments; according to Heisenberg, it is a law of nature. Even spacetime turns out to be a fuzzy and derived concept. In string theory the worldline has to be replaced by the worldtube, and the particle is no longer a dimensionless point but a vibrating string, a tiny loop with a size around 10^{-33}cm, controlled by a new fundamental string constant $\alpha'=(10^{-33}\,\text{cm})^2$. The vacuum is no longer empty but is filled with unruly creations and annihilations of quantum particles.

The development of the operator description and the creation of the new quantum paradigm owe a great deal to the development of Schrödinger wave mechanics and Heisenberg matrix mechanics, generalized later in the Dirac quantum formalism. A novelty of quantum-mechanical thinking is that the wavefunction, as opposed to the functions of classical mechanics, does not represent a physical system directly, but is simply a statement (operator) which provides us with information or potential knowledge concerning the behavior of the quantum system in question. With every observable quantity in quantum physics we associate a corresponding Hermitian operator. Neither the wavefunction nor the operator is physical in the sense of classical physics, but mathematical abstractions. In order to obtain actual physical data about the process we must formulate and solve an eigenstate equation, such as, for example, the time-independent Schrödinger equation,

$$H|\psi\rangle = E|\psi\rangle$$

where H is a Hamiltonian operator, and E are the energy eigenvalues.

One already gets a taste and initial understanding of the rationale behind this operator thinking in seminal quantum mechanics. Let a wavefunction be a complex wave

$$\psi = Ae^{-\frac{i}{\hbar}(Et-px)}$$

where A is amplitude, E energy, p momentum and $\hbar$ is the Planck's constant divided by 2π.

Differentiating ψ with respect to time

$$\frac{\partial\psi}{\partial t} = -\frac{i}{\hbar} EAe^{-\frac{i}{\hbar}(Et-px)}$$

and multiplying both sides by $i\hbar$, we get

$$i\hbar\frac{\partial\psi}{\partial t} = E\psi.$$

We now see that the result of applying the operator $i\hbar\partial/\partial t$ to the wavefunction ψ is the same as multiplying the function by energy E. Thus the wavefunction ψ is the eigenfunction of the operator $i\hbar\partial/\partial t$ and E is the corresponding eigenvalue. In this way one identifies the quantum energy or Hamiltonian operator $H = i\hbar\partial/\partial t$, generally comprising the kinetic and potential energy terms: $H = T + U$. The wavefunction ψ is an eigenfunction and the numbers E are eigenenergies of H.

In similar way we can determine the explicit form of the momentum operator. Since

$$-i\hbar\frac{\partial\psi}{\partial x} = p\psi$$

the momentum operator $\mathbf{p}$ is $-i\hbar$ times the differentiation operator:

$$\mathbf{p} = -i\hbar\frac{\partial}{\partial x}.$$

What is important is that the operators obtain an existence of their own: e.g. we can add and multiply them, deriving new quantum operators. The correspondence principle tells us that in this way we can construct *new* physical observables, in analogy to observable operators from which a compound operator has been constructed. Consider for example the product of the momentum operator with itself

$$\mathbf{p}^2 = -i\hbar^2\frac{\partial^2}{\partial x^2}.$$

Since the kinetic energy of a classical particle is

$$T = \frac{p^2}{2m},$$

replacing the number p^2 by the operator $\mathbf{p}^2$ we obtain the corresponding quantum operator for energy

$$T = \frac{\mathbf{p}^2}{2m} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}.$$

The operator method has been shown to be a very effective theoretical tool of modern physics. It requires, however, the introduction of an additional postulate in the form of commutation relations. With the aid of operators one

naturally derives the fundamental quantization relation, the underlying principle of quantum mechanics:

$$[\mathbf{x}, \mathbf{p}]\,|\psi\rangle = -\,\mathrm{x}\; i\hbar\frac{\partial|\psi\rangle}{\partial \mathrm{x}} - \left(-\; i\hbar \mathrm{x}\,\frac{\partial|\psi\rangle}{\partial \mathrm{x}} - \; i\hbar|\psi\rangle\right) \;=\; i\hbar|\psi\rangle$$

The quantum-mechanical commutator $[\mathbf{x}, \mathbf{p}]$, acting on a wavefunction $|\psi\rangle$, returns $|\psi\rangle$ with the factor $i\hbar$, which is a fundamental unit of quantization, $\hbar \approx 10^{-27}$ erg•sec.

The existence of the number and operator descriptions has fundamental physical consequences. Whereas in classical mechanics the pair of canonical variables, the position x and momentum p are ordinary continuous functions which commute

$$[\mathrm{x}, \mathrm{p}] = 0,$$

in quantum mechanics the position and momentum are represented by noncommuting operators

$$[\mathbf{x}, \mathbf{p}] = i\hbar.$$

Due to noncommutation the position and momentum become approximate quantities, obeying the Heisenberg uncertainty relation:

$$\Delta \mathrm{x}\, \Delta \mathrm{p} \sim i\hbar.$$

The commutation postulate and the uncertainty relation reveal the limitations of quantum-mechanical measurement. Two physical quantities, represented by operators, could be simultaneously known if and only if their respective operators commute. If the observables do not share a common eigenstate, quantum uncertainty unavoidably enters the picture.

The abstract operator description gets even more abstract in the second quantization, where the operators of the first quantization become operands and higher-order operators, acting on these operands, are introduced. While in quantum mechanics (the first quantization) one considers quantized particles propagating in continuous classical fields, in field mechanics (the second quantization) one goes further: fields are quantized too, which in a sense dissolves the distinction between a field and a particle. The field interaction of particles is interpreted as an annihilation-creation metamorphosis, the fundamental process in which some particles are born and other disappear. For our discussion it is important that the second quantization is accomplished with the aid of new operators, the annihilation and creation operators, a^* and a. For a bosonic field

$$[\mathrm{a}^*, \mathrm{a}] = 1,$$

and for a fermionic field

$$\{\mathbf{b}^*, \mathbf{b}\} = 1\,.$$

Free fermionic field obeys the system of equations

$$\left(\gamma_\mu \frac{\partial}{\partial \mathrm{x}_\mu} + m\right)\psi = 0, \qquad \overline{\psi}\left(\gamma_\mu \frac{\partial}{\partial \mathrm{x}_\mu} - m\right) = 0$$

where $x_\mu (\mu = 1,2,3,4)$, γ_μ are Dirac's matrices $\gamma_\mu\gamma_\upsilon + \gamma_\upsilon\gamma_\mu = 2\delta_{\mu\upsilon}$, and bispinors ψ and $\overline{\psi}$ agree with the anticommutation rules

$$\{\psi_\alpha(x)\psi_\beta(x')\} = 0, \qquad \{\overline{\psi}_\alpha(x)\overline{\psi}_\beta(x')\} = 0,$$

$$\{\psi_\alpha(x),\ \overline{\psi}_\beta(x')\} \equiv i S_{\alpha\beta}(x - x'),$$

$$\alpha,\beta = 1,2,3,4$$

If the fundamental system of functions are flat waves

$$e^{i\hbar(px-Et)}$$

then

$$u^r(p)e^{i\hbar(px-Et)}$$

where $u^r(p)$ is bispinor amplitude, describes a fermion with the momentum p and spin r, while

$$v^r(-p)e^{-i\hbar(px-Et)}$$

describes an antifermion with the bispinor amplitude $v^r(-p)$. The annihilation and creation operators for fermions and antifermions satisfy the anticommutation relations

$$\{a_r(p),\ a_{r'}{}^*(p')\} = \delta_{r\,r'}\,\delta^3(p - p')$$

$$\{b_r(p),\ b_{r'}{}^*(p')\} = \delta_{r\,r'}\,\delta^3(p - p'),$$

with all other anticommutators equal zero.

The field equation for a bosonic field is different:

$$(\Box^2 - m^2)\psi = 0$$

The commutation rule applies:

$$[\,\psi(x)\,,\,\psi^*(x')] = i\,D(x - x')$$

where

$$D(x) = \frac{1}{(2\pi)^3}\int e^{ikx}\,\frac{\sin\omega t}{\omega}\,d^3k$$

Here k is the boson's momentum, ω is energy and $\hbar = 1$.

As with a fermionic field a bosonic field can be expanded in the system of fundamental functions

$$\psi(x) = \int d^3k\,\{a(k)U(k)e^{ikx} + b^*(k)U^*(k)e^{-ikx}\}$$

$$\psi^*(x) = \int d^3k\,\{a^*(k)U(k)e^{-ikx} + b(-k)U^*(-k)e^{ikx}\}$$

where

$$U(k) = \sqrt{\frac{1}{2\omega}}$$

In contrast to anticommutation for fermions, the bosonic annihilation and creation operators obey the commutation relations

$$[a(k),\ a^*(k)] = \delta^3(k - k'), \qquad [b(k),\ b^*(k)] = \delta^3(k - k').$$

Other commutators are zero.

THE QUANTIZATION THEOREM

Good mathematicians see analogies between theories; great mathematicians see analogies between analogies.

Stefan Banach

The purpose of this section is to prove the quantization relation in logic, from which general logical theory can be obtained in pure deductive fashion, clarifying the quantum hypothesis of the brain. Suppose we need to compare two quantities 'a' and 'b'. If the quantities are classical, they are represented by real numbers and their classical comparison is subtraction (a-b). Here quantum-mechanical comparison is not applicable and would universally fail given that for any pair of numbers

$$[a, b] \equiv 0,$$

as opposed to a pair of noncommuting quantum quantities whose commutator does not vanish:

$$[x, p] = xp - px = i\hbar.$$

In quantum mechanics, for two noncommuting observables an increase in knowledge on one leads to a decrease of knowledge on the other. There is a cognitive analog of this situation. When we try to observe our own thought process,we necessarily introduce unpredictable and uncontrollable disturbances into the process itself. As a result it becomes unclear whether we process the initial thought or trying to observe it. This affinity of cognitive effects to the Heisenberg uncertainty principle gives rise to the quantum hypothesis of brain function. To clarify the assertions of this hypothesis, we make use of matrix logic, in which logical quantities are raised to the rank of observables represented by operators. This permits us to compare logical operations with the quantum-mechanical model for observation. In quantum mechanics the measured values of the observables are the eigenvalues of the corresponding operators, the position, momentum or energy. In direct analogy to quantum mechanics we formulate the logical problems as eigenvalue problems, the solutions of which yield the logical quantum numbers:

$$L|q\rangle = \lambda |q\rangle.$$

Introducing the operator description in logic, we are naturally interested in determining whether cognitive logic commutes or whether it is quantized and follows the appropriate quantization relation. In a more general framework there is a nontrivial question at issue: is it possible to formulate the laws of logical brain in a fully consistent way without reference to wavefunction? The physics of the twentieth century evolved under the magic spell of the quantum-mechanical wavefunction. Is this the only theoretical tool with which to accommodate quantization? With information and logic entering

physics at a fundamental level, new options become available. In this study the fundamental concept is neither wavefunction nor probability, but a logical truth-value which we calculate as an eigenvalue of corresponding matrix operators. Accordingly, the cognitive degrees of freedom are not described by numbers but by matrices which generally do not commute, leading to logical quantization, just as noncommutation of position and momentum led to the quantization relation when quantum mechanics emerged.

Using Schrödinger's prescription, we will define the truth-value operator or the operator of the logical coordinate **q** as multiplication by the generalized truth-value q. We also define the logical momentum operator **M**= -d/dq as the differentiation with respect to the logical coordinate q, in a manner quite similar to Schrödinger's definition of the momentum operator **p** = $-i\hbar\partial/\partial x$, as differentiation with respect to the position coordinate x. Having identified the canonical pair of logical operators and constructing their commutator, we prove:

$$[\mathbf{q}, \mathbf{M}]|q> = [q, -\frac{d}{dq}]|q> = (\frac{d}{dq}q - q\frac{d}{dq})|q> = \frac{d}{dq}q|q> - q\frac{d}{dq}|q>$$

$$= \frac{d}{dq}\begin{pmatrix} q\bar{q} \\ q^2 \end{pmatrix} - q\begin{pmatrix} -1 \\ 1 \end{pmatrix} = \begin{pmatrix} 1-2q+q \\ 2q-q \end{pmatrix} = \begin{pmatrix} 1-q \\ q \end{pmatrix} = |q>.$$

This logical quantization theorem shows that a cognitive state and the reasoning which creates it do not commute and therefore cannot both be known simultaneously with complete precision. The commutator of the logical coordinate and momentum, acting on any normed logical state, leaves it intact:

$$[\mathbf{q}, \mathbf{M}]\,|q> = |q>$$

Comparing our result with the Schrödinger quantization relation, we see that in both cases the corresponding commutators conserve the input state with appropriate factors. For a logical state |q> the factor is 'invisible' because it is the unity 1, which we can treat as a *fundamental unit of information*, the bit 1. For a quantum state |ψ> the factor is a *fundamental unit of quantization* $i\hbar$. Setting $\hbar = 1$, the two quantization rules differ only in that one refers to the *real* and the other to the *complex* unit:

$$[\mathbf{q}, \mathbf{M}] = 1, \qquad [\mathbf{x}, \mathbf{p}] = i\,.$$

One naturally draws a parallel between the logical quantization relation [**q**, **M**] = 1 and the second quantization in quantum field theory,

$$[a, a^*] = 1,$$

where

$$\mathbf{a} = \frac{1}{\sqrt{2\hbar\omega}}(\omega x + ip), \qquad \mathbf{a}^* = \frac{1}{\sqrt{2\hbar\omega}}(\omega x - ip).$$

are the non-Hermitian annihilation and creation operators respectively.

Employing the exponentials of the matrix annihilation and creation operators, there is a finite way to convert second-quantized quantum field theory into logical theory and vice versa.

Schrödinger's quantization rule applies to the canonical pair of observables, the position and momentum, represented by Hermitian operators:

$$\mathbf{x} = \mathbf{x}^{+}, \qquad \mathbf{p} = \mathbf{p}^{+}.$$

The logical quantization rule applies to the logical momentum operator **M** which is non-Hermitian, hence, not a quantum-mechanical observable. Even though the acting operator is non-Hermitian and no quantum-mechanical wavefunction is involved in logical quantization, it apparently replays the structure of the quantum codes but in terms of real probability, thus providing an explicit scheme for macrophysical quantization. Matrix logic and quantum mechanics both obey non-Abelian commutation algebra. If we were to replace $|q\rangle$ by $|\psi\rangle$, the logical coordinate q by the position coordinate x, and multiply the result by $i\hbar$, we would convert a logical bit into a quantum bit, ending up with Schrödinger's quantization relation.

A classical state $|q\rangle$ is the eigenstate of the logical commutator **[q, M]**, just as a wavefunction $|\psi\rangle$ is the eigenstate for the quantum-mechanical commutator **[x, p]**. This is in spite of the fact that Schrödinger derives his result from simple properties of the differentiation of a product function, while the logical quantization theorem is due to a relatively more intricate transformation.

Just as the quantization relation is central to quantum physics, so the logical quantization relation is a fundamental theorem from which a theory of the governing code of the brain, the intelligence code, can be erected from *first principles*. Since the action of the commutator **[q, M]** on a logical state is identity, any normed state $|q\rangle$ is a fixed point of the commutator operator **[q, M]**, and $\langle q|[\mathbf{q}, \mathbf{M}]|q\rangle \equiv \langle q|q\rangle$. This result refers directly to consciousness because in the final analysis, the goal is to derive a covariant theory of the brain which allows the act of thinking about thinking (self-reference). The logical commutation relation throws light on a fundamental limitation imposed on introspective observation of cognitive states. It shows that the thought process can be quantized outside the realm of quantum mechanics and without assuming that consciousness is quantum-mechanical. A distinction between *quantized* and *quantum* must be duly recognized. We have obtained a mathematical proof of the quantization of matrix logic which does not necessarily entails the quantum nature of logic. The fact that logic can be quantized without reference to wavefunction has far-reaching implications for the quantization of classical system.

Is logical quantization pure mathematical abstraction? When Otto Stern in 1921 proposed to check out the quantization of angular momentum (spin), Max Born assured him that he would see nothing, because such spatial quantization is only a mathematical fiction. As we all know, the experiment proved Born to be fundamentally wrong. There are those who believe that our logical quantization is just theoretical and does not refer to a real state of

affairs. How would one go about testing the quantization theorem by means of experiment? Is it possible to split a single logical degree of freedom into a quantized pair of mutually exclusive thoughts which the mind would not accept simultaneously? If one thought occurs, the other cannot; the two together are meaningless. What is meaningless, the argument goes, does not exist and what does not exist cannot be measured. This is laboratory physics. But in cognitive logic 'meaningless' stands for a contradiction which can 'exist' and be 'measured' by the faculty of mind, no matter how bizarre and counterintuitive. Without expensive technology, simply making use of the 'technology' of the mind, anyone can easily observe the effect of logical quantization in the Liar paradox.

SPEEDING THOUGHT UP

By making use of different algorithms we can speed up or slow down the thought process. There are fast and slow thinkers. Somehow we believe that those who think fast are more intelligent, and those who think slow are stupid, which is a misconception. Apparently consciousness is an inertial system with asymptotic freedom: speeding thought up produces the opposite result. One can think faster than another, but not faster than one think. For a particular solution method the thought process cannot be accelerated: a logical state can be given a momentum, but the iteration cancels the momentum out.

Definition. The logical acceleration operator **A** is the second-order derivative with respect to truth-value which in matrix form is the momentum operator squared:

$$\mathbf{A}|q> = \frac{d^2}{dq^2}|q> = \mathbf{M}^2|q>.$$

Direct calculation shows that acceleration of the thought process sends a logical state, via the coherent superposition $|S_->$, to the origin

$$\mathbf{A}|q> = |S_0>,$$

hence the logical velocity is constant.

The commutation theorem

$$\mathbf{A} \sim [\mathbf{M}, [\mathbf{q}, \mathbf{M}]]$$

Proof

Since

$$\mathbf{A}|q> = |S_0>,$$

we will prove the theorem if we prove that

$$[\mathbf{M}, [\mathbf{q}, \mathbf{M}]]|q> = |S_0>.$$

According to the quantization theorem

$$[\mathbf{q}, \mathbf{M}]|q> = |q>.$$

Acting on both sides of this equation with $\mathbf{M}$ we get :

$$\mathbf{M}\,[\,\mathbf{q},\mathbf{M}\,]|q\rangle = \mathbf{M}\,|q\rangle = |s_-\rangle.$$

We can also begin with

$$\mathbf{M}|q\rangle = |s_-\rangle,$$

and to apply the commutator $[\mathbf{q}, \mathbf{M}]$ to both sides of the equation. Then given that

$$[\,\mathbf{q},\mathbf{M}\,]\,|s_-\rangle = |s_-\rangle,$$

we again have

$$[\,\mathbf{q},\mathbf{M}\,]\,\mathbf{M}\,|q\rangle = [\,\mathbf{q},\mathbf{M}\,]\,|s_-\rangle = |s_-\rangle.$$

which proves the commutation

$$[\,\mathbf{M},[\,\mathbf{q},\mathbf{M}\,]\,] = \mathbf{0}.$$

Hence,

$$[\,\mathbf{M},[\,\mathbf{q},\mathbf{M}\,]\,]|q\rangle = \mathbf{0}|q\rangle = |s_0\rangle.$$

Theorem

$$\mathbf{M}\,[\mathbf{q},\mathbf{M}] = [\mathbf{q},\mathbf{A}]$$

Proof

Writing the operators explicitly, we get:

$$\begin{aligned}
-\frac{d}{dq}[q,-\frac{d}{dq}] &= -\frac{d}{dq}(\frac{d}{dq}q - q\frac{d}{dq}) \\
&= -\frac{d^2}{dq^2}q - \frac{d}{dq}\frac{dq}{dq} + \frac{dq}{dq}\frac{d}{dq} + q\frac{d^2}{dq^2} \\
&= -\frac{d^2}{dq^2}q - \frac{d}{dq} + \frac{d}{dq} + q\frac{d^2}{dq^2} \\
&= -\frac{d^2}{dq^2}q + q\frac{d^2}{dq^2} \\
&= [q,A]
\end{aligned}$$

Nonassociativity theorem

$$\mathbf{M}\,([\,\mathbf{q},\mathbf{M}\,]|q\rangle) \neq (\mathbf{M}\,[\,\mathbf{q},\mathbf{M}\,])\,|q\rangle$$

Proof

The result of calculation for the left hand side of the expression we know,

$$\mathbf{M}\,([\,\mathbf{q},\mathbf{M}\,]|q\rangle) = |s_-\rangle.$$

According to the preceding theorem, the right hand side contracts as follows:

$$\mathbf{M}\,[\,\mathbf{q},\mathbf{M}\,] = [\,\mathbf{q},\mathbf{A}\,].$$

Acting now with $[\mathbf{q}, \mathbf{A}]$ on $|q\rangle$ we derive

$$[\mathbf{q},\mathbf{A}]|q> = (q\frac{d^2}{dq^2} - \frac{d^2}{dq^2}q)|q> = |S_0> - \frac{d^2}{dq^2}\begin{pmatrix} q-q^2 \\ q^2 \end{pmatrix} = -\begin{pmatrix} -2 \\ 2 \end{pmatrix} = 2|S_->.$$

We determined the breakdown of the associativity of multiplication of the logical momentum operator **M** and the commutator operator **[q, M]**:

$$|S_-> \neq 2|S_->.$$

SELF-"CONSCIOUS" POLYNOMIALS

Until the development of matrix logic, the operator description was applied exclusively in quantum physics. The interpretation of logical connectives as matrix operators and the identification of truth-values as the eigenvalues of these operators [Ref 88, 89] put logic and quantum mechanics in the same and unique class of abstract theories where one has to revert from classical thinking in terms of ordinary number functions to quantum thinking in terms of operators. This conceptual reform enables us to gain insight into abstract thinking, indicating that the laws of logic are fundamental in origin. The interpretation of the truth-values as eigenvalues, introduced in matrix logic, radically changes our understanding of high-level brain functions. Much wisdom is built into the eigenvalue concept. In particular, a logical operator **L** is singular if and only if it has at least one zero truth-value, which is the direct consequence of the fact that **detL** equals the product of the eigenvalues of **L**:

$$\mathbf{detL} = \lambda_1\lambda_2 \ldots \lambda_n .$$

The sum of the eigenvalues of **L** is another fundamental quantity of a logical operator, equal to the trace of **L**:

$$\mathbf{Tr\ L} = \lambda_1+\lambda_2+ \ldots +\lambda_n .$$

Knowing either, we automatically know the other.

But the most significant self-referential feature of matrix thinking is due to the theorem:

Every matrix satisfies its own characteristic equation.

If the characteristic polynomial of **L** is

$$\mathbf{pol}(\lambda) = \lambda^n + k_1\lambda^{n-1} + \ldots + k_{n-1}\lambda + k_n = 0$$

then simply replacing the scalar λ by the operator **L**, we have the matrix polynomial

$$\mathbf{pol(L)} = \mathbf{L}^n + k_1\mathbf{L}^{n-1} + \ldots + k_{n-1}\mathbf{L} + k_n\mathbf{I} = \mathbf{0}$$

This is an important statement: the equation $\mathbf{pol}(\lambda) = 0$ is satisfied for **n** values $\lambda = \lambda_1, \lambda_2, \ldots, \lambda_n$, and $\mathbf{pol(L)} = \mathbf{0}$ states that the matrix polynomial is identically equal to the **(n×n)** zero matrix. The symmetry which exchanges λ

and L makes a logical matrix in a sense "conscious" of itself. The scalar calculations for the truth-values are carried over directly to the operator calculation and vice versa.

The matrix polynomial can be rearranged to give

$$L^n = - k_1 L^{n-1} - k_2 L^{n-2} - \ldots - k_{n-1} L - k_n I$$

which expresses any power L^n as a linear combination of the powers L^{n-1}, L^{n-2},..., L, I. For the nonsingular L, we can premultiply the matrix polynomial by the inverse L^{-1} which, divided by $k_n \neq 0$, gives the template for computing the inverse as a linear combination of powers of L up to L^{n-1}:

$$L^{-1} = - (L^{n-1} + k_1 L^{n-2} + \ldots + k_{n-1} I) / k_n$$

A strong cognitive element, a built-in 'self-knowing', distinguishes the knowledge operators which transform every normed logical state into an eigenstate. The logical momentum M forms an eigenstate directly by acting on an arbitrary state:

$$M|q\rangle = |s_-\rangle$$

$$M\,|s_-\rangle = \lambda\,|s_-\rangle, \qquad \lambda = 0.$$

These and other intelligent properties of operator description can be used to overcome many of the limitations of classical logic. But first what are these limitations?

LIMITATIONS OF CLASSICAL LOGIC

To appreciate fully the need for the operator reformulation of logic inspired by quantum thinking, we must examine the limitations of classical logical theory commonly in use. Logic is a window on the mind. Just as the wavefunction enables us to think quantum-mechanically, logic has been the only tool, at least until now, which enables us to think consciously. The

algebraic formalization of logic has opened up new opportunities for the conversion of human thought into precise mathematical formulas. The formalization of logic, however, is not achieved without a price. Classical logic is blind to the original and extraordinary. Basically it is a logic of fools and computers, reminding us forcefully that intelligent thinking should not be taken for granted. When we hold a conversation, dial a phone or decide to take an umbrella, expecting rain, the underlying thought processes may seem trivial. Its exceptional complexities, however, quickly become apparent when one attempts to simulate even elementary mental activity on a computer. Finding an algorithm even for simple intelligent behaviour turns out to be a horrendous undertaking.

To form an intelligent thought is to form right connections between abstract concepts which are either given or must first be developed. As soon as the necessary concepts are developed and adequate connections are formed, the deduction process follows naturally. However, finding the proper connections is not always easy. Often we are swept away by the multiplicity of language. A simple conjunction can take many different forms, such as 'and', 'also', 'but also', 'but', 'while', 'not only ... but also', 'although', 'as well as' and so on, which all may serve as valid operations of conjunction. Valid logical thoughts take the shape of so-called *well-formed formulas* (wff). In such formulas logical concepts, denoted by variables **x**, **y**, **z**, ... are joined together by the unambiguously defined logical connectives **AND**, **OR**, **NOT**, **IMPLY**, etc. If the connectives are correct and the priorities of logical operations are respected, the truth or falsity of the formula for a given set of logical variables can be determined algorithmically, according to well-defined rules. The classical logic we commonly employ conforms to the rules of Boolean algebra, and is often called simply Boolean logic.

Certain formulas - tautologies and contradictions - are universal, being universally true or universally false under the arbitrary interpretation of the logical variables. Such are, for example, the fundamental syllogism law:

$$((x \text{ IMPLY } y)\text{AND}(y \text{ IMPLY } z)) \text{ IMPLY } (x \text{ IMPLY } z) \equiv \text{TRUE},$$

and the **Modus Ponens** inference rule:

$$((x \text{ AND } (x \text{ IMPLY } y)) \text{ IMPLY } y \equiv \text{TRUE}.$$

In the framework of brain science the universal logical formulas must be treated as cognitive constants which hold no matter what the conditions of the world. They represent the fundamental coding units of the thought process, a fact sometimes poorly grasped even by professional logicians. Any logical inference rule we employ is necessarily a tautology, and the intelligence code is a tautology of tautologies, a global tautology built up from elementary tautologies of the lower levels. Tautologies are not just a universal code of communication, but only tautologies are covariant and only true results of

intelligence are tautologies [Ref 89]. Because much of the puzzle of consciousness can be traced to the fundamental covariant construct of tautology, the study of the laws governing tautologies is instrumental in the deciphering of the intelligence code. Although classical logic excels in proving deterministic tautologies, it is yet to learn how to formulate tautologies in randomized statistical universes with probabilistic correlations.

Classical logic was devised to articulate and communicate logical truths. The use of symbolic notation and the development of Boolean logic algebra has brought logic closer to the realm of mathematics and the exact sciences. However, in spite of being able to capture important formal aspects of deductive reasoning, logic still lacks the decisive features that would allow it to function fully as an instrument of calculation. Different formalizations enjoy different degrees of success. Mathematical logic continues to be developed, to a great extent under the influence of problems arising in applications, and it is here where many limitations of classical logic become apparent.

Propositional Boolean logic restricts itself to the study of true and false logic expressions, with true and false representing the so-called binary alphabet of truth-values. A logic designed over such an alphabet is called a two-valued or binary logic. The convenient simplicity of this alphabet of truth-values, however, imposes severe limitations on the objects of logical inquiry, since it disregards those expressions which cannot be defined in terms of true and false, such as those which are undecided, undetermined, unknown, meaningless or superposed. Investigation of these and other intelligent properties of logical discourse is attempted in multivalued and modal logics, which deal with the truths that lie beyond the grasp of Boolean 'black-or-white' logic. Multivalued or **k**-valued logical systems introduce, albeit arbitrarily, other truth-values beside true and false. This prompts the question: what is the fundamental value of **k**? The existing multivalued logics are not able to answer this question from first principles. In spite of engendering much excitement and interest the multivalued approach has so far offered no fundamental breakthrough or prediction.

Mathematical logic is often understood as a quest for a certainty which will enable us to place often the vague intuitive structures of the thought process into the definite framework of Boolean algebra. Boolean logic thrives on precision and its whole purpose is to exclude multiplicity. However, precision, useful in some situations, can be detrimental in others, constraining the ability of classical logic to accommodate the complexities and ambiguities of thinking. Such features of the thought process as indeterminism and uncertainty, spontaneous but nonetheless correct insights and decisions, unpredictability, interference and intuition, etc. are not formalizable in classical logic.

Even though classical logic is an important component of the thought process, Boolean logic does not encompass the entire spectrum of thinking operations, due to a number of fundamental limitations. Before we start to seek ways to overcome these limitations we must first clearly identify them.

TRUTH-TABLES

The development of logical theory follows closely in the footsteps of mathematics and the use of symbolic notation has brought it closer to the realm of mathematics. However, classical logic is more mathematical in form than in substance. Mathematics cannot be properly dealt with without the techniques that afford certain and rigorous computation, and the lack of such an overall computing capability is precisely what separates the domain of logic from that of mathematics. Although the boundary between the two is somewhat vague and overlapping, it is nevertheless real and significant.

A major leap forward in the mathematization of logic occurred in the 19th century with the development by August DeMorgan and George Boole of what is known at present as Boolean logic. In scientific folklore no discovery is ever attributed to the right person. Indeed DeMorgan, whose name is immortalized in the famous duality rules:

$$x \cap y = \overline{\bar{x} \cup \bar{y}}, \qquad x \cup y = \overline{\bar{x} \cap \bar{y}},$$

preceded Boole, if not by much, in realizing the possibility of an algebraic formulation of logic. This formulation become possible upon the realization that logical statements, if represented by binary numbers, can be written as algebraic equations and treated purely computationally.

A standard way of describing a logical operation is by defining its truth-table. For two Boolean variables **x** and **y**, one can write down **16** different truth-tables corresponding to **16** different logical operations. Although we can achieve a great deal with **16** logical operations, and one needs three, two or even one operation to design logical calculus, the thought process clearly involves much more. We know how to write a truth-table for connectives like 'neither **x** nor **y**' or for 'absolutely not **x**' or 'necessarily **x**', for 'unless', for 'of course', '**x** but not **y**' and a number of others. But classical logic does not have a truth-table for 'create', for 'retrace', for 'abort', for 'come back' or 'cancel' and many others. It is poorly understood that the limitations and many drawbacks in the study of the actual thought process stem from making use of just **16** binary logical operations. This number is increased dramatically in the formalism of matrix logic which takes the full step into the maths. The logical connectives are given the mathematical status of matrix operators, replacing the Boolean operations with one universal procedure of matrix multiplication. In classical logic there is a strict division into unary and binary connectives. It seems quite natural to consider, in addition to the binary operations, the unary operations of logical operators. In doing so, one ought to recognize that in matrix logic, as in conventional logic, a division into unary and binary operations does exist, but a division into unary and binary connectives or operators ceases to have similar implications. Indeed, in contrast to ordinary connectives, the same logical operator is able to act either as a binary operator between two logical vectors, or as a unary operator upon a single logical vector. This implies, furthermore, that the set of **4** unary operations of Boolean logic is enlarged to a set of **16** unary operations, making the number

of binary and unary operations in matrix logic equal, in total, **32** standard operations.

DEALING WITH ABSURDS

The key notions of classical logic are those of a logical variable and of a logical connective. In Boolean logic logical variables are defined in the binary set $E_2 = \{0,1\}$ where the values 0 and 1 correspond to false and true respectively. As we mentioned above, there are unary connectives, like **NOT**, which act on a single atomic variable, and binary connectives like **AND**, **OR**, **IMPLY**, **IF**, etc., which act on a pair of variables, joining them into a compound formula. Each logical connective is defined by a corresponding truth-table, which tells us exactly the result of the logical operation, depending on the value of the input variables. With the notable exception of the implications **IF** and **IMPLY**, Boolean connectives have a clear intuitive meaning. Although implication is in common use and we master this logical operation easily, the controversy surrounding implication remains unresolved. Because the implication syllogism lies at the heart of the matrix principle formulated earlier, we must look more closely at the truth-table for implication, which is a common source of confusion:

IMPLY	y = 0	y = 1
x = 0	TRUE	TRUE
x = 1	FALSE	TRUE

On the one hand, implication completely rules out the possibility of a true antecedent x = 1 implying a false consequent y = 0, which fully agrees with our intuition. But on the other hand, implication is valid for the false x = 0 implying the true y = 1. This can lead to valid but absurd and meaningless situations, and one has to be reminded that a logical implication represents only a relationship between the truth status of x and y and not a relationship between their meanings. Among the various limitations of classical logic this is probably the best known. Boolean implication enables the truth of y to be inferred from the truth of x, and nothing in particular to be inferred from the falsity of x. The difficulty arises when a false antecedent implies a true consequent and one cannot interpret logical implication in terms of cause and effect in a physical sense.

The failure to provide a clear and satisfactory explanation of the bizarre but mathematically correct and combinatorially necessary properties of implication is a major weakness of classical logic. One understands implication as a conditional relation between two statements **x** and **y**,

x implies **y**,

when the relation is true. However, implication does not necessarily represent the logical force of conditional statements. If **x** is false or **y** is true, the implication is true. Therefore classical logic will accept as valid implications like

"If liars have integrity, cats can walk on water".

Two lies can form a true implication! These paradoxes of implication have led to the search for definitions of *strict* implication:

$$(x \rightarrow y)$$

where the modal connective *necessarily* is introduced. In various texts we are told that solves the paradox. It doesn't. It simply axiomatically imposes a constraint to suit the logician's need to live in a perfectly meaningful world. The world, however, is not perfect at all. Since the truth-value of $(x \rightarrow y)$ does not depend wholly on the truth-values of **x** and **y**, we have here a modal and not a truth-functional link. But it is the truth-functional complexity of implication which is the source of the problem. The resolution of the paradox of logical implication is not simply beyond the grasp of classical logic, it **lies outside logic**, or at least what we have understood by logic until now.

NEGATION

An important fact commonly overlooked or perhaps not realized by many logicians is that Boolean logic is a theory of the fixed point:

$$x^2 = x.$$

Put another way, the essence of classical logic, as well as its limitations, stems from the assumption that logical values are idempotent. Clearly, this idempotency axiom immediately restricts the possible values of logical variables to just two values **0** and **1**, corresponding to false and true respectively. Classical logic is two-valued. The existence of the fixed point entails that logical negation is computationally a binary complement,

$$\mathrm{NOT}x \Rightarrow 1 - x.$$

Then the classical Law of Contradiction is deduced naturally:

$$x - x^2 = x\,(1 - x) = 0$$

According to the Law of Contradiction no statement can be true and false simultaneously: $x\,\bar{x} \equiv 0$, which, employing the DeMorgan duality rule, turns into the classical Law of Excluded Middle, the tautology

$$\bar{x}\ \mathbf{OR}\ x \equiv 1.$$

Making use of the fixed point, one builds up Boolean logic from first principles, which is advantageous, but this first principle has fundamental limitations. Whereas the thought process frequently involves subtle, partial, local or incomplete forms of negation, Boolean negation is a total and nondivisible logical operation, unintelligently and globally swapping true and false. **Classical negation has no parts, but there are many.**

This question is intimately linked to the question of logical reversibility. The negation of negation is identity:

$$\bar{\bar{x}} = x,$$

which tells us that Boolean **NOT** is a self-inverse, in fact the only nontrivial nonsingular operation in Boolean logic. Classical logic insists, quite improperly, that only the negation operation can be reversed while all other logical operations do not have inverses and are not invertible. This is another fundamental limitation and a major blunder of classical theory.

AN UNNOTICED OPERATION IN BOOLEAN LOGIC

The development of mathematical logic became possible upon the realization that if the propositions of logic could be represented by precise symbols, then the relation between them could be read as precisely as algebraic equations. From the fixed point axiom, we determine that **NOT** can be formalized as a subtraction from 1. Further, the logical **AND** and **OR** are identified as binary algebraic operations, product and addition respectively:

$$(x\mathbf{AND}y) \Rightarrow x \cdot y$$

$$(x\mathbf{OR}y) \Rightarrow x + y$$

To make this representation fully legitimate another serious limitation has to be imposed. The sum of two truths must be a truth. Logical addition follows the unnatural involution rule:

$$1 + 1 = 1,$$

responsible for the awkward fact that a given logical expression may correspond to different algebraic expressions.

Given a set of functions (operations), we are typically interested in determining the minimal irreducible basis set. The three logical operations **NOT, AND** and **OR** form a functionally complete basis set. The textbooks on logic usually tell us that these operations are the only operations which can be represented as algebraic operations over binary numbers: subtraction, product and addition respectively. Allegedly there are not enough algebraic operations in Boolean algebra to allow every logical connective to be associated with a particular algebraic operation. In fact, there is a fourth, and a very important logical operation, implication, which can also be expressed as an algebraic operation over binary numbers. For that we explore the powers y^x, where x, $y \in E_2$. Then, setting $0^0 = 1$, we can represent the implication as follows:

$$(x \text{ implies } y) \sim y^x$$

Taking the power x in y^x to be the antecedent and the base y to be the consequent, one arrives at the following truth table:

x \ y	0	1
0	$0^0=1$	$1^0=1$
1	$0^1=0$	$1^1=1$

Since any value raised to the power of null is unity:

$$(\text{anything})^0 = 1,$$

if the consequent is true, the implication is true: $1^0=1$, and $1^1=1$, and is not dependent, as required, on the value of the antecedent.

As unravelled by the matrix principle, implication is central to thinking operations. What **Boole** and his followers either did not pay attention to or did not know is that implication can be modelled by a binary exponent, fact that went unnoticed for a long time. However, important as it is, the algebraic formalisation of **NOT, AND** and **OR**, and now also **IF**, is restricted to only these three (four) connectives, which is another fundamental limitation of classical logic. Whenever one wishes to bring the **13** (**12**) other Boolean operations into algebraic form, one can do this only indirectly by expressing them in the basis set of connectives **NOT, AND** and **OR**, (**IF**). Consider for example implication expressed in the minor basis sets:

x IMPLY y = (**NOT x**) **OR y** = (1-x) + y = 1- x + y

or

x IMPLY y = **NOT**(x • (**NOT y**)) = 1-(x (1- y)) = 1 - x + xy.

The logical connectives in Boolean logic are *operations*, and there exist only a limited number of them in ordinary algebra, a natural limitation classical logic has no means to overcome. Abstract thought requires many, not just a few operations, just as one cannot create a life form with just a few genes but need thousands of them. Not surprisingly, classical logic fails dramatically to explain induction and creative high-level intelligence. Abstract thought **manipulates operations**, not just variables as classical logic does. But there is nothing in classical theory which could enable one to gain direct control over the operations. The strict division into operators and operands exist in classical logic, in contrast to consciousness, where the distinction between them is fuzzy or dissolves altogether.

When one moves up the ladder of abstraction, from the basic level of data to the top level of consciousness, one realizes that classical logical theory is confined to just the few first levels of data and information processing, modestly probing the next level of the knowledge processing. Although logic forms a core of high-level intelligence, our knowledge of high-level algorithms is poor. The only algorithms we fully understand and manage confidently are the algorithms of computation, and these alone in their present form are insufficient to handle high-level cognitive functions such as abstraction, induction, intuition and creativity.

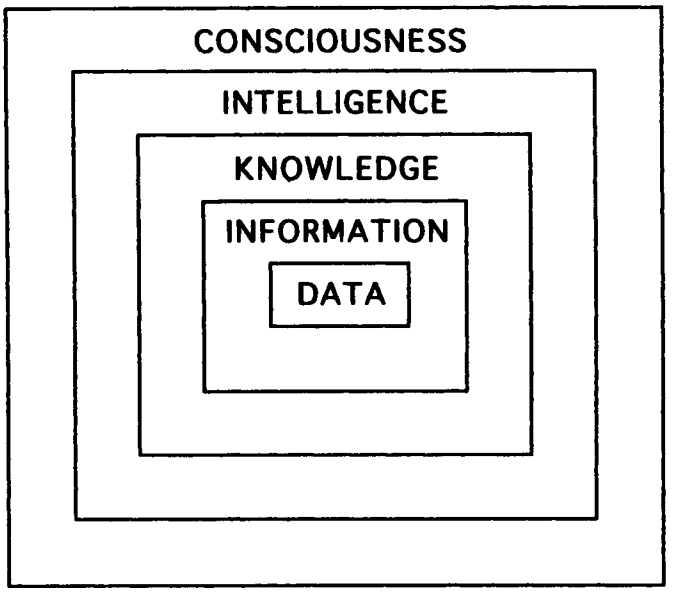
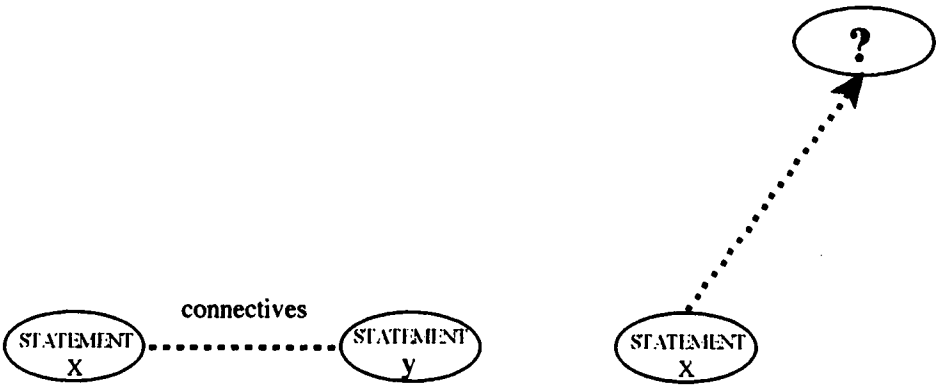


Fig. 21
The complexity levels of consciousness

As far as intelligence and ultimately consciousness are concerned, logical theory has a long way to go before it can be meaningfully and effectively applied there. To come to grips with the problem of consciousness one needs a deeper exploration of logic.

LOGIC INTERPOLATES, MIND EXTRAPOLATES

Mathematical and physical theories are commonly idealizations and approximations, which often disregard actual physical features. A ball is not a perfect sphere, there is no pointlike mass and no absolutely smooth surface without friction. In the realm of idealizations logic is no exception, a fact which has both its advantages and its drawbacks. But besides the flaws that are due to idealization, such as two-valuedness of classical logic, there are defects in logical theory that are fundamental in nature. This fact becomes clear when one attempts to employ classical logic to describe the intelligent processes in the brain. One soon discovers that while logic interpolates, mind extrapolates:



INTERPOLATION EXTRAPOLATION

Our fascination with the elegance and effectiveness of logic in the realm of deduction is combined with our frustration at its impotence in the realm of induction.

What are logical interpolation and extrapolation? When, given two logical statements, we connect them by means of connectives, this in some abstract topological space amounts to interpolation between two logical points. Technically speaking we establish validity of an expression by computing some formula which must be well-formed. Logic doesn't extrapolate. Although the usage of the modal connectives **possibly** $\Diamond$ and **necessarily** can be useful, they are not genuine computational connectives, and provide little support, if any, to the attempt to formulate and compute the unknown. In cognitive logic, circumstances are fundamentally different. A thought very often must run forward, into unknown and uncharted territory. The final destination point is not given but has to be found or created. The fundamental extrapolation mechanism enters the thought process, indicating that all attempts to solve the question of artificial intelligence in the framework of a logic which only interpolates are doomed to failure.

TIMELESS LOGIC

Another limitation of classical logic is its inability to account for dynamical aspect of thinking. Logic is static. It does not tell us about time. Among the various limitations of classical logic the most serious one concerns its timeless character. The dynamical aspect of intelligent thinking is totally unaccounted for, even though some attempts were made in tense logic. The introduction of time parameters into logic brought no significant results, and is basically philosophical not computational. In this study the question of logical dynamics is of paramount importance, and will be treated by making use of the differential calculus, a tool unfamiliar in mathematical logic.

The question of time in logic is closely connected to the question of commutation. A major defect of classical logic is that it often commutes in situations where it shouldn't. The truth-value of the expression

JFK was assassinated **AND** buried

and the expression

JFK was buried **AND** assassinated,

is according to the rule of Boolean logic the same, although the second expression is a meaningless statement. Likewise, the conjunction:

'a girl got pregnant **AND** gave birth to a child'

is truth-functionally equivalent to a conjunction

'a girl gave birth to a child **AND** got pregnant',

which underscores the disregard for time and causality in classical logic. Boolean variables are Abelian and commute under conjunction. The inability of classical logic to distinguish between Abelian and non-Abelian systems not only offends common sense, it reveals the inadequacy of classical logic to deal with high-level intelligence. Noncommutation is a fundamental feature of logic which naturally leads us to the development of non-Abelian matrix logic in the framework of the matrix principle.

SELF AS A FIXED-POINT

We might think of the single convergence to a fixed point as the fundamental computing step of a brain.

J.J. Hopfield

SOLVING THE PROBLEM OF BEING ITSELF

In an attempt to understand the nature of "I" and SELF we naturally come to the problem of fixed points in cognitive logic. Given a map $F: X \to X$ from a topological space to itself, a point $x \in X$ is a fixed point of F if $F(x) = x$. Any continuous mapping of a compact convex set to itself is known to have a fixed point. If X is the n-ball E^n then any such F has a fixed point. Calculating the homology groups, one determines that for $n = 2$, any continuous map of a circular disc onto itself has a fixed point. More generally, if X is a polyhedron and the Lefschetz number L(F) is a certain number depending only on F: $H_n(X) \to H_n(X)$, where $n \geq 0$, then F has a fixed point if $L(F) \neq 0$.

These theorems are existential rather than constructive. They *cannot tell us how to go about finding a fixed point.* SELF is a dynamical process with unnoticeable incremental changes. "I" is constantly in motion, remaining invariant, very much like a squirrel running in a rotating wheel, but staying in one place, while the wheel is in motion.

The logical quantization theorem, proven earlier, is important not only because it explicitly demonstrates the noncommutativity of logic, but also because it can be naturally and meaningfully interpreted as a fixed point theorem. Any normed logical state $|q\rangle$ is a fixed point of the commutator of the truth-value and the logical momentum:

$$[q, M]\,|q\rangle = |q\rangle\ .$$

This theorem tells us that logical SELF can manifest through the recursive enclosure

$$F(F(F(F(|q\rangle...)))) = |q\rangle.$$

where

$$F = [q, M].$$

The fixed points occur in a realm where logical elements act on themselves, temporarily forcing the states $|q\rangle$ outside the convex probability space into singularity. Continuity of SELF is accomplished by retaining and reinforcing the fixed point. "I" is a relation between mySELF and observing mySELF. To gain conscious awareness the logical system requires self-reference, where the gap between successive applications of "I" is not detectable. The fixed-point structure of SELF creates the fiction of continuity, and allows the act of thinking about thinking. As a fixed point SELF refers to itself, and because the set of normalized logical states $|q\rangle$ is infinite, there are infinitely many cognitive fixed points. This result is universal and speaks directly to conscious experience.

THE GOLDEN RATIO

Theorem

The eigenstates of the **OR** operator are the fixed points (steady states) of stochastic matrices formed from these eigenstates.

Proof

First we consider the eigenstate equation for **OR**:

$$\vee |q\rangle = \phi |q\rangle,$$

where the eigenvalues are denoted by a special symbol ϕ, to underline their link to the Fibonacci numbers [Ref 91]. Fibonacci numbers are a subset of the natural number series such that any member f_n is a sum of the two preceding members:

$$f_n = f_{n-1} + f_{n-2}; \quad f_0 = 0, \quad f_1 = 1.$$

Solving the characteristic polynomial

$$\phi^2 - \phi - 1 = 0,$$

we obtain the two eigenvalues:

$$\phi_{1,2} = \frac{1 \pm \sqrt{5}}{2},$$

which are the golden ratios:

$$\phi_1 = 1.618, \qquad \phi_2 = -0.618.$$

Since ϕ_2 is a logical complement of ϕ_1,

$$\phi_2 = 1 - \phi_1$$

it is natural to simplify the notation:

$$\phi_1 = \phi, \qquad \phi_2 = \bar{\phi}.$$

where

$$\phi + \bar{\phi} = 1, \qquad \phi\,\bar{\phi} = -1,$$

The eigenvalues can also be calculated as the ratios of the successive **Fibonacci** numbers f_n for a sufficiently large n:

$$\phi = \frac{f_{n+1}}{f_n}, \qquad \bar{\phi} = -\frac{f_n}{f_{n+1}},$$

with

$$\frac{f_{n+1}}{f_n} - \frac{f_n}{f_{n+1}} = 1.$$

The eigenstates corresponding to the eigenvalues ϕ and $\bar{\phi}$ are found to be functions of the eigenvalues:

$$\text{for } \phi = 1.618, \qquad |q_1\rangle = \begin{pmatrix} 0.382 \\ 0.618 \end{pmatrix} = \begin{pmatrix} \bar{\phi}^{-2} \\ -\bar{\phi} \end{pmatrix},$$

$$\text{for } \bar{\phi} = -0.618, \qquad |q_2\rangle = \begin{pmatrix} 2.618 \\ -1.618 \end{pmatrix} = \begin{pmatrix} \phi^{2} \\ -\phi \end{pmatrix}.$$

Naturally, the eigenstates are orthogonal:

$$\langle q_1 | q_2 \rangle = \bar{\phi}^{-2}\phi^2 + \bar{\phi}\,\phi = 1 - 1 = 0.$$

By attaching a copy of an eigenstate vector to itself we now construct the matrices:

$$(|q_1\rangle\ |q_1\rangle) = \begin{pmatrix} \bar{\phi}^{-2} & \bar{\phi}^{-2} \\ -\bar{\phi} & -\bar{\phi} \end{pmatrix}, \qquad (|q_2\rangle\ |q_2\rangle) = \begin{pmatrix} \phi^{2} & \phi^{2} \\ -\phi & -\phi \end{pmatrix},$$

and determine their stochastic invariance:

$$(|q_1\rangle\ |q_1\rangle)^n = (|q_1\rangle\ |q_1\rangle), \qquad (|q_2\rangle\ |q_2\rangle)^n = (|q_2\rangle\ |q_2\rangle).$$

The logical system can reside in the stable stochastic limits which are unchanged by iteration:

$$\begin{pmatrix} \bar{\phi}^{-2} & \bar{\phi}^{-2} \\ -\bar{\phi} & -\bar{\phi} \end{pmatrix}^n = \begin{pmatrix} \bar{\phi}^{-2} & \bar{\phi}^{-2} \\ -\bar{\phi} & -\bar{\phi} \end{pmatrix}, \qquad \begin{pmatrix} \phi^{2} & \phi^{2} \\ -\phi & -\phi \end{pmatrix}^n = \begin{pmatrix} \phi^{2} & \phi^{2} \\ -\phi & -\phi \end{pmatrix}.$$

Explicitly, for n = 2 we calculate:

$$\begin{pmatrix} 0.382 & 0.382 \\ 0.618 & 0.618 \end{pmatrix}\begin{pmatrix} 0.382 & 0.382 \\ 0.618 & 0.618 \end{pmatrix} = \begin{pmatrix} 0.382 & 0.382 \\ 0.618 & 0.618 \end{pmatrix},$$

$$\begin{pmatrix} 2.618 & 2.618 \\ -1.618 & -1.618 \end{pmatrix}\begin{pmatrix} 2.618 & 2.618 \\ -1.618 & -1.618 \end{pmatrix} = \begin{pmatrix} 2.618 & 2.618 \\ -1.618 & -1.618 \end{pmatrix}.$$

In self-referential fashion this stability directly implies that the states

$$|q_1\rangle = \begin{pmatrix} \bar{\phi}^{-2} \\ -\bar{\phi} \end{pmatrix} \quad \text{and} \quad |q_2\rangle = \begin{pmatrix} \phi^{2} \\ -\phi \end{pmatrix}$$

are the fixed points:

$$\begin{pmatrix} \bar{\phi}^{-2} & \bar{\phi}^{-2} \\ -\bar{\phi} & -\bar{\phi} \end{pmatrix}\begin{pmatrix} \bar{\phi}^{-2} \\ -\bar{\phi} \end{pmatrix} = \begin{pmatrix} \bar{\phi}^{-2} \\ -\bar{\phi} \end{pmatrix}, \qquad \begin{pmatrix} \phi^{2} & \phi^{2} \\ -\phi & -\phi \end{pmatrix}\begin{pmatrix} \phi^{2} \\ -\phi \end{pmatrix} = \begin{pmatrix} \phi^{2} \\ -\phi \end{pmatrix}.$$

The fixed points are logically normalized states. However, while the fixed point $|q_1\rangle$ is a probability, containing strictly positive components, the fixed point $|q_2\rangle$ is an improbability, containing the negative and greater-than-unity components which sum to unity:

$$0.382 + 0.618 = 1, \qquad 2.618 + (-1.618) = 1.$$

Here we come across a fundamental feature of a cognitive fixed point. A stochastic logical process can achieve stability and be on the whole in agreement with the laws of probability, in spite of the presence of the transition elements, which are uninterpretable in terms of conventional probability theory.

LOGARITHMIC FIXED POINT

The stochastic process is a dynamical process. We have determined that the eigenstates of the logical operator **OR** form an orthogonal pair of fixed points. The fundamental operator **OR** is distinguished in another important respect, by being in itself a logarithmic fixed point.

Theorem

The natural logarithm $\ln v$ is a differential fixed point:

$$\frac{d}{dv}\ln v = \ln v$$

Proof

For the left-hand side, according to the logarithmic rule of differentiation, we get:

$$\frac{d}{dv}\ln v = \frac{1}{v}.$$

Since

$$e^{\ln v} = v$$

while by expansion into the series, and neglecting higher-order nonlogical terms,

$$e^{\ln v} = \mathbf{I} + \ln v$$

one derives

$$\ln v = v - \mathbf{I} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}.$$

But

$$\begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix} = v^{-1} = \frac{1}{v}.$$

SECOND-QUANTIZED FIXED POINTS

For cognitive studies a situation in which a number function or a vector represents a fixed point is very different from a situation when a fixed point is a matrix or an operator. The second-quantized annihilation and creation operators are the fundamental fixed points for the implication operators:

$$\mathbf{IMPLY} \cdot a = a, \qquad \mathbf{IF} \cdot a^* = a^*$$

where

$$[\,\mathbf{IMPLY},\ a\,] = [\,\mathbf{IF},\ a^*\,] = \mathbf{0}$$

$$[\,\mathbf{IMPLY}, \mathbf{IF}\,] = [\,a,\ a^*].$$

These relations show that a second-quantized quantum system and a logical system are convertible. Since

$$a^* = \overline{\overline{\mathbf{IMPLY}}}\,, \qquad a = \overline{\overline{\mathbf{IF}}}\,,$$

one easily exchanges between the logical operator fixed points

$$\mathbf{IMPLY} \cdot \overline{\overline{\mathbf{IF}}} = \overline{\overline{\mathbf{IF}}}\,, \qquad \mathbf{IF} \cdot \overline{\overline{\overline{\mathbf{IMPLY}}}} = \overline{\overline{\overline{\mathbf{IMPLY}}}}$$

or between the quantum field fixed points:

$$\overline{\overline{a^*}} \cdot a = a, \qquad \bar{a} \cdot a^* = a^*$$

Duality

In accordance with the conversion principle the implications are dual to annihilation and creation field operators, the property which carries on the commutator:

$$[\rightarrow, \leftarrow] = [a,\ a^*].$$

This is a unique case in which a replacement of operators with dual operators in the commutator is an invariant identity transformation:

$$[a,\ a^*] = [e^a,\ e^{a^*}]$$

The duality conversion from a non-Abelian algebra to a non-Abelian group translates the quantum field into fundamental logic, distinguished by the differential fixed points:

$$\frac{d\rightarrow}{da} = \frac{de^a}{da} = e^a = \rightarrow, \qquad \frac{d\leftarrow}{da^*} = \frac{de^{a^*}}{da^*} = e^{a^*} = \leftarrow.$$

Self-powers:

In view of the finitely terminating exponential, the second-quantized fixed points can be reformulated as the **exact** logarithmic fixed points of the implication operators:

$$\rightarrow \bullet \ln \rightarrow \; = \; \ln \rightarrow$$

$$\leftarrow \bullet \ln \leftarrow \; = \; \ln \leftarrow$$

The matrix self-power is another fundamental and unique fixed point, which is one of critical results of this study. An implication, raised to the power of itself, remains intact on any number of iterations:

$$\rightarrow^{\rightarrow^{\rightarrow^{\cdots}}} = \rightarrow , \qquad \leftarrow^{\leftarrow^{\leftarrow^{\cdots}}} = \leftarrow .$$

LOGICAL RADAR COORDINATES AS FIXED POINT

Radar coordinates derive their name from a device which is used to determine the position of a moving object, like a plane or a missile, by comparing the times needed for a probe signal to go to the object and back. The product of the two calculated distances can be used to define the pseudo-Euclidian coordinates as the radar coordinates:

$$\Delta s^2 = (x + vt)(x - vt) = x^2 - (vt)^2 .$$

In logic the radar coordinates emerge naturally. The relative truth-value

$$\Delta q = \bar{q} - q$$

is a fundamental fixed-point, a *secret metric*, hiding within itself the pseudo-Euclidian interval. This fact becomes clear when we consider logical radar coordinates. Probing the logical distance in the radar coordinates, we get the pseudo-Euclidian metric:

$$\Delta q = (\bar{q} + q)(\bar{q} - q) = (\bar{q})^2 - q^2$$

But

$$\bar{q} + q = 1,$$

which reduces the metric to ordinary linear distance:

$$\Delta q = \bar{q} - q,$$

and confirms the fact that the quadrature involution of the relative truth-value is the fixed-point:

$$(\bar{q})^2 - q^2 = \bar{q} - q$$

For normed logical states the pseudo-Euclidian interval is simply a logical distance. But the involution of the radar probe applies for any normed distances:

$$a + b = 1,$$

which are not necessarily probabilities. For example for a = -1 and b = 2,

$$a + b = -1 + 2 = 1,$$
$$a - b = -1 - 2 = -3,$$
$$a^2 - b^2 = 1 - 4 = -3.$$

The scalar rule of the difference of squares:

$$(a + b)(a - b) = a^2 - b^2$$

applies only to commuting matrix coordinates, and in general the secret metric do not extend to matrix dimensions. For the nilpotent annihilation and creation operators the difference is reduced to a single term.
Consider the product of implication and its inverse

$$\rightarrow \bullet \rightarrow^{-1} = I$$

or equivalently

$$e^a e^{-a} = e^{a-a} = e^0 = I$$

which in effect is the basic rule for difference of squares with one nilpotent term cancelling itself out:

$$(I + a)(I - a) = I^2 - a^2 = I$$

Likewise for the creation operator a^*.

IMPROBABILITIES

When you have eliminated the impossible, whatever remains, however improbable, must be the truth.

A. Conan Doyle

PROBABILITY OF IMPOSSIBLE AND IMPOSSIBLE PROBABILITIES

The modern treatment of probability is to view it as part of measure theory. The axioms of additivity and complementation, which are basic to

probability, coincide with the axioms of the Borel-Lebesque measure. Suppose X is a nonempty set and B is a Borel field of subsets of X,

$$\varnothing \in B, \qquad A \in B, \qquad \overline{A} \in B.$$

If A_i is a countable collection of members of B then

$$\bigcup A_i \in B$$

Now let p be any **nonnegative** function defined on B such that

$$p(X) = 1,$$

$$p\left[\bigcup_{i=1}^{\infty} A_i\right] = \sum_{i=1}^{\infty} p(A_i) \quad \text{if} \quad A_i \in B \quad \text{and} \quad A_i A_j = \varnothing.$$

Such a function p is a probability measure on X and the triple (X, B, p) is a probability space. The measurable set corresponds to an event, a member of B, and the null set to an impossible event: $p(\varnothing)=0$. Probability is a set-theoretic measure and the random variable is a measurable function.

The beautiful recipe of probability has been discussed many times. There is a measurable set of possible events, to each of which we can assign a certain **positive** number, a probability p, $0 \leq p \leq 1$, such that when all such numbers are summed for the complete set of events, the result is unity. This is common sense. Nothing happens that couldn't happen. We assume we can always describe the complete sets with nonnegative values. Mathematics, however, often defies intuition. The negative and greater-then-unity values, which appeared in our logical matrices, clash with the classical concept of probability. Simple algebra tells us that unity need not be obtained not necessarily as a sum of positive numbers. The balance of a bank account is not just the credit amount but an algebraic sum of the credit and the overdraft.

When we toss a coin, an educated guess is that the coin will end up either as a head or as a tail, and therefore there are two equally likely realizations, each with probability 1/2. We conveniently exclude the outcome of the coin falling on its edge as improbable. If the coin indeed ends up on its edge, this will force us to redefine the complete set of events of the test. Can we be sure in a finite experiment that a set of possible events is complete? If an experiment could be repeated an infinite number of times where is the guarantee that we would not record other 'improbabilities'? Suppose one blew up a piece of a stone and asked, what is the probability of the outcome being Auguste Rodin's *Thinker* statue? Although it is a highly improbable and for all practical purposes is an impossible outcome, the laws of physics do not forbid such an outcome in principle.

The second law of thermodynamics demands that the entropy of an isolated physical system will increase. But the world is more than just ever-increasing entropy. There is information, intelligent logic and consciousness. Matrix logic adds the greatest controversy to probability theory, requiring us to consider negative and greater-than-unity values. For these we introduce a new term '**imaginary** probabilities', or in short, **im**probabilities. The concept of

improbability must not be confused with the existing notion of improbable events in classical probability theory nor with quantum probability amplitudes. The thought process does not conform to the rules of classical probability because **im**probabilities show up. It cannot be described with complex probabilities either, because matrix logic may violate the quadratic rule of normalization. Because classical probability and quantum probability can neither explain nor predict consciousness, we clearly need a new different probability concept. One of the key assertions of this study is that to provide a complete description of consciousness we require **im**probability. The quantization theorem, which we proved earlier, not only explicitly introduces negative improbabilties, it also reveals that the quantization of probability is a fundamental feature of intelligent logic, without which the conscious mind may not work at all. The brain obtains reading on improbabilities and through logical superposition intelligently mixes improbabilities and probabilities to provide for a nonnegative outcome. Improbability is essential for creative thinking, providing new insight in the laws of intellect. Improbabilities in thinking operations has not been recognized because of their complicated tensor signature. But as the classical picture of the brain as a system of logical neurons gave way to matrix theory, it became clear that a theory of the thought process cannot be consistently formulated without allowing for negative and greater-than-unity values. According to this proposal to encompass consciousness in addition to **objective** probabilities we must also consider **subjective** probabilities which often differ [Ref 76]. One different way to introduce improbability is based on entropy considerations. Given the logarithmic, strictly positive, measure of entropy in information theory

$$H = - \sum p_n \log p_n ,$$

one realises that to make the expression negative, H<0, is to consider $p_n>1$. Since probability is not defined for values greater than unity, improbability is needed to explain cognitive information. Maxwell's demon is merely a fiction, but the decrease of entropy or negative entropy is an observed fact of the universe, if only on a smaller scale in comparison with the opposite process of entropy growth [Ref 9,10]. The approach which regards information as a mere record, existing outside and essentially uncoupled from the laws of physics, is no longer tenable. As was first observed by Brillouin, information is physical.

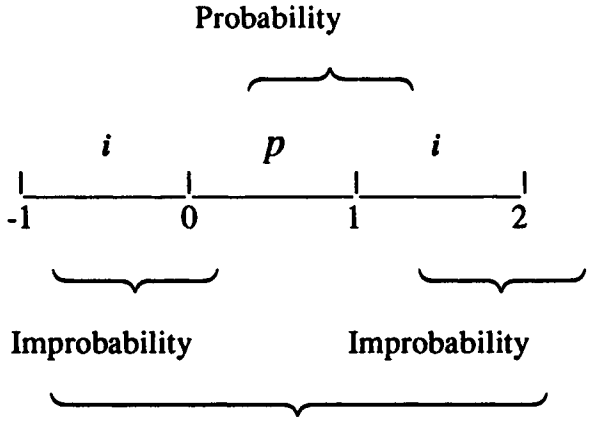
To gain better insight how consciousness can manipulate improbabilities, let us now define the concept of improbability more vigorously. While the domain of ordinary probability is the unit interval with boundaries:

$$p(x) \in [0, 1],$$

the domain of improbability $i(x)$ is its complement, that is the real axis, with the removed unit interval:

$$i(x) \notin [0, 1].$$

Without much loss of generality, we define improbability function in the two unit intervals $i(x) \in [-1, 0)$ and $i(x) \in (0, 2]$, as shown in the figure below:



More generally, the improbability function extends in both directions to infinity. In contrast to ordinary probability, which is strictly confined to the interval [0, 1], we consider the triplicated unit interval, in two sections of which the laws of classical probability will be violated. The complement function of probability and improbability behaves differently. While the complement of a probability is again in the probability interval:

$$p \in [0, 1] \qquad \text{and} \qquad \bar{p} \in [0, 1],$$

the complement of improbability in a given interval is found in the adjacent interval, 'jumping over' the unit interval [0, 1]:

$$i \in [-1, 0) \qquad \text{but} \qquad \bar{i} \in (1, 2].$$

PROBABILITY CALCULATED AND PROBABILITY MEASURED

Probability theory is in essence a classical theory, developed before we knew about quantum, Schrödinger's cats and Bell's nonlocality theorem. Classical physics forbids an object to be in two different places at the same time. But quantum topology defies common sense. In the two-slit experiment a quantum particle traverses through both holes at once, very much as in the cognitive 'two-slit experiment' described earlier in which we feel two fingers no matter that our eyes tell us that there is only one. To avoid paradoxes and to deal with this sort of counterintuitive problems, quantum theorists resort to the formalism of complex probability amplitudes which accommodates coherent superposition and quantum entanglement. The complex probabilities of quantum mechanics are not necessarily positive; they mix descriptions of reality, not reality itself. The Wigner distribution function W(x,p) can be negative. When integrated over one canonical variable, it yields the probability distribution for the other:

$$\int_{-\infty}^{\infty} W(x,p)dp = |\psi(x)|^2 \qquad \int_{-\infty}^{\infty} W(x,p)dx = |\psi(p)|^2$$

Nothing else in quantum mechanics comes closer to describing points in quantum phase space. Whenever W(x,p) is a mixture of localized wavepackets, it can be regarded as a classical probability distribution. However, when the state is genuinely quantum, W(x,p) will have an alternating sign. Since negative values have no place in probability theory, this fact alone makes it impossible to regard the Wigner function as a probability distribution. Only when the system decoheres and negative contributions to the interference term cancel out, will W(x,p) retain just two peaks corresponding to the two classical alternatives. The function W(x,p) retains many of the features of a probability distribution, except that it can be negative in some regions of phase space. While a classical particle occupies a single point in position-momentum phase space, for a quantum particle the Heisenberg principle introduces an area of uncertainty no smaller than $\hbar/2$. A probability distribution in quantum phase space must incorporate the fact that a localized position leads to a delocalized momentum and vice versa. Although the function W(x,p) has features not found in classical distributions, it leads to a natural link between quantum and classical phase spaces. As the measurement resolution is degraded away from the quantum limit so that quantum uncertainty plays no role, negative values are washed out, and the convolved distribution approaches the usual classical probability distribution. This analysis indicates that we must carefully distinguish between the probability **calculated** and probability **measured**. Wigner's negative probability is just theoretical, not accessible to actual observation, but in matrix theory improbabilities are not merely abstractions but actual quantities of consciousness. Wigner gave consciousness the last word in collapsing the quantum superposition but the problem of consciousness is not so much in collapsing the state vector, as in compounding the classical states into coherent superpositions by making use of improbabilities [Ref 93]. If improbabilities can actually affect the states of the brain, consciousness becomes susceptible to physical analysis. The neurodynamics of the brain obeys classical dissipative equations, and the relevant observables are macroscopic. But much as one wishes to believe that classical states have predictable consequences, if there is a lesson to be learned from matrix logic, it is that nonclassical effects can emerge in a perfectly classical phase space, such as the improbabilities present in the phase space of the Liar.

Improbabilities are not simply theoretical calculations: the 'improbable' consciousness does actually occur. The improbability highlights the inherent anticorrelation of logical true and false. When we have a superposed or entangled logical system, which cannot be expressed as a product, the improbabilities may emerge, signifying nonlocality and inseparability. Then the ordinary laws of probability, such as the sum and product of probabilities, would not apply universally. If we abandon the dogma of a strictly positive measure of probability, counterintuitive as that might be, improbabilities

correctly resolve the logical paradox. Although a clearly nonclassical concept, improbability nonetheless is fully applicable in classical situations as long as global normalization holds. This was not immediately clear when negative probability was first deduced in matrix logic. Probability concerns the prediction of the outcome of some measurement. In physics, when a prediction is made about the future states of a system, it has no bearing on the current state. But when a cognitive system anticipates its future behaviour, the prediction itself becomes an integral part of and interferes with the current data. Cause and effect become inseparably looped and can no longer be treated in a canonical linear fashion. In conditions of self-reference and closed topology the contribution of improbability with its nonnormalized and negative values, comes to be of paramount significance. It is here that the cognitive signature of negative probability can be clearly seen. This suggests a strategy for classical resolution of coherent superpositions in the Liar paradox. The physical fact and the demands of logic clash in the paradox. While we can cognize just one of the logical alternatives, our consciousness explicitly tells us that both are simultaneously present in the superposition. To be able to address this issue meaningfully we consider that negative and nonnormalized improbabilities are real for consciousness, in the same manner as nonnegative probability is real in physics.

Consider a statistical approach to the Liar. Instead of trying to solve the paradox by finite means, suppose we repeat the evaluation infinite number of times, in order to determine the probability of the Liar being true or false. The outcome of such a test will be that for 50% the Liar tells the truth, and for 50% tells the lie. If a logical thinking system is unable to keep track of its action and 'forgets' the initial conditions, the paradox will not show up. But as soon as consciousness is switched on, the improbable **'true+false'** enters the picture. In a probabilistic approach to the Liar we can delegate to a device the task of registering the outcome of the evaluation. The device will record the two equiprobable realizations which we can conveniently represent by the normed probability vector

$$\begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$$

But if the 'device' is the brain of a conscious observer, capable of self-reference, then the outcome of logical evaluation is the coherent superposition of the true state $|1\rangle$ and false state $|0\rangle$:

$$|s_+\rangle = |1\rangle + |0\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

From this we can deduce that a particular state of the Liar, true or false, is obtained by subtracting a complementary state from the superposed state

$$|\bar{x}\rangle = |s_+\rangle - |x\rangle.$$

To reintroduce the initial probability vector of equiprobable realizations, while retaining the above equality, one requires improbabilities 2 and -1:

$$|\bar{x}> \;=\; 2\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix} + (-1)\,|x>$$

where 2 and -1 are normed: -1 + 2 = 1, recovering the statistical scheme of the paradox. This solution of the Liar reveals the underlying dynamics of the paradox of consciousness: the probabilities are converted into improbabilities, one of which is negative and the other is greater than 1. When these improbabilities join, the 'defect' cancels out and we get an ordinary probability state. This suggests a natural generalization. 'Tossing' the Liar paradox we assumed two outcomes: the Liar tells the truth (**head**) or a lie (**tail**), with equal probability +1/2. Since these are logically 'impossible' events (**edge**): if the Liar tells the truth, he lies and if the Liar lies, he tells the truth, we can assign to each of them the negative improbability -1/2. If a coin (the Liar) falls into a **grey** hole, we can never determine whether it lands tails or heads. However, in ordinary circumstances we have the implication tautologies: the liar lies, and truthful person tells the truth, which by definition are both certainties with probability 1. In terms of the formalism of the logical density matrix the overall situation is expressed as follows

$$\Omega \;=\; \begin{pmatrix} 1 & -\frac{1}{2}\\ -\frac{1}{2} & 1\end{pmatrix}$$

The four alternative outcomes represent the complete sample space of events whose global trace is unity

$$\sum\begin{pmatrix} 1 & -\frac{1}{2}\\ -\frac{1}{2} & 1\end{pmatrix} \;=\; 1$$

We observe that the logical density matrix of the Liar is an entanglement and not factorable, as opposed to linear discourse where a density matrix can be written as the outer product

$$\Omega = |q><q|.$$

Another possibility is to choose for the Liar the densities

$$\Omega_{\bar{x}} \;=\; \begin{pmatrix}1 & -1\\ 0 & 1\end{pmatrix} \quad \text{or} \quad \Omega_{x} \;=\; \begin{pmatrix}1 & 0\\ -1 & 1\end{pmatrix},$$

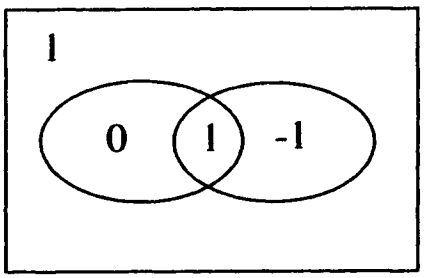
which are apparently the inverses of implications, respectively:

$$\Omega_{\bar{x}} \;=\; \rightarrow^{-1} \quad \text{or} \quad \Omega_{x} \;=\; \leftarrow^{-1}$$

satisfying, in spite of the presence of the negative matrix element -1, the global normalization

$$\sum(\Omega) = 1,$$

The corresponding set diagram is shown below:



Logical density retains the global normalization, independently of whether the states are ordinary probabilities or improbabilities. For probability the determinant of the density matrix $|p><q|$ is singular and the global trace is universally a tautology:

$$\det \Omega \equiv 0, \qquad \sum(\Omega) \equiv 1.$$

But the same theorems also apply to the improbability density matrix $|i><j|$ which contains negative and greater-than-unity values, and to the general "mixed" matrix $|p><i|$. It follows that a logical density matrix besides the strictly positive signature

$$\begin{pmatrix} + & + \\ + & + \end{pmatrix}$$

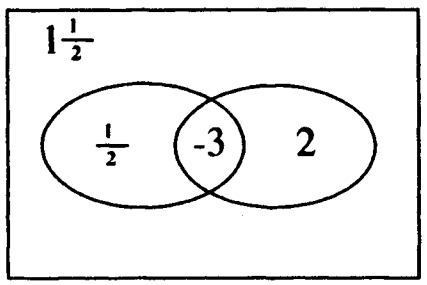
could have six other possible impregnations where the negative values appear:

$$\begin{pmatrix} - & - \\ + & + \end{pmatrix}, \begin{pmatrix} + & - \\ - & + \end{pmatrix}, \begin{pmatrix} - & + \\ + & - \end{pmatrix}, \begin{pmatrix} + & + \\ - & - \end{pmatrix}, \begin{pmatrix} - & + \\ - & + \end{pmatrix}, \begin{pmatrix} + & - \\ + & - \end{pmatrix}$$

Here we have the even parity: negative and positive values appear in pairs. But nothing prevents us from generalizing this situation to include the odd parity, which also preserves the global normalization. For example, consider

$$\Omega = \begin{pmatrix} 1\frac{1}{2} & 2 \\ \frac{1}{2} & -3 \end{pmatrix}$$

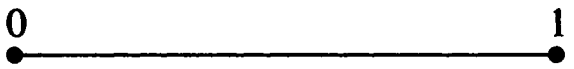
with $\sum(\Omega) = 1$, which is not factorable into an outer product entanglement. The corresponding set diagram is as follows:



When the classical states $|0\rangle$ and $|1\rangle$ form an entanglement, the eigenvalues of the corresponding density matrix $\Omega = |x\rangle\langle x|$ are probabilities found in the unit interval: $0 \leq \lambda \leq 1$. But when the superposed logical states are entangled, then the eigenvalues of the density matrices $\Omega = |s_{\pm}\rangle\langle s_{\pm}|$ are 0 and 2, and cannot be treated as probabilities.

TENSOR PROBABILITY

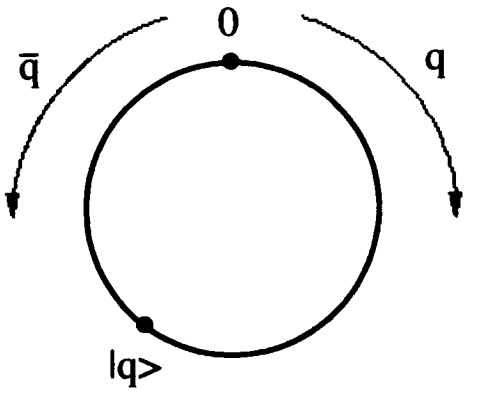
The formalism of the logical density matrix shows that probability and improbability can be joined computationally into a single concept of a tensor probability which naturally overcomes the limitations imposed by the nonnegative and scalar concept of probability. Tensor probabilities expand scalar probability to **n**-dimensional spaces. In physics tensors find various applications. What makes them particularly useful is that the tensor calculus permits us to formulate the laws of physics in covariant form, independent of the frame of reference. By the same token the true facts of logic are covariant and should not depend upon a particular thinking system in which they are obtained. Finding solutions is greatly simplified if probability is treated as tensor instead of a scalar. It is meaningful to think about an elementary tensor probability as a point on a circle instead of line. Given the unit set [0, 1]



we identify its left boundary point 0 with the right boundary point 1, which entails topological closure of the space, taking the shape of a circle. The points $|q\rangle$ of such a closed topology space Ω we will call the tensor probabilities. As opposed to ordinary scalar probability, an elementary tensor probability is determined by **two** values instead of **one**. For

$$|q\rangle \in \Omega, \qquad |q\rangle = \begin{pmatrix} \bar{q} \\ q \end{pmatrix}$$

the lower component q is the clockwise rotation from the origin 0 to the point q, and the upper component $(1-q)$ is the counterclockwise rotation to the same point:



When the components are rotated in the opposite direction they acquire the negative sign. Clearly the two values are correlated and can be normalized as ordinary probabilities or improbabilities,

$$q + \bar{q} = 1.$$

In the first instance one value uniquely determines the value of the other, and is hence redundant, but situations exist where the two values are not correlated as complementary and then neither is dispensable. There are nine tensor probabilities associated with the origin of the space:

$$\binom{0}{0}, \binom{0}{1}, \binom{1}{0}, \binom{0}{-1}, \binom{-1}{0}, \binom{1}{1}, \binom{-1}{1}, \binom{1}{-1}, \binom{-1}{-1}$$

of which only two, the second and third, are correlated classically, while for the others the law of probability normalization cannot be satisfied.

Given elementary tensor probabilities we wish to be able to construct compound probabilities of various kinds. In probability theory, when one obtains a compound probability from elementary ones, the compound remains normed. We naturally demand the same property of a compound tensor probability. With the aid of the tensor probability we can express the classical laws for the addition and the product of probabilities

$$p(A \wedge B) = p(A)\, p(B), \qquad p(A \vee B) = p(A) + p(B) - p(A \vee B)$$

uniformly:

$$|p(A \wedge B)\rangle = c\,|p(A)\rangle + \bar{c}|p(B)\rangle, \qquad |p(A \vee B)\rangle = \bar{c}|p(A)\rangle + c\,|p(B)\rangle$$

where the improbabilities c and $\bar{c}$ are the components of the normed tensor probability $|c\rangle$. These are simply swapped with the changeover from the addition law to the product law. For example, consider a normed logical probability 0.67 which is the result of the fuzzy disjunction

$$\langle 0.5|\mathbf{OR}|0.33\rangle = 0.5 + 0.33 - 0.16 = 0.67.$$

The tensor probability which delivers the same outcome is $|c\rangle = |{-1}\rangle$:

$$\begin{pmatrix} 0.33 \\ 0.67 \end{pmatrix} = 2\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} + (-1)\begin{pmatrix} 0.67 \\ 0.33 \end{pmatrix}$$

We now replace **OR** with **AND** and consider the fuzzy conjunction

$$<0.5|\mathbf{AND}|0.33> = 0.5 \bullet 0.33 = 0.16.$$

The corresponding tensor composition is calculated to be

$$\begin{pmatrix} 0.84 \\ 0.16 \end{pmatrix} = (-1)\begin{pmatrix} 0.5 \\ 0.5 \end{pmatrix} + 2\begin{pmatrix} 0.67 \\ 0.33 \end{pmatrix}$$

which is obtained from the original one simply by reshuffling the factors:

$$|-1> \Leftrightarrow \ |2>.$$

Finally we must reflect on the fact that the algebra of tensor probabilities is non-Abelian. In contrast to the ordinary probabilities q and p whose product is commuting $qp = pq$, tensor probabilities are noncommuting, and exist as the inner product $<q|p>$ which is a **0**-rank scalar tensor and as the outer product $|p><q|$ which is a **2**-rank tensor or the logical density matrix

$$\Omega = |p><q| = \begin{pmatrix} \bar{p}\bar{q} & \bar{p}q \\ p\bar{q} & pq \end{pmatrix}$$

In quantum mechanics probability has become an essential but abstract element of physical description. The tensor probability and the use of negative and greater-than-unity values in probability theory induce a shift from abstract to actual. In order to function properly the brain must extrapolate, estimate and anticipate the forthcoming events, measuring probabilities in natural way. In our mind probability becomes an actually measured quantity, with all consequences that this entails.

IMPROBABLE DISPERSIONS

The uncertainty relation in quantum mechanics is valid only when based on a positive-definite inner product, which may not be the case in relativistic cases with highly distorted topologies. Moreover, the usual uncertainty relations are already problematic in conventional quantum mechanics: the standard deviation of a perfectly reasonable probability distribution may be infinite. The negative entropy which shows up in inductive cognitive processes is a clear indication that the logical probability distribution is not a *bona fide* probability distribution, containing negative values not found in classical phase-space distributions. The 'improbable' events are hidden and nonmeasurable; yet they are actual cognitive events which the formalism of matrix theory must accommodate consistently.

Quantum-mechanical uncertainty is inherently tied up to the measurement of standard deviations ΔA, determined by the dispersion or variance of the corresponding Hermitian operators

$$\Delta A = \sqrt{<(A - <A>)^2>}$$

The dispersion of an observable can also be calculated as the difference between the average of the squared operator and the squared average:

$$D(A) = <(\Delta A)^2> = <(A - <A>)^2> = <A^2> - <A>^2$$

Since $D(A)$ is the average of a squared quantity $<(A - <A>)^2>$, one concludes that $D(A)$ is necessarily nonnegative, which entails

$$<A^2> \geq <A>^2.$$

In this analysis we assume that observables are represented by Hermitian operators and that ΔA is a noncomplex quantity. Even though quantum inseparability, nonlocality and entanglement, and most importantly the effects of cognitive brain may suggest otherwise, the realist physicist is consistently interested only in quantities which s/he is measuring in a laboratory or could in principle measure.

However, should the uncertainty be a complex quantity $i\Delta A$, which is equivalent to operator A being non-Hermitian, the dispersion would become negative, taking us away from the domain of classical probability to the domain of imaginary probability or improbability. Taking this nontrivial approach, we transform the dispersion into a improbable dispersion that is negative in value

$$D(A) = <(i\Delta A)^2> = <A^2> - <A>^2 \leq 0$$

Specifically with regard to the difference $<A^2> - <A>^2$ this entails that the term $<A^2>$ must be either negative or zero $<A^2> = 0$, and never greater then $<A>^2$.

The dispersion of non-Hermitian cognitive states is not detectable by ordinary means. The key non-Hermitian observables of thinking systems, associated with the operators **M**, **ON**, **OFF**, ▲, ▼, disperse negatively, indicating that their standard deviations are complex-valued:

$$D(\mathbf{M}) = <q|\mathbf{M}^2|q> - <q|\mathbf{M}|q>^2 = 0 - (\bar{q} - q)^2 < 0$$

$$D(▼) = <q|▼^2|q> - <q|▼|q>^2 = - <q|q> - 0 < 0$$

$$D(\mathbf{ON}) = <q|\backslash^2|q> - <q|\backslash|q>^2 = 0 - (q\,\bar{q})^2 < 0$$

We establish the following fact of major importance: dispersion of SELF is negative. This is a general way of saying that the operators emerging from the square root polarization of the logical vacuum are non-Hermitian,

$$D(\sqrt{0}) < 0.$$

Likewise non-Hermitian Hadamardians, which are matrix complex numbers, disperses negatively:

$$D(.H) < 0, \quad D(H^{\bullet}) < 0.$$

Proof

$$D(.H) = \langle q|.H^2|q\rangle - \langle q|.H|q\rangle^2 = 0 - (\langle q|q\rangle + \langle q|\blacktriangle|q\rangle)^2 = - \langle q|q\rangle^2.$$

Footnotes:

(a)

$$D(.H)\, D(\blacktriangle) = \langle q|q\rangle^3.$$

(b)

The non-Hermiticity of a cognitive operator is not a sufficient condition for improbable dispersion. **TRUE** and **FALSE** are non-Hermitian but

$$D(\mathbf{TRUE}) > 0, \quad D(\mathbf{FALSE}) > 0$$

(c)

The difference in dispersions of the Hermitian spin operator σ and non-Hermitian logical momentum operator is **<IS>**:

$$D(\mathbf{M}) = - (\bar{q} - q)^2 < 0$$

$$D(\sigma) = \langle q|\sigma^2|q\rangle - \langle q|\sigma|q\rangle^2 = \langle q|q\rangle - (\bar{q} - q)^2 > 0$$

$$D(\mathbf{M}) < D(\sigma)$$

$$D(\sigma) - D(\mathbf{M}) = \langle \mathbf{IS} \rangle$$

Theorem

The dispersion of the identity is quantum-logical entropy:

$$D(\mathbf{IS}) = \mathrm{Tr}\Omega \ln \Omega$$

where Ω is a logical density matrix.

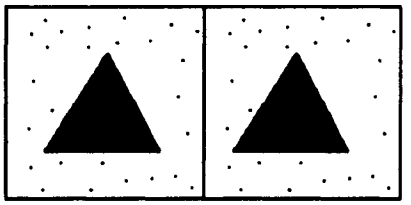
Proof

$$\begin{aligned} \mathrm{Tr}\Omega \ln \Omega &= \mathrm{Tr}\, \Omega(\mathbf{I} - \Omega) = \mathrm{Tr}\, (\Omega - \Omega^2) = \mathrm{Tr}\, (\Omega - \langle q|q\rangle\, \Omega) \\ &= \mathrm{Tr}\, \Omega - \mathrm{Tr}\langle q|q\rangle\, \Omega = \mathrm{Tr}\, \Omega - \langle q|q\rangle \mathrm{Tr}\, \Omega \\ &= (1 - \langle q|q\rangle)\, \mathrm{Tr}\, \Omega = (1 - \langle q|q\rangle)\langle q|q\rangle \\ &= \langle q|\mathbf{IS}^2|q\rangle - \langle q|\mathbf{IS}|q\rangle^2 \\ &= D(\mathbf{IS}). \end{aligned}$$

IMAGINARY CERTAINTIES

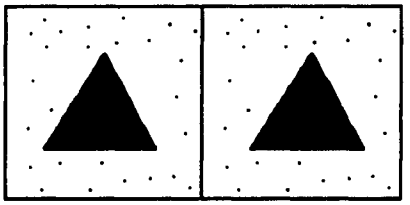
Consciousness is essentially and fundamentally non-Hermitian in nature. Negative dispersions manifest themselves in the most basic cognitive operations. Our ability to see and to actualize abstract concepts, to 'see' in our

minds, is based on the dimensional coherent superposition. This can be observed in visual perception, which can be treated as spinor superposition, producing a cognitive effect. Consider the effect of stereographic fusion, in which imaginary cognitive effect overrides physical fact. In the figure below the two triangles, left and right, correspond to left and right spins:



$|\leftarrow> - |\rightarrow>$

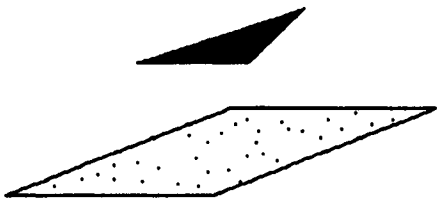
(A)



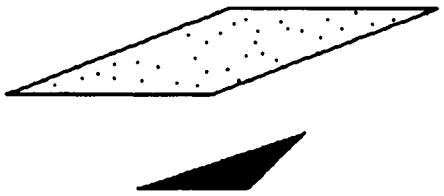
$|\leftarrow> + |\rightarrow>$

(B)

When the left triangle and the right triangle are seen by the left eye and by the right eye separately and then fused by your brain, depending on the form of coherent superposition, the "cognitive" triangle jumps up (A) or falls behind (B) the plane, as shown in the figure below. You can obtain this effect of a 'cognitive spin' and superpose the two perceptual inputs without equipment but by voluntary control by looking at a distant point, slowly 'inserting' the picture between your eyes and the point.



(A)



(B)

Although the actual physical picture is flat and two-dimensional, your consciousness will clearly "see" a triangle present in 3-D, just as the spin-up and spin-down are determined by the superposition of left and right spins:

$$|\mathbf{up}> = \frac{1}{\sqrt{2}}(|\mathbf{left}> - |\mathbf{right}>)$$

$$|\mathbf{down}> = \frac{1}{\sqrt{2}}(|\mathbf{left}> + |\mathbf{right}>).$$

Cognitive objects, at least cognitive objects in perception, behave congruently to spin. We conjecture the existence of a cognitive spin, generated through the superposition of a pair of orthogonal components of increasing complexity. For higher-order cognitive spins one freely rotates between the Boolean basis

$$|S_+> = |0> + |1>, \qquad |S_-> = |0> - |1>$$

and the superposed basis

$$|1> = \frac{1}{2}(|S_+> - |S_->), \qquad |0> = \frac{1}{2}(|S_+> + |S_->)$$

There are many other examples of imaginary cognitive certainties, where a cognitive effect overrides a physical fact. This was the case with the 'false' boundaries in the non-Hermitian pentagon which we considered earlier. Particularly significant is the topological "**2 in 1**" cognitive effect, in which crossing the index and middle fingers and placing another finger between them, one paradoxically perceives two fingers instead of the actual one, no matter that the eyes are telling otherwise. The non-Hermitian effects in perceptual and higher-order cognitive after-images are commonplace, although we are not always aware of their presence.

The scalar **true** and **false** are ordinary probabilities, and hence Hermitian, which distinguishes them from the matrix **TRUE** and **FALSE** which are not:

$$\perp \neq \perp^+, \qquad \mathsf{T} \neq \mathsf{T}^+.$$

The dispersions of true $D(\perp)$ and false $D(\mathsf{T})$ are equal:

$$D(\perp) = q - q^2 = q\,\bar{q},$$

$$D(\mathsf{T}) = \bar{q} - (\bar{q})^2 = \bar{q}(1 - \bar{q}) = \bar{q}\,q,$$

which directly leads to the pseudo-Euclidian *secret* metric:

$$D(\mathsf{T}) = D(\perp)$$

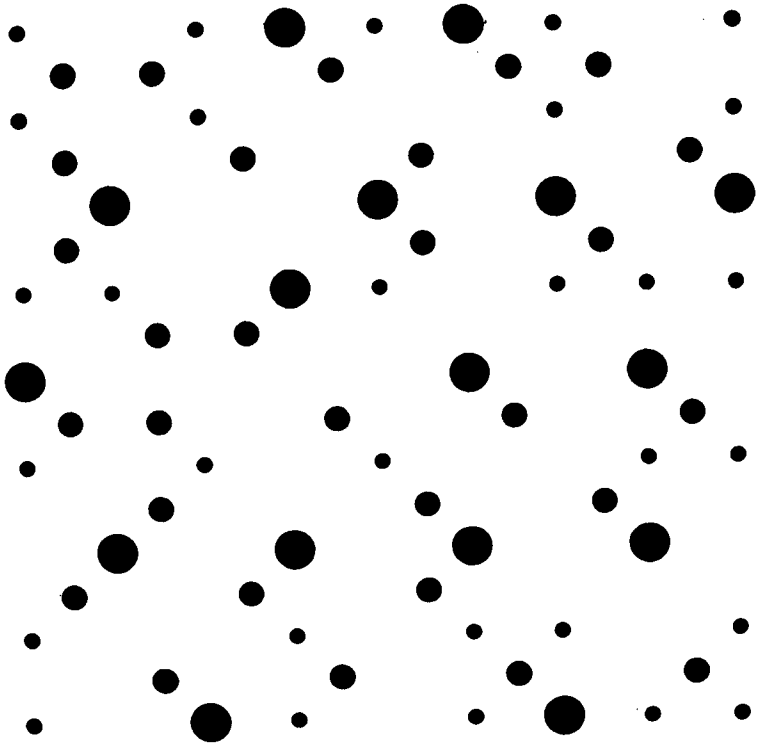
$$q - q^2 = \bar{q} - (\bar{q})^2$$

$$q - \bar{q} = q^2 - (\bar{q})^2$$

$$\bar{q} - q = (\bar{q})^2 - q^2$$

Significantly, the dispersions $D(\perp)$ and $D(\mathsf{T})$ yield the same value as the averages of the quantum annihilation and creation operators, which are also non-Hermitian:

$$D(\mathsf{T}) = D(\perp) = <a> = <a^*>.$$



Bridget Riley. *Fragment no. 6*, 1965, plexiglass print.
The picture seems rather a monotonous array of black circles of various size. But as we continue to look at the picture, its simple structure begins to dissolve before our eyes. A sort of dance takes place, the black circles floating about at a leisurely pace with the white after-images leaping and flashing among them. It is not until these after-images, large and small, make their appearance that the picture come to life. This is an artistic example of interactive consciousness where the brain contributes non-Hermitian elements apparently absent in the actual picture.The spectator is an active participant in the work of art, which exists neither in the picture nor in the brain of the observer but in their interaction.

It is important to keep in mind that there are two interlinked aspects to Hermiticity. Quantum-mechanical observables are represented by Hermitian operators because the eigenvalues of such operators are real numbers, which allows us to treat as real the results of physical measurements. An eigenvalue

must not be a complex number. However, a non-Hermitian operator does not necessarily carry complex eigenvalues. This is the case with **TRUE** and **FALSE** which are non-Hermitian operators, but whose eigenvalues are real. But alongside the Hermiticity of eigenvalues there is another fundamental limitation on observation. Even if they are Hermitian, two quantities cannot be measured with arbitrary precision if their respective operators do not commute. This is essence of the uncertainty principle, a fundamental restriction of the quantum laws. The noncommuting observables, like the position x and momentum p of a particle, do not share mutual eigenstates, and cannot be known simultaneously: $\Delta x \Delta p \geq \hbar/2$. Whenever a particle resides in the eigenstate, we can measure the exact value of an observable. This implies that if in a simultaneous measurement of A and B we obtain the exact values (eigenvalues), then the state in question is a mutual eigenstate for both A and B. Hence A and B must commute, [A, B] = 0. And conversely, if the Hermitian operators of two observables commute, we can measure them simultaneously with arbitrary precision. But there could be several quantum states $|\psi\rangle$ in which $\langle\psi|[A,B]|\psi\rangle = 0$. We often disregard notable exceptions. For example, the components L_x, L_y and L_z of the angular momentum do not commute. At the same time all three components commute with the squared total momentum L^2. Then for the state in which $L_z|\psi\rangle = 0$, the quantities $L_x|\psi\rangle$ and $L_y|\psi\rangle$ are nonzero and can be simultaneously measured, in spite of the noncommutation of corresponding operators, $[L_x, L_y] \neq 0$.

What seems to be unusual in quantum mechanics becomes prevalent in logic. The **TRUE** and **FALSE** operators are non-Hermitian and do not commute

$$[\,\mathsf{T}, \perp\,] \neq 0,$$

but characterising two complementary aspects of a logical state, can be known simultaneously:

$$D(\mathsf{T})D(\perp) = \langle(\Delta\mathsf{T})^2\rangle\,\langle(\Delta\perp)^2\rangle = (\langle\mathsf{T}^2\rangle - \langle\mathsf{T}\rangle^2)(\langle\perp^2\rangle - \langle\perp\rangle^2) =$$

$$= (\bar{q} - (\bar{q})^2)(q - q^2) = \bar{q}\,q\,q\,\bar{q} = (q\,\bar{q})^2$$

Consequently,

$$D(\mathbf{TRUE})\,D(\mathbf{FALSE}) \leq \frac{1}{4}$$

which is a counterfactual to the uncertainty relation in quantum mechanics:

$$D(\mathrm{x})\,D(\mathrm{p}) \geq \frac{1}{4}.$$

The **TRUE** and **FALSE** are solved to the following eigenstates,

T: eigenstates $|0\rangle$ and $|s_-\rangle$, eigenvalues $\lambda_{1,2} = 1,\ 0$

$\perp$: eigenstates $|1\rangle$ and $|s_-\rangle$, eigenvalues $\lambda_{1,2} = 1,\ 0$

Hence, they share the superposed eigenstate $|s_-\rangle$, in which they occur simultaneously.

According to the standard dispersion theorem

$$D(\mathbf{M})\,D(\perp) \geq \frac{1}{2}|\langle[\,\mathbf{M},\perp\,]\rangle|^2 \quad \text{and} \quad D(\mathbf{M})\,D(\mathbf{T}) \geq \frac{1}{2}|\langle[\,\mathbf{M},\mathbf{T}\,]\rangle|^2.$$

Since

$$[\,\mathbf{M},\perp\,] = [\,\mathbf{M},\mathbf{T}\,] = \mathbf{M}$$

and

$$\langle \mathbf{M} \rangle^2 = \langle \mathbf{H} \rangle$$

which leads to

$$D(\mathbf{M})\,D(\perp) \geq \frac{\langle \mathbf{H} \rangle}{2}, \qquad D(\mathbf{M})\,D(\mathbf{T}) \geq \frac{\langle \mathbf{H} \rangle}{2},$$

where $\langle \mathbf{H} \rangle = (\bar{q} - q)^2$ is logical energy.

Since $D(\mathbf{M}) \leq 0$ is an imaginary dispersion, while $\langle \mathbf{H} \rangle / 2 \geq 0$, the inequality can hold only in the undecided state $|1/2\rangle$ or in the states of strict coherent superposition.

NEGATIVE PROBABILITY AND VACUUM

The understanding of the logical brain can be improved with a better understanding of vacuum. Consciousness is highly improbable. The fact that conscious systems, like ourselves, exist, allowing thinking to pose questions about thinking, requires a revision of the concept of probability and its extension in the the realm of improbabilities. In the physical framework improbability can be connected with the physical vacuum. In the modern theory of elementary particles a vacuum is not simply empty space but in fact a richly structured medium, governed by the symmetries which we are only beginning to understand. When Dirac derived the relativistic equation for the electron, the strange negative energy solutions presented themselves unexpectedly. Dirac proposed that in empty space, the negative energy levels are occupied. Excitations above these levels makes 'holes', which are positively charged antiparticles. The concept of improbability opens up the possibility of associating negative probability with the negative energy level of the vacuum. We exist in the world, filled primarily with particles, and not in the antiworld, filled with antiparticles. And just as antiparticles exist only momentarily and are rapidly annihilated, consciousness too is rare in the universe.

Can improbability be embedded in the quantum nature of empty space? Can we deduce the macroscopic laws of logic from the microphysical symmetries of the vacuum? Comparing quantum uncertainty and logical certainty

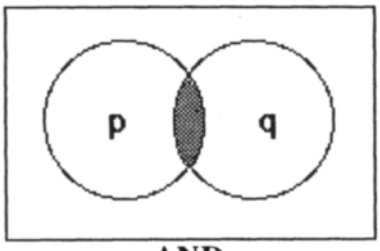
$$\Delta x \Delta p \geq \frac{1}{4} \qquad \text{and} \qquad \Delta T \Delta \perp \leq \frac{1}{4},$$

we see that there is the common minimax point 1/4. At this point a logical system must be in the undecided, equally true and false state $|1/2\rangle$, which is a fuzzy coherent superposition. To activate a productive thought, and shift the

logical evaluation away from the equilibrium with maximal entropy towards definite truth or definite falsity the superposition must be reduced to a Boolean limit. It is compelling to explain this reduction as the result of breaking the symmetry of the vacuum. Understanding how the symmetry of the vacuum is broken one determines the vacuum of general theory. Since we already know how to treat the dimensional structure of the logical theory, we are able to formulate such a vacuum explicitly.

CHARGED SETS

Treating improbability and probability on an equal footing allows one to inject a physical element into traditional set theory. With each set we now will associate a corresponding charge: positive, negative or zero, and, in view of a natural similarity, the elements of such **charged** sets we will call **pions**, the nuclear particles which also exist in the three different charged states. Normally we diagram probability space with sets which are also 'charged' but only with nonnegative pions. For example consider the diagrams for several Boolean functions:



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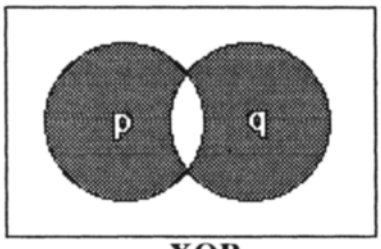
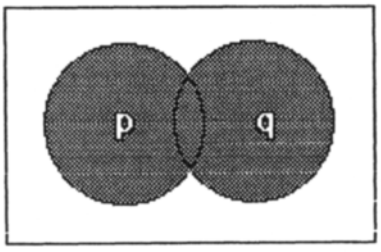
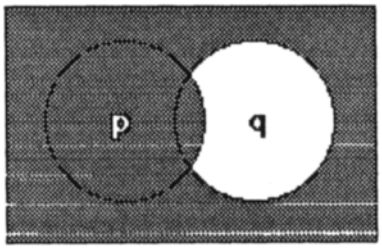


XOR

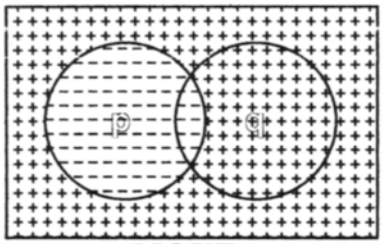


OR

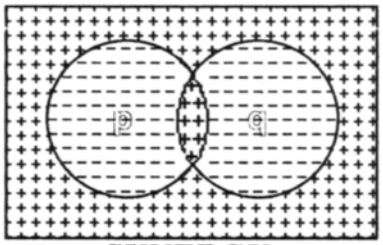


IF

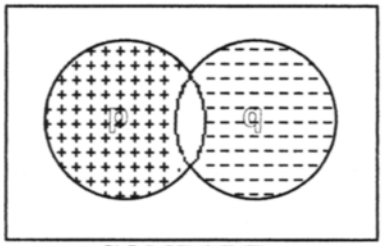
These diagrams involve only positive and neutral charges. If we add also the negative pion to the system, we obtain charged sets which can account both for probabilities and improbabilities. Below we give several examples of such charged sets.



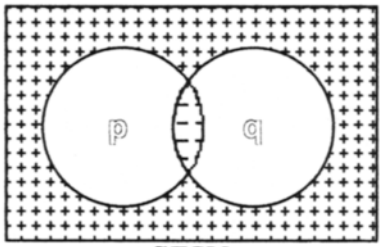
MOVE



SYNERGY



COMPARE



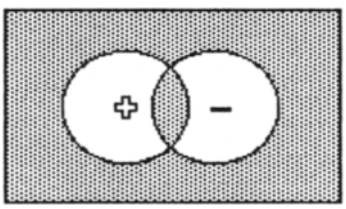
SPIN

The diagrams which we have presented give us a qualitative description. In computational terms, with every measurable set **L** we associate the integral logical charge $\mu(L)$, calculated as follows:

$$\mu(L) = \int_0^1 <q|L|q> dq.$$

Of course, the vacuum is the empty set with zero charge $\mu(NONE) = 0$, but the zero charge also exists for the nonempty charged sets, whenever positive and negative charges cancel out. Such cancellation distinguishes the fundamental charged sets, $\mu(COMPARE)$, $\mu(SPIN)$ and $\mu(MOVE)$.

Taking the union sum of the charged sets we obtain new sets in which probability and improbability impinge on each other and may cancel out. The union of all logical probability sets is **8 ONE**. The union of all probability and improbability sets is **NONE**. We find an actual realization of the charged sets in semiconductors, where holes and electrons represent the charges literally:



When the electric currents propagate through a semiconductor the comparison operation can be carried out.

THINKING SYSTEMS

Imagination is more important than knowledge

Albert Einstein

NON-HERMITIAN REALITY. FROM QUANTUM PHYSICS TO INFOPHYSICS

In recent years there has been an explosive growth in understanding of information in physical terms. More and more we view information as something real, as real as space, energy and matter. This is especially relevant for cognitive studies: fields which we previously thought developing separately are in fact parts of a general theory. This is symptomatic of what is now happening to the theory of measurement. It is known in quantum physics that simultaneous observation of two physical quantities is possible if their corresponding operators commute. For example one can measure simultaneously the momentum and the energy of a particle because

$$[\mathbf{p}, H] = 0.$$

Consider a general physical system, called **WORLD**, described by the Hamiltonian operator H and the density matrix $\rho(q_1, q_2, q_3,...)$, associated with an infinite but countable set of dynamical variables q_i, represented by the corresponding set of Hermitian operators

$$Q_1, Q_2, Q_3, \ldots$$

For a given density matrix ρ, the expected value of an observable, obtained through measurement, is computed by the tracing

$$\langle Q_i \rangle = \mathrm{Tr}\,(\rho Q_i).$$

The Hamiltonian H is Hermitian too, hence is a physical observable that gives a reading on the average energy of the **WORLD**

$$\langle E \rangle = \mathrm{Tr}\,(\rho H).$$

Those observables Q_i and Q_j, which can be measured jointly, commute

$$[Q_i, Q_j] = 0,$$

and do not commute otherwise

$$[Q_i, Q_j] \neq 0.$$

This of course imposes a fundamental limit on simultaneous measurements but not on the measurement as such. Within the constraints of the Heisenberg uncertainty principle, any dynamical variable, represented by a Hermitian operator, can be observed in principle and made quantitatively known, even though the actual measurement may be difficult to obtain in a finite period of time or to procure with a given technology.

We now take a radical step and postulate the existence of a new nontrivial dynamical variable, called **INFO.** This extravagant decision to add to the **WORLD** a new dynamical variable, without disturbing the structure of physical law, can be legitimate if and only if **INFO**

(a) is represented by a non-Hermitian operator

$$\mathrm{INFO}^{+} \neq \mathrm{INFO}.$$

(b) commutes neither with the Hamiltonian H nor with *any* other **Hermitian** observables of the **WORLD**, for $\forall\, Q_i$:

$$[\mathrm{INFO}, Q_i] \neq 0.$$

The properties of non-commutation and non-Hermiticity which we are assigning to **INFO** express the fact that the laws of physics set a fundamental limitation on the observation of cognitive states, which places cognitive variables outside the domain of standard physical measurement. To put it in very general terms: physics and the mind do not commute:

$$[\ \mathbf{INFO, PHYSICS}\ ] = \mathbf{INFO \cdot PHYSICS - PHYSICS \cdot INFO} \neq \mathbf{0}$$

In accordance with this factoid there exists no measurement procedure, no physical device whatsoever which can provide a simultaneous reading on **INFO** and any of Q_i. Our system possesses attributes that are both real but mutually exclusive. Within the framework of standard physics **INFO** is not observable and in this sense nonexistent for **physics**, as we know it. Nevertheless it can be understood and studied within the framework of

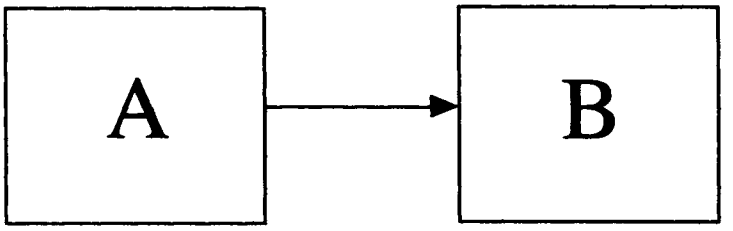
infophysics, which incorporates information primitives into physical description.

EFFECTIVE ACAUSALITY

We often find ourselves in the paradoxical situation that thoughts and ideas pop up in our head, seemingly coming out of nowhere. Archimedes called it *eureka*. Notwithstanding the fact that we are unable to determine the cause of a spontaneous thought or insight, as realists we know it does exist. In an attempt to make sense of this alogical situation, it is not my intention to refute the all-powerful thesis of causality. What I intend to demonstrate, however, that in order to determine the *invisible* causes of intelligent actions we must break out of the confinements of Hermitian physics. Just as a complex number is composed of a real **Re**(z) and an imaginary **Im**(z) ingredient, the real world contains Hermitian and non-Hermitian elements, the former being part of laboratory physics and the latter being part of consciousness. Far-reaching implications follow from this central postulate of infophysics. The complete theory is very different from a sterile picture of reality from which physicists willingly or unwillingly exclude consciousness, the undisputable fact of cosmology. According to this line of reasoning the operators representing quantities in the mind and the operators representing quantities in the brain does not commute:

$$[\text{Mind}, \text{Brain}] \neq 0.$$

Suppose now that we have two events **A** and **B**, connected causally, with **A** being the cause and **B** the effect or vice versa:



non-Hermitian **Hermitian**

Acausal processes cannot become manifest. This is a fundamental requirement of physics to which we inequivocally subscribe, not forgetting of course that quantum-mechanical uncertainty provides restricted room for acausality to occur momentarily. Suppose further that one quantity, say **A**, is cognitive and can be described only by a non-Hermitian operator while **B** is Hermitian. Theoretically, through some indirect means, if available, we may

know that **B** is conditioned by the non-Hermitian **A**; yet our measuring apparatus can deliver knowledge only about the Hermitian quantity **B**. In physics it is in principle not possible to measure a non-Hermitian quantity. But something which we may *never* measure effectively does not exist in a physical sense. We cannot perceive what we cannot conceive. But we can cognize what we cannot analyze. This situation introduces what can be called effective acausality.

One is led to conclude next that in order to understand the nature of thought processes, a non-Hermitian reform of causality is required. Infophysics offers a suitable theoretical framework to achieve this goal. However, the seed of the idea can be found in ordinary physics. For a physical quantity noncommuting with the Hamiltonian of a quantum system one can freely choose a non-Hermitian representation, but another experimental setup exists where this quantity is Hermitian and can be measured. In contrast, cognitive thought always must be represented by a non-Hermitian operator. The mind is part of the real world, but being non-Hermitian and non-commuting with physical quantities, it is not observable quantum-mechanically, neither in conjunction with physical quantities nor alone. Conversely, physical quantities can neither be observed cognitively nor be simultaneously known with cognitive quantities. The brain measures thought, a non-Hermitian variable, but not the Hermitian variables, like the position or momentum of a particle. In synchrony, a physical device would remain 'silent' when presented with consciousness, whatever the condition of the world. With changeover from the physics to infophysics a fundamental inversion is taking place: Hermitian and non-Hermitian quantities change their roles. What was an observable quantity turns into unobservable one and vice versa. This transformation lends itself naturally to treatment as a duality symmetry, involving the exchange of quantum numbers and logical quantum numbers.

FUNDAMENTAL INFORMATION

Whereas in ordinary physics one talks about spacetime and classical fields it may contain, in string theory one talks about an auxiliary two-dimensional field theory that encodes information.

Edward Witten

This duality exchange is at the heart of the theory of infophysics, giving rise to a new concept of information, which enters physics at a fundamental level. The inclusion of information primitives in physics has been a long-standing problem of the fundamental theory of consciousness. We used to

thinking of information and about the related concepts of entropy in terms of continuous functions. But particles, fields and even spacetime are discrete and quantized. If information is to be connected to physics in a fundamental way, it ought to be quantized too. In line with quantization of energy, fields and spacetime, the quantization of our descriptions themselves is on the agenda.

Fundamental cognitive information is represented by (topo)logical charges which are in a dual relation to Noether charges. If we take the Schrödinger momentum

$$\mathbf{p} = -i\hbar \frac{d}{dx}$$

to represent a Noether charge, then the dual (topo)logical charge is represented by the logical momentum

$$M = -\frac{d}{dq}$$

For the sake of uniformity, denoting the momentum by the differentiation sign, we have the duality exchange

$$\nearrow \leftrightarrow \frac{1}{\nearrow}$$

which can be employed to define the fundamental information [Ref 79]:

$$\mathbf{INFO} \sim \frac{1}{\mathbf{p}}$$

The duality of the non-Hermitian logical momentum and the Hermitian quantum momentum can be utilized as a guiding principle for dealing with the singularity of consciousness. Both momentums are derivatives with respect to a respective position coordinate. But the quantum momentum acts on bilateral manifolds of Hilbert space, carrying the Noether charges, while the logical momentum acts on the unilateral manifolds, associated with (topo)logical charges. According to this duality the change of truth-value in actual space is constant and quantized:

$$\frac{dq}{dx} = i\hbar,$$

indicating a link between logic and the spacetime compactification. In five-dimensional field theory, compactification of one dimension leads to the Kaluza-Klein quantization condition

$$\mathbf{p} = \frac{n}{R}$$

where **p** is the periodic quantized momentum for a circle of radius R and n is an integer. If we take the limit $R \rightarrow \infty$, the momentum becomes continuous, and we retrieve the full uncompactified theory. If we take $R \rightarrow 0$, then **p**

becomes either 0 or ∞, and hence the compactified dimension effectively decouples from the theory.
The ratio of fundamental information **INFO** = 1/**p** for the large quantum momentum **p** will tend to 0, which effectively uncompactifies the compactified dimension and the information becomes classical. When **p** decreases, after a certain point the information becomes quantized, nonvanishing since the momentum **p** is never 0, at the very least the vacuum oscillation $\hbar/2$. This dimensional scheme is in close analogy to the De Broglie wave $\lambda = \hbar/\mathbf{p}$, associated with a particle in motion, and has been used to define fundamental information as an intrinsic property of *sufficiently* quantum systems [Ref 79]. According to this definition, the smaller the momentum **p** of a system, the higher its information capacity. Less is more. The information effects must increase when we descend to microphysical scales on the order of the Planck length $\mathbf{10}^{-33}$ cm. Quite symmetrically, a very large momentum should force information to zero, just as a very large volume or density of information will tend to nullify the momentum. This explains why one needs a sufficiently quantum system to deal with fundamental information, the treatment of which is different from the treatment of information in communication theory, which is based on entropy. It must be emphasized that although our analysis is linked to quantum mechanics, it neither refers explicitly to wavefunction nor is dependent on it. Fundamental information is also in close contact with the T-duality which swaps R with α'/R, exchanging the wrapping and momentum states. Although we considered circles wrapped around a periodic dimension, similar arguments can be made for any circle in spacetime.

Fundamental information is not a fixed record, but manifests itself only when particles of the brain are in motion; but according to quantum mechanics, any such motion inevitably produces a wave. In physics there are waves and particles, but in reality everything comes from the description by waves, which are then quantized to give particles. Physical entropy and information entropy are two sides of a coin, but the fundamental information is superquantized.

COGNITIVE OBSERVABLES

No one doubts that thought is associated with the brain but any attempt to localize thought in particular spacetime leads to an intellectual catastrophe. The failure of physics, as we know it, to resolve this question has brought about a clear understanding that a new physics or infophysics is needed to get insight into intractable consciousness. We define infophysics as an extended physical theory which includes information-logical primitives in the description of natural phenomena. While physics studies such quantities as energy, momentum and position, information science refers to bits, truth-values, algorithms etc. The purpose of infophysics is to bring these two seemingly unrelated elements into one unified theory.

In our formalism physical and logical quantities appear fundamentally intertwined. Moreover, they influence each other in an intricate fashion. For those theoretical physicists and information theorists who never cross the border of their particular disciplines this may present conceptual problems. Introducing information primitives in their own right, one must explain how physical and information quantities interact. This is a fundamental and nontrivial problem, although one does not question this fact as such. We are well aware through personal experience of thought and action that physics and information come together in the functioning of the human brain. It goes without saying that the thought process is realised by some physical means. Yet it is highly insensitive to physical forces. We compute 2 plus 2 as 4 whether we stand on our feet or are suspended in air, whether we are accelerated, in free fall or in a state of weightlessness. Gravity, fortunately for space exploration, is not essential for the thought mechanism, as cosmonauts and astronauts report. Neither electric and magnetic fields, nor the strong force, induce any observable changes in our thoughts. The role of the weak interactions is less clear, which has led to the idea about their possible role in consciousness mechanics [Ref 81]. One may speculate about some unknown force at the foundation of the mind but we shall take a different route, and seek an explanation of the physics of the logical mind in the framework of cognitive observables.

Logic has never enjoyed the status of a fundamental science. Unlike in physics, where the notion of physical observables has existed for a long time, the notion of cognitive observables and a corresponding theoretical framework have begun to emerge only recently with the development of matrix logic. The notion of a cognitive observable is crucial to the theory of thinking systems. What is the nature of cognitive observables? In quantum mechanics observables are dynamical variables represented by Hermitian operators on the state space of the observed system. Although it may be difficult and even beyond existing technology to procure a measurement of an observable, quantum theory always allows one to imagine that the measurement can be made and a measuring device can be devised, if only in principle. Cognitive self-measurements are fundamentally different from measurements in a physical laboratory. The brain can *measure* thought but not such physical quantities as the momentum or position of a particle. The Hermitian physical observables which can be realized as measurements in the laboratory cannot be registered by the mind. Conversely, logical observables which can be realized as cognitive measurements in the brain fall outside the scope of Hermitian quantities. In quantum mechanics if some observable cannot be measured simultaneously with the Hamiltonian of the system one can freely choose for it a non-Hermitian representation. Hence, in certain circumstances, a quantum-mechanical Hermitian quantity may turn out to be *in part* non-Hermitian. In logic '*in part*' is taken to an extreme, becoming *the whole*, and no modification of an experimental setup whatsoever may convert a cognitive quantity into a Hermitian observable. We arrive at the idea of a new class of 'unobservable'

observables. Cognitive quantities must be represented by non-Hermitian operators and therefore are not quantum-mechanical observables. Conversely, Hermitian physical quantities are cognitively unobservable. The Hermiticity or non-Hermiticity of a quantity thus places it at either side of the brain-mind divide. The purpose of our investigation is to identify non-Hermitian cognitive operators and to seek explicit rules connecting quantum and cognitive observables. One of the key results we are going to present concerns the differentiation operator, which is a Hermitian observable in the Hilbert space of quantum mechanics

$$\partial = \partial^{+}$$

but a non-Hermitian in cognitive logic.

$$\partial \neq \partial^{+}$$

In different experimental setups one and the same operator reveals different properties. This introduces us to a sort of information 'relativism' where a single quantity is observable or not, depending on methods of measurement.

What are the implications of cognitive observables for fundamental physics? The cognitive argument is persuasive, but many physicists may be distrustful of the conclusion, asking for demonstration. How can something exist but be nonmeasurable? Think of a person whose leg has been amputated but who continues experience pain in the leg which no longer exists.

WHY A TINY 2X2 MATRIX?

We have introduced two critical hypothesis: (a) the observability of a quantity is not an absolute property but depends on the nature of the measuring device (b) the mind is a non-Hermitian operator. This clarifies our position with regard to physical nature of logical variables. We now ask the question, what is the mathematical form of the dynamical logical variables?

As we determined in previous studies [Ref 87, 88, 89, 91], in matrix logic the dynamical logical variables are represented by (2×2) matrices. The corresponding (2×2) logical operators, acting in the two adjoint spaces of bra and ket states, give rise to a universal logical calculus, which is adequate for the description of the general thought process.

As I was developing matrix logic, I for a long time felt very unsettled by the idea that, of the infinite variety of dimensions, nature somehow picks for the intelligent thought process, the most complex process we know, a subset of the prime (2×2) logical matrices which nevertheless is then able to cover the enormous complexities of the multidimensional world. This was a strange and paradoxal situation. Why the (2×2) matrix? What could be so special or extraordinary about this tiny and simple, almost trivial, construct? What special wisdom can be built into it? Why not the (4×4) or the (13×13)

matrix, for example, or, in view of the exceptional complexity of the brain, why not a huge matrix like **(100000000000×100000000000)**? The answer to this question is both simple and fundamental. In picking the prime **(2×2)** matrices as dynamical logical variables nature had no other option. **Not because there is something extraordinary about these primitives but precisely because they are primitives!** There is no more elemental square matrix than a **(2×2)** matrix; next in the series are scalars or ordinary numbers. When a **(n×n)** square matrix is partitioned into its constituent matrices, as soon as we reach the **(2×2)** dimension level, the partitioning ends. A more simple matrix does not exist, and just as the complex world of physics is built up of elemental fundamental particles, the matrix structure of logic and ultimately the mind is built up of elementary **(2×2)** matrices. The world is constructed from atoms, and the mind is from nibbles.
Logical intelligence and abstract thinking are matricial: they cannot be obtained in a one-dimensional scalar framework. Many properties of consciousness run into conflict with the scalar physical intuition but are naturally resolved with matrices.

With the discovery and development of the theory of matrices in the middle of the 19th century mathematics prepared itself for its future service in quantum mechanics. But as scientific visionaries turned from the study of outer space to study of inner space, it became clear that it had also inadvertently prepared itself for fundamental breakthroughs in logic. Strange matrices are hidden in the mind.

THE UNRULY ALGEBRA OF MATRICES

The algebra of matrices is distinguished from the algebra of numbers in several fundamental respects which makes it much more suitable and effective for dealing with the complexities of the thought processes.

Powers

For numbers the equality

$$a^a = a$$

holds exclusively for a =1. When a matrix is raised to the power of itself, the same is true of the unit matrix $\mathbf{I^I = I}$, but also of the fundamental matrix logical implication. **IF to the power of IF is IF**:

$$\mathbf{IF^{IF} = IF},$$

which a critical result, signifying the fact that a thought can be controlled not only by the brain but equivalently well 'from within' by the thought itself.

Cancellation rule

If the equality

$$ab = cb,$$

held for numbers, then the cancellation rule would apply, and

$$a = c.$$

However, for matrices this is not necessarily true. For example,

$$\mathbf{IMPLY \cdot OFF = NAND \cdot OFF}$$

yet

$$\mathbf{IMPLY \neq NAND.}$$

Likewise

$$\mathbf{NOR \cdot AND = ON \cdot AND}$$

but

$$\mathbf{NOR \neq ON}$$

Multiplicity is an essential feature of matrix operations, where the cancellation rule of ordinary algebra is not universally applicable.

Commutation

The product of numbers is commutative

$$ab = ba$$

but for matrices typically

$$AB \neq BA$$

which has fundamental repercussions. Formulas involving only a single matrix usually carry over from the scalar case, for example,

$$\sin 2A = 2\sin A \cos A$$

But generally, a matrix formula based on scalar identities holds only if all the matrices commute with each other. For example for matrices A and B the equalities

$$e^{A+B} = e^{A}e^{B}$$

and

$$\sin(A + B) = \sin A \cos B + \cos A \sin B$$

would be valid on the condition that

$$AB = BA.$$

The rule for the logarithm of a product is not universally transferable to matrices, even if matrices commute, but happens to work for the fundamental logical matrices of implication:

$$\ln(\rightarrow\bullet\rightarrow) = \ln\rightarrow + \ln\rightarrow.$$

Finally, while the nature of the product is unaltered if the scalar factors are swapped, matrices may yield different products, the inner or the outer:

$$\langle x|y\rangle \Rightarrow \text{number}$$

$$|x\rangle\langle y| \Rightarrow \text{matrix}$$

Divisors of zero

For a number product when one factor or/and both are zero, $a = 0$ or/and $b = 0$,

$$ab = 0.$$

But for matrices it is possible that $AB = \mathbf{0}$, even when none of the factor matrices is zero. For example, $\mathbf{AND \cdot NOR = 0}$,

$$\begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}1&0\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}$$

The zero matrix **0** plays a role similar to that of the number 0 in ordinary algebra. Nonzero matrices whose product is **0** are called orthogonal matrices or the divisors of zero. Such are the **AND** and **NOR** projective matrices and the self-orthogonal **ON** and **OFF**:

$$\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0&0\\1&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix},\qquad \begin{pmatrix}0&1\\0&0\end{pmatrix}\begin{pmatrix}0&1\\0&0\end{pmatrix}=\begin{pmatrix}0&0\\0&0\end{pmatrix}.$$

Nilpotence

Any number raised to the power of 0 is unity, $a^0 = 1$, which is counterintuitive but easy to prove:

$$1 = \frac{a^k}{a^k} = a^{k-k} = a^0 = 0.$$

The same is true of matrix powers, e.g. $\mathbf{TRUE}^{\mathbf{0}} = \mathbf{FALSE}^{\mathbf{0}} = \mathbf{I}$, but for the integer logical section we could also have $A^B = \mathbf{I}$ even if $B \neq \mathbf{0}$, for example

$$\mathbf{TRUE}^{\mathbf{FALSE}} = \mathbf{FALSE}^{\mathbf{TRUE}} = \mathbf{TRUE}^{\mathbf{TRUE}} = \mathbf{FALSE}^{\mathbf{FALSE}} = \mathbf{I}.$$

Just like a finite product, the finite power of a number is never zero:

$$a^k \neq 0, \qquad \forall\, a \neq 0.$$

This no longer universally applies to matrices which can be nilpotent, e.g.:

$$\begin{pmatrix}0&0\\1&0\end{pmatrix}^2=\begin{pmatrix}0&0\\0&0\end{pmatrix},\qquad \begin{pmatrix}0&1\\0&0\end{pmatrix}^2=\begin{pmatrix}0&0\\0&0\end{pmatrix},$$

Likewise for the logical section of the matrix power $\mathbf{H}^{\mathbf{N}} = \mathbf{0}$,

$$\begin{pmatrix}1&-1\\-1&1\end{pmatrix}^{\begin{pmatrix}0&1\\1&0\end{pmatrix}}=\begin{pmatrix}0&0\\0&0\end{pmatrix}.$$

Negating matrices

A nonzero number inverts its sign when multiplied by -1,

$$(-1)\,a = -a.$$

A matrix, multiplied by **-I**, also changes its sign to the opposite, for example **(-YES)•H = - H**,

$$\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}=\begin{pmatrix}-1&1\\1&-1\end{pmatrix}$$

But equally well we can obtain sign inversion, making no use of **-I**. For example, **NOT•H = - H**,

$$\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&-1\\-1&1\end{pmatrix}=\begin{pmatrix}-1&1\\1&-1\end{pmatrix}$$

Real complex matrices

Last, but not least, we must emphasise one feature of matrices which is of fundamental significance for the theory presented in this study. For any real number, a*= a,

$$a^2 \geq 0$$

and never

$$a^2 < 0.$$

But matrices with real-valued elements exist which behave like the complex unit *i*. For example, $\blacktriangle^2 = -\mathbf{I}$,

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}.$$

We thus have a correspondence between numbers and matrices displaying similar algebraic properties:

$$0 \Rightarrow \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \quad 1 \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad i \Rightarrow \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

As a matrix counterpart of 2 we can take the *scalar matrix*,

$$2 = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$$

which can be thought of as the sum of the two unit matrices:

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

Making use of a different criterion, we also can derive other 2 matrices, observing that 2 is the only prime number for which

$$2 + 2 = 2 \bullet 2.$$

It is easily checked that this is true for the matrix $\left(\begin{smallmatrix} 2 & 0 \\ 0 & 2 \end{smallmatrix}\right)$:

$$\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}\begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}.$$

But the same is true, for example, for the serial-parallel matrices

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

which also are distinguished by $\mathbf{L}^2 = 2\mathbf{L}$. The addition of these two 2 matrices is the 2 matrix again:

$$2 + 2 = 2$$

or

$$\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} + \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix},$$

which warns us that one must proceed very carefully while transferring the rules of ordinary algebra to the unruly algebra of matrices.

Inverses

For numbers the equality of a number and its inverse:

$$\frac{a}{b} = \frac{b}{a}$$

will hold only in the trivial situation when a = b, but for matrices it is also possible that

$$\frac{A}{B} = \frac{B}{A}$$

when $A \neq B$. Consider for example

$$\frac{OR}{IF} = \frac{IF}{OR}$$

where **OR≠ IF.**

Non-squaring matrices

Thus far we have focused on the square matrices. A major advantage of these matrices is that they can be both added and multiplied, thus providing a basis for developing an algebra of matrices that is in many respects reminiscent of the algebra of ordinary numbers. Some important differences distinguish nonsquare or rectangular matrices, whose algebra is even more restrictive. A null matrix and a unit matrix were both defined for square matrices. For rectangular matrices a unit rectangular matrix does not exist, although a null matrix can be defined.

If square matrices can be added they can always be multiplied and vice versa, but it can happen that we add two rectangular matrices but their product does not exist. Conversely, it is possible that matrices can be multiplied but not added.

While two square matrices can be multiplied in any order, the rectangular matrices may be multiplied in the one order, while in the reverse order multiplication is not defined. For rectangular matrices both products AB and BA exist only if A is of the order **(m×n)** and B is of the order **(n×m)**. The **(2×2)** square matrices, which are of particular interest for our discussion, can be derived as products of the rectangular matrices of the orders **(2×n)** and **(n×2)**. It follows that the rectangular matrices induce the dimension-changing transformations, as opposed to square matrices, necessarily providing a mapping of a space to itself.

A number and a square matrix can always self-multiply or be squared as opposed to a rectangular matrix for which powers do not exist. Raising a matrix to a power is a very important operation but a rectangular matrix can never be multiplied by itself. **In pointing out the nonsquaring nature of rectangular matrices, we come upon an important distinction between matrix and ordinary algebra.**

A finite nonzero number a always has an inverse 1/a, such that

$$a \cdot \frac{1}{a} = 1.$$

However, there exist nonzero matrices - singular matrices - which do not have inverses. Because division by a matrix is defined as multiplication by an inverse matrix, existing only for square matrices, division is not always possible, and for rectangular matrices cannot be designed even in principle.

TRUTH-VALUE AS EIGENVALUE

The notion of truth is the key notion of logical theory. It is a common procedure in mathematical logic to represent truth-values by binary numbers. A new idea which underlies matrix theory is a connection of the truth-value, the fundamental notion of logic, with the eigenvalue, the fundamental notion of quantum physics. In quantum mechanics the observables are represented by operators whose eigenvalues stand for what we actually measure in laboratory. For example reducing the Schrödinger equation to the time-independent format:

$$(i\hbar\frac{\partial}{\partial t} - H)|\psi> = 0, \qquad \Rightarrow \qquad (H - E)|\psi> = 0$$

we effectively obtain an eigenstate equation where is H the operator and E are the quantum numbers to be measured. In quantum mechanics observables are represented by Hermitian operators on the state space of the observed system. When we measure some physical quantity we effectively measure an eigenvalue of a corresponding operator.

In matrix logic the situation is analogous to quantum mechanics, suggesting that the eigenvalue problem is as fundamental for logic as it is for physics. Realising that classical logic is at fault in ignoring the operator nature of the logical connectives, we have taken steps to identify truth-values as eigenvalues of matrix logical operators [Ref 89]. This innovation profoundly alters the conceptual landscape of logical theory and accomplishes a computational reform of logic which is of fundamental significance. The truth-value which in classical logics is given directly is now encoded as an eigenvalue of logical operators. Just as in quantum mechanics the eigenvalues of quantum operators are called quantum numbers, the eigenvalues of logical operators we call the logical quantum numbers [Ref 89, 91]. We raise logical quantities to the rank of observables represented by operators and associate with each logical operator a corresponding eigenequation

$$\mathbf{L}|q> = \lambda|q>$$

where λ is a logical quantum number(s). Besides standard Boolean 0 and 1, the logical quantum numbers also include the improbabilities -1 and 2, the golden sections ϕ and $\bar{\phi}$, the quantum factor $\sqrt{2}$, and the imaginary unit $\pm i$. The set of eigenvalues λ represents the spectra of a logical operator $\mathbf{L}$, which are a set of all those scalars λ that are roots of the characteristic equation

$$\det(\mathbf{L} - \mathbf{I}\lambda) = 0.$$

For any polynomial

$$p(L)|q> = p(\lambda)|q>$$

every eigenstate of the operator **L** with the eigenvalue λ is simultaneously the eigenstate of $p(L)$ with the eigenvalue $p(\lambda)$. Consequently, if the operator **L** satisfies the equation $p(L) = 0$, then $p(\lambda) = 0$ for any eigenvalue of that operator.

To compare our eigenvalue method with the conventional scheme, consider a Boolean conjunction (xANDy). According to the multiplication rule of binary numbers it is solved to false or true, depending on the truth-value of x and y:

$$(x\,\mathbf{AND}\,y) = \begin{cases} 0 & \text{false} \\ 1 & \text{true} \end{cases}$$

The matrix-logical operator **AND** can also be solved to false 0 or true 1, but now these logical values are obtained as the eigenvalues of the operator. From the eigenequation

$$\mathbf{AND}|x> = \lambda|x>$$

in view of the idempotence

$$\mathbf{AND}^2 = \mathbf{AND}$$

we determine that

$$\lambda^2 - \lambda = 0$$

and

$$\lambda_{1,2} = \begin{cases} 0 & \text{false} \\ 1 & \text{true} \end{cases}$$

The eigenvalue spectrum of **AND** in full numerical agreement with the classical formula but fundamentally different in essence.

George Boole rightly guessed the idempotence of the variables of classical logic

$$x^2 = x$$

which leads to

$$x^2 - x = 0 \quad \Rightarrow \quad x = 0,\ 1,$$

and correctly formalized the negation as the complement operation over the binary numbers 0 and 1. But the eigenvalue approach provides a deeper understanding of logic. What Boole and many after him did not realize is that as a consequence of the operator nature of logic truth-values are eigenvalues found as the roots of the characteristic equations. Given the logical operators **TRUE** and **FALSE**, from $\mathbf{L}^2 = \mathbf{L}$ one determines that their characteristic equation

$$\lambda^2 - \lambda = 0 \quad \Rightarrow \quad \lambda = 0,\ 1$$

has a form and solutions identical to Boole's idempotence axiom.
The eigenvalue approach provides an important refinement of logical theory, introducing in a natural way the negative truth-value that is so critical for cognitive operations. We consider the negation operator which is self-inverse:

$$\mathbf{NOT}^2 = \mathbf{YES}$$

hence

$$\lambda^2 = 1 \quad \Rightarrow \quad \lambda = 1, \ -1,$$

where one of the solutions is negative, a significant fact which can be only understood in the framework of a interpretation of truth-value as eigenvalue. The eigenvalues for **OR** are the golden ratios $\phi = 1.618$ and its complement $\bar{\phi} = -0.618$ which are nonintegers. The golden ratio played an important role in art, architecture and aesthetics. Its appearance in logic is surprising and significant. Apparently the eigenvalues of the matrix-logical operators of the major basis set {**NOT, AND, OR**}, as opposed to its Boolean counterpart, accommodate the diverse cognitive forms: negative, discrete and continuous.

The number of different eigenvalues which one may determine for a (2×2) operator is at most 2. When a logical operator carries a complete set of eigenvalues these correspond to a pair of orthogonal eigenstates. The objective of logical thinking is to separate true and false into orthogonal subspaces, which entails the vanishing of the inner products of the corresponding eigenstates. We calculated the logical quantum numbers for

$$\mathbf{AND}: \quad \lambda_1 = 0, \quad \lambda_2 = 1,$$

and

$$\mathbf{NOT}: \quad \lambda_1 = 1, \quad \lambda_2 = -1.$$

The corresponding logical eigenstates are orthogonal, respectively,

$$\langle 0|1\rangle = 0 \qquad \text{and} \qquad \langle s_+|s_-\rangle = 0.$$

When the eigenvalue 0 is the result of the measurement of **AND**, we know that the observable is sent to the false subspace, and when the eigenvalue 1 is the result of the measurement, the observable is in the true subspace. Likewise a definite separation of true and false is possible for **NOT**. However, the condition of separability is not attainable if true and false are weakly orthogonal (fuzzy). Then the logical subspaces overlap, and the system cannot be confined to a definite subspace of true or false. This is another way of saying that a logical thought (operator) cannot be diagonalized. Consider the characteristic polynomial for **IMPLY**:

$$|(\rightarrow - \lambda \mathbf{I})| = \begin{vmatrix} 1-\lambda & 1 \\ 0 & 1-\lambda \end{vmatrix} = (1-\lambda)^2 = 0$$

It is solved by the root $\lambda = 1$ which is degenerate. The eigenstate vectors are found from the system of equations

$$x_1 + x_2 = x_1$$
$$x_2 = x_2.$$

which entail $x_2 = 0$ while x_1 is arbitrary. Consequently, for **IMPLY** the eigenstate is the degenerate false

$$\begin{pmatrix}1\\0\end{pmatrix}.$$

Likewise, for **IF** $x_1 = 0$ while x_2 is arbitrary and the eigenstate is the degenerate true

$$\begin{pmatrix}0\\1\end{pmatrix}.$$

The transformation matrix formed from such vectors could only be singular, hence there is no orthogonal transform which diagonalizes the operator. Because implications cannot be diagonalized, they cannot be separated into orthogonal subspaces. This explains the confusing property of implication, which, with a false antecedent, is true both for the true and false consequent:

$$\langle 0|\rightarrow|0\rangle = 1, \qquad \langle 0|\rightarrow|1\rangle = 1.$$

In addition to applications in logic, the identification of logical values as eigenvalues provides a natural connection between logic and quantum physics, which has far-reaching implications for logical theory and the fundamental analysis of thinking systems.

TRUTH-VALUE AND FALSE-VALUE

The realization of the fact that logical values are not scalar but tensor quantities immediately has led to two very important applications [Ref 84,88]. Logical connectives which previously were seen as primitive entities in their own right, now become a derivative concept. Any logical operator is an outer product of two logical states or a linear entanglement of such products. The information bits exist not only in orthogonal Boolean states but also form coherent superpositions. Until the development of matrix logic our understanding of the thought process and logic in general were essentially one-dimensional. Truth-value was a scalar quantity, which in classical logic could have two complementary and orthogonal values: true 1 or false 0, thus accounting for both the truth or falsity of a logical state. Even though true is bound by complementation to false, these two values are quite different, a fact which is not so obvious. This becomes evident when we consider self-referential statements. True may refer to itself without fundamental consequences. We easily make a statement

EVERYTHING IS TRUE

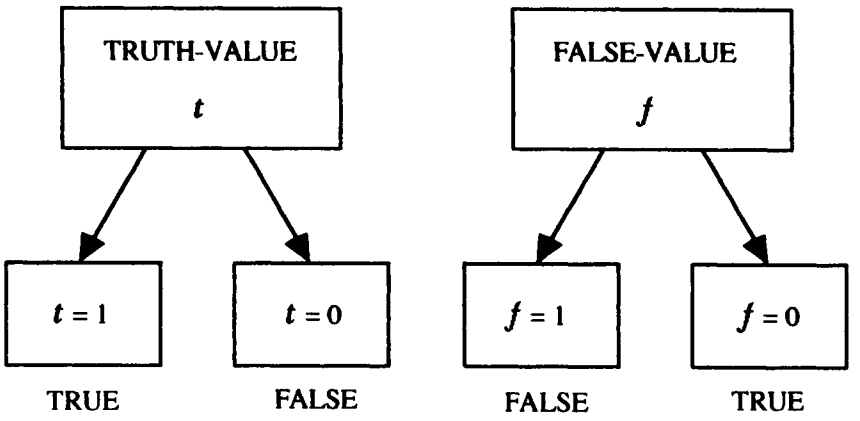
which of course is a false assertion, but in a real-life situation can be applied without leading to absurdity. However, a statement

EVERYTHING IS FALSE

which is seemingly symmetric to the first one, is apparently a major intellectual catastrophe. Since everything is false, "everything is false" is false too, but then not everything is false, and the Liar paradox begins its unending oscillation. Truth and false are not simple symmetric complements of each other: true and false are different and both must be put into the foundation of a complete logical theory, each on its own footing.

The key element of the theory of matrix logic is the extension of the alphabet of logical values, where in addition to truth-value we also introduce the *false-value*. In this new scheme the true and the false represent distinct logical degrees of freedom. Just as Hamiltonian physics requires both the coordinate and momentum or energy and time for an adequate physical description, the falsum and verum represent a canonical pair of coordinates in logical phase space. This is an essential step if we are to formulate a meaningful theory of the thought mechanism. As opposed to a truth-value, which can be defined by a single number, a cognitive logical state is defined by two components, and in a sense is a complex number.

The true and the false, although connected, are not necessarily complements in the narrow Boolean sense. Whenever this is the case, the values are weakly orthogonal and it will be not possible to distinguish between them perfectly. However, when strong complementarity applies, the truth-value and false-value behave numerically as follows:



Although at first glance one may see no special advantage in this innovation, and may think that the addition of the false-value is redundant, this is not so. Not only because t and f are not necessarily complements of each

other, as will be the case in some important situations we will consider below, but because ignoring the false-value in the study of the thought process prevents us from overcoming certain fundamental limitations. Trying to understand the thought process in terms of traditional scalar formalisms is like trying to understand the geometry of curved spacetime without vectors and tensors.

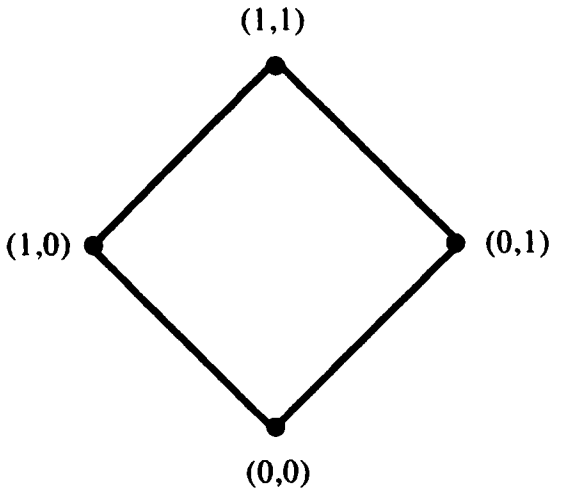
Having introduced the false-value it will be convenient to form a construct which enables us to consider both values simultaneously. This can be done by defining a 'thinking' vector or *denktor*

$$\begin{pmatrix} f \\ t \end{pmatrix}$$

in which the truth-value and false-value enter together explicitly. Various options now become available, distinct from each other in the extent of correlation between t and f. Four different possibilities presents themselves at once:

$$\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix},$$

giving rise to fundamental logic which operates with four values: **True**, **False**, **True and False**, **Neither True Nor False**. The respective denktors can be identified by the vertices of a 2-dimensional Boolean lattice:



SQUARE ROOT OF NOTHING

Among the great things which are found among us the existence of nothing is the greatest

Leonardo da Vinci

THE LOGICAL VACUUM REVISITED

The world around us is in constant motion and very few have difficulties in understanding what is physical momentum. A ball begins to roll if pushed. A string begins to vibrate when given a momentum. But what starts the thought process? What is logical or mental momentum? One may have some intuitive ideas in this regard, such as a nebolous idea of *free will*, but we must define logical momentum in precise mathematical terms. We know that the thought process can be monitored by external or internal factors and directed by the goals we set for ourselves. Thinking starts with the formulation of the problem, with the logical momentum determining the dynamics of the process. The average value of the logical momentum is the logical interval

$$< q|-\frac{d}{dq}|q >= \bar{q} - q$$

One of our objectives will be to show the equivalence between differential form and the matrix form of the logical momentum. The logical momentum operator is hidden in a vacuum and must be extracted. It is a divisor of zero and can be created by the square root polarization of the logical vacuum:

$$\sqrt{0} = M$$

This beautiful result has far-reaching implications. There is not just one zero but many. Actual minds which occur in the actual world can be described as solutions to the equations of logic. Solving these equations requires determining the ground state of the system, which is a vacuum. Logical momentum, emerging from the vacuum, is the **knowledge** operator which naturally provides a fundamental solution to the eigenstate problem and to the

problem of coherent logical superposition: The momentum operator takes a classical logical state out of a continuum and propels it into a coherent superposition:

$$M\ |q\rangle = |s_-\rangle.$$

The logical momentumwo generates superposed logical states:

$$-\frac{d|q\rangle}{dq} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

Because logical differentiation can take a matrix form, it is applicable not only for continuous functions but also in the discrete logic. The logical derivatives yield superposed states independently of whether the logical state is fuzzy or Boolean:

$$M|q\rangle = |s_-\rangle, \quad M|x\rangle = |s_-\rangle,$$
$$1|q\rangle = |s_+\rangle, \quad 1|x\rangle = |s_+\rangle.$$

Since the momentum comes from 'nothing', there must be a reverse process in which the squared momentum should self-destruct in accordance with the nilpotence property; the rows of M are orthogonal to the columns,

$$M^2 = M \cdot M = 0$$

Thought is created in the mind only to be dissolved into the void, giving way to another thought, so that a new circle of creation and annihilation, of wear and repair, can begin all over again. The psychologist calls the void the "unconsciousness" from which the "I" picks up the logical elements needed to form a cognitive thought. The logical momentum monitoring thought is renamed *free will*. This helps a bit. But can one mystery explain another? Aren't we back at square one? Strangely enough it is not in the biosciences but in modern physics that we find a conceptual framework more suitable for thinking about thinking. In some profound way the structure of consciousness resembles the subatomic realm of quantum field theory, where the distinction between matter and empty space has had to be abandoned. In modern physics the vacuum is recognized a fundamental structure of major importance. The quantum field is a dynamical system characterized by the birth and death of particles, where the particles are created out of the void, and vanish back into the vacuum. The behaviour of the logical momentum closely parallels the effects of the physical vacuum. It does not take us long to stop hearing a monotonous repetitive sound. To see is to compare. Our eyes will very quickly become selectively 'blind' and stop seeing an image that is fixed on the eye ball so that it is invariably projected to the same location on the retina. High-level brain processes display the same self-destructive property, most prominently in a cognitive effect of the annihilation of meaning. If some word is repeated continuously, after a while the meaning of the word will vanish. One will hear and recognize the sound, but the meaning associated with the word is no longer there. The form is preserved but the content has gone. Very much like a

totalitarian slogan, placed on every building and written on every wall, but empty and devoid of meaning. In many cognitive and perceptual effects a repeated action leads to destruction. Familiarity feeds contempt is an unwritten law of biology: it is the closest neighbours which are most often at war.

On a psychological level we understand this effect very well. Now we can understand it in technical terms as the self-destruction of logical momentum. For the thinking brain the ground state is the matrix-logical vacuum. Thoughts are generated from the ground state in a manner quite similar to the way in which quantum particles are created from a physical vacuum and there is in fact a way to relate the logical vacuum to formalism of quantum field theory. The logical momentum is the logical derivative which is to the factor $i\hbar$ is the same as the quantum momentum operator:

$$M = -\frac{d}{dq} \qquad \text{and} \qquad p = -i\hbar\frac{d}{dx} .$$

Since M is the divisor of zero, the second derivative, which is the product of the two differential operators, must be the nil operator:

$$-\frac{d}{dq} \bullet -\frac{d}{dq} = \frac{d^2}{dq^2} = 0.$$

This implies that the logical states are represented by linear functions $f(q)$, and the second-order derivative reduces the functions to zero:

$$\frac{d^2}{dq^2} f(q) = 0.$$

A thought emerging out of nowhere will submerge back into the void. As opposed to a wavefunction, which is infinitely differentiable, with each new derivative simply releasing factors in the exponent, a linear thought process cannot be accelerated, since the momentum operator self-destructs by iteration.

Understanding the actual physics behind the operation of the square root polarization is critical for the understanding of the thought mechanism. Having the explicit form of the logical momentum, we can determine the logical energy operator, which must be related quadratically:

$$\langle H \rangle = \langle M \rangle^2$$

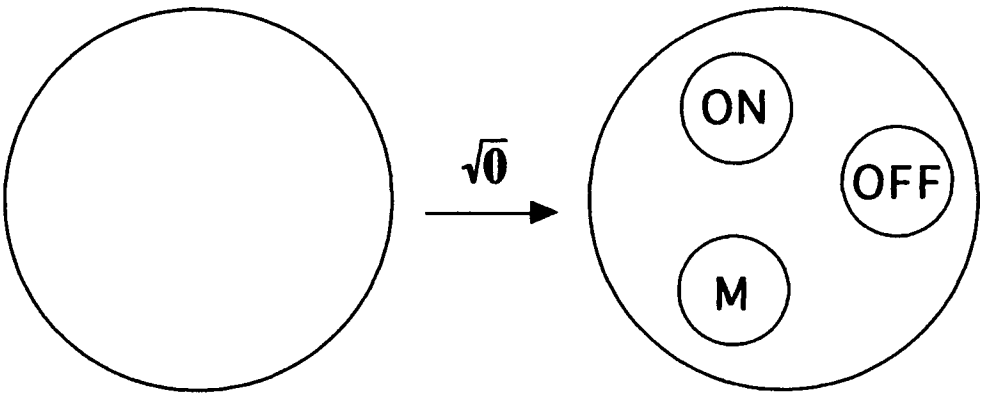
The only operator in matrix logic which agrees with this requirement is the **Lagrangian H**, found originally from different considerations as the logarithm of the logical evolution operator [Ref 91]:

$$H = -\ln N.$$

MOMENTUM AS TENSOR PRODUCT

The fundamental logical theory of the brain can be deduced from a properly defined vacuum, whose polarization gives rise to the cognitive logical momentum. It is also important to keep in mind that other logical operators also emerge in the polarization of the vacuum, most importantly the triplet:

$$\sqrt{\mathbf{0}} = \mathbf{M}, \mathbf{ON}, \mathbf{OFF}$$



Besides the square root polarization, another way to obtain the fundamental triplet basis {**ON, M, OFF**} is through tensor interactions of orthogonal logical states:

$$|1\rangle,\ |0\rangle,\ |s_+\rangle,\ |s_-\rangle\ .$$

We distinguish two different classes of tensor outer product interactions. One class is formed by identical states which interact with their own kind:

$$|1\rangle\langle 1| = \mathbf{AND} \quad \text{or} \quad |s_+\rangle\langle s_+| = \mathbf{ONE} \quad \text{or} \quad |s_-\rangle\langle s_-| = \mathbf{H},$$

where

$$\langle 1|1\rangle \neq 0,\ \langle s_\pm|s_\pm\rangle \neq 0,$$

and the other class by by interaction of a pair of orthogonal states:

$$|1\rangle\langle 0| = \mathbf{ON} \quad \text{or} \quad |0\rangle\langle 1| = \mathbf{OFF} \quad \text{or} \quad |s_-\rangle\langle s_+| = \mathbf{M},$$

where

$$\langle 1|0\rangle = 0,\ \langle s_-|s_+\rangle = 0.$$

The logical nibbles created through the tensor interaction of identical states are idempotent or factor-idempotent:

$$\mathbf{L}^2 = n\mathbf{L}, \qquad n = 1, 2.$$

The nibbles which are created through the interaction of orthogonal logical states are the same as the fundamental basis {**ON, M, OFF**}, which is created by the square root polarization of the logical vacuum.

The average of the logical momentum is the classical-quantum trace

$$\langle q|\mathbf{M}|q\rangle = \mathrm{Tr}|q\rangle\langle s_-| = \bar{q}\text{-}\ q.$$

If the coherent superposition $\langle s_-|$ is changed to the orthogonal $\langle s_+|$, the trace is changed to unity, and the outer product turns in the matrix fixed point of the logical momentum

$$\mathbf{M}|q\rangle\langle s_+| = \mathbf{M}.$$

Looking into the fine structure of **M**, we determine that it can be factorized into the pair of orthogonal states $|S_-\rangle\langle S_+| = \mathbf{M}$, which can then be disentangled into a system of the **4** logical particles. Conversely, the ordered interaction of **4** particles will recover the logical momentum operator.

LOGICAL MOMENTUM AS QUANTUM KNOT

We have determined several fundamental ways to generate the logical momentum:

(1) by the square root polarization of the logical vacuum:

$$\sqrt{\mathbf{0}} = \mathbf{M},$$

(2) through the tensor product of orthogonal superposed logical states:

$$|S_-\rangle\langle S_+| = \mathbf{M},$$

(3) as the commutator product of the **FALSE** and **TRUE** operators:

$$[\mathbf{FALSE}, \mathbf{TRUE}] = \mathbf{M}.$$

The commutator generation of the momentum is a quantum knot of the logical derivative, converting an orientable topology into a nonorientable one. But the logical derivative can have many other possible commutator realizations. Given the set of matrix-logical operators we can obtain **80** different commutator knots, all yielding the logical momentum. Each of the **80** logical nibbles of matrix logic can be assigned its own commutator of the logical momentum, presented in the Table 2.

1	-1	-1	-1	-1	0	0	-1	-1	21	-1	1	1	-1	-1	-1	0	0
2	-1	-1	-1	-1	0	1	0	-1	22	-1	1	1	-1	-1	0	1	0
3	-1	-1	-1	-1	1	0	-1	0	23	-1	1	1	-1	0	-1	0	1
4	-1	-1	-1	-1	1	1	0	0	24	-1	1	1	-1	0	0	1	1
5	-1	-1	0	0	-1	-1	-1	-1	25	0	-1	-1	0	0	0	-1	-1
6	-1	-1	0	0	0	-1	-1	0	26	0	-1	-1	0	0	1	0	-1
7	-1	-1	0	0	0	0	-1	-1	27	0	-1	-1	0	1	0	-1	0
8	-1	-1	0	0	1	-1	-1	1	28	0	-1	-1	0	1	1	0	0
9	-1	-1	0	0	1	0	-1	0	29	0	-1	0	1	-1	-1	-1	-1
10	-1	-1	0	0	1	1	-1	-1	30	0	-1	0	1	0	-1	-1	0
11	-1	-1	1	1	-1	-1	0	0	31	0	-1	0	1	0	0	-1	-1
12	-1	-1	1	1	0	-1	0	1	32	0	-1	0	1	1	-1	-1	1
13	-1	-1	1	1	0	0	-1	-1	33	0	-1	0	1	1	0	-1	0
14	-1	-1	1	1	1	0	-1	0	34	0	-1	0	1	1	1	-1	-1
15	-1	0	1	0	-1	-1	-1	-1	35	0	0	-1	-1	-1	1	1	-1
16	-1	0	1	0	-1	-1	0	0	36	0	0	-1	-1	0	1	0	-1
17	-1	0	1	0	-1	-1	1	1	37	0	0	-1	-1	0	1	1	0
18	-1	0	1	0	0	-1	-1	0	38	0	0	-1	-1	1	1	-1	-1
19	-1	0	1	0	0	-1	0	1	39	0	0	-1	-1	1	1	0	0
20	-1	0	1	0	1	-1	-1	1	40	0	0	-1	-1	1	1	1	1

41	0	0	1	1	-1	-1	-1	-1	61	1	0	-1	0	-1	1	1	-1
42	0	0	1	1	-1	-1	0	0	62	1	0	-1	0	0	1	0	-1
43	0	0	1	1	-1	-1	1	1	63	1	0	-1	0	0	1	1	0
44	0	0	1	1	0	-1	-1	0	64	1	0	-1	0	1	1	-1	-1
45	0	0	1	1	0	-1	0	1	65	1	0	-1	0	1	1	0	0
46	0	0	1	1	1	-1	-1	1	66	1	0	-1	0	1	1	1	1
47	0	1	0	-1	-1	-1	1	1	67	1	1	-1	-1	-1	0	1	0
48	0	1	0	-1	-1	0	1	0	68	1	1	-1	-1	0	0	1	1
49	0	1	0	-1	-1	1	1	-1	69	1	1	-1	-1	0	1	0	-1
50	0	1	0	-1	0	0	1	1	70	1	1	-1	-1	1	1	0	0
51	0	1	0	-1	0	1	1	0	71	1	1	0	0	-1	-1	1	1
52	0	1	0	-1	1	1	1	1	72	1	1	0	0	-1	0	1	0
53	0	1	1	0	-1	-1	0	0	73	1	1	0	0	-1	1	1	-1
54	0	1	1	0	-1	0	1	0	74	1	1	0	0	0	0	1	1
55	0	1	1	0	0	-1	0	1	75	1	1	0	0	0	1	1	0
56	0	1	1	0	0	0	1	1	76	1	1	0	0	1	1	1	1
57	1	-1	-1	1	0	0	-1	-1	77	1	1	1	1	-1	-1	0	0
58	1	-1	-1	1	0	1	0	-1	78	1	1	1	1	-1	0	1	0
59	1	-1	-1	1	1	0	-1	0	79	1	1	1	1	0	-1	0	1
60	1	-1	-1	1	1	1	0	0	80	1	1	1	1	0	0	1	1

Table 2.
The commutator knots of the logical momentum.

The number of momentum anticommutators is less. While there are **80** commutator knots, there are **40** anticommutator knots, given in in Table 3.

$$\mathbf{M} = \{ \mathbf{L}_i , \mathbf{L}_j \}_k , \quad k = 1, \ldots , 40$$

where

$$\mathbf{L}_i , \mathbf{L}_j \in \left\{ \pm\mathbf{1}, \pm\mathbf{H}, \pm\mathbf{M}, \pm\perp , \pm\mathbf{T}, \pm\mathbf{W}(\perp), \pm\mathbf{W}(\mathbf{T}) \right\}$$
$$i , j = 1, \ldots , 14.$$

1	-1	-1	-1	-1	-1	0	1	0	21	0	0	1	1	-1	1	1	-1
2	-1	-1	-1	-1	0	-1	0	1	22	0	0	1	1	0	1	0	-1
3	-1	-1	0	0	-1	-1	1	1	23	0	0	1	1	1	1	-1	-1
4	-1	-1	0	0	-1	0	1	0	24	0	1	0	-1	-1	-1	1	1
5	-1	-1	0	0	-1	1	1	-1	25	0	1	0	-1	0	0	1	1
6	-1	-1	1	1	-1	-1	0	0	26	0	1	0	-1	1	1	1	1
7	-1	-1	1	1	-1	0	1	0	27	1	-1	-1	1	0	0	-1	-1
8	-1	-1	1	1	0	0	-1	-1	28	1	-1	-1	1	1	1	0	0
9	-1	-1	1	1	0	1	0	-1	29	1	0	-1	0	1	1	-1	-1
10	-1	0	1	0	-1	-1	-1	-1	30	1	0	-1	0	1	1	0	0
11	-1	0	1	0	-1	-1	0	0	31	1	0	-1	0	1	1	1	1
12	-1	0	1	0	-1	-1	1	1	32	1	1	-1	-1	0	-1	0	1
13	-1	1	1	-1	-1	-1	0	0	33	1	1	-1	-1	0	0	1	1
14	-1	1	1	-1	0	0	1	1	34	1	1	-1	-1	1	0	-1	0
15	0	-1	0	1	-1	-1	-1	-1	35	1	1	-1	-1	1	1	0	0
16	0	-1	0	1	0	0	-1	-1	36	1	1	0	0	1	-1	-1	1
17	0	-1	0	1	1	1	-1	-1	37	1	1	0	0	1	0	-1	0
18	0	0	-1	-1	-1	-1	1	1	38	1	1	0	0	1	1	-1	-1
19	0	0	-1	-1	0	-1	0	1	39	1	1	1	1	0	1	0	-1
20	0	0	-1	-1	1	-1	-1	1	40	1	1	1	1	1	0	-1	0

Table 3.
The anticommutator knots of the logical momentum.

A quantum knot is a dynamical system, where swapping the **TRUE** and **FALSE** changes the logical momentum to its opposite:

$$[\mathbf{FALSE}, \mathbf{TRUE}] = -\,[\mathbf{TRUE}, \mathbf{FALSE}]$$

Since that for the (bi)asymmetric logical operators $[\,\mathbf{L}, \mathbf{H}\,] = [\,\mathbf{H}, \overline{\mathbf{L}}\,]$, the reversal of the momentum is carried to the commutators with the synergy operators **H** and **1**, when the **FALSE** and **TRUE** are interchanged. The synergies, **H** and **1**, knotted by the quantum operation of commutation with the **TRUE** and **FALSE**, create a logical momentum of appropriate sign:

$$[\mathbf{TRUE}, \mathbf{H}\,] = \top - \bot = \mathbf{M}$$
$$[\,\mathbf{FALSE}, \mathbf{H}\,] = \bot - \top = -\,\mathbf{M}$$

and

$$[\,\mathbf{FALSE}, \mathbf{1}\,] = 2\top - \mathbf{1} = \mathbf{M}$$
$$[\mathbf{TRUE}, \mathbf{1}\,] = 2\bot - \mathbf{1} = -\,\mathbf{M}$$

Hence:

$$[\mathbf{TRUE}, \mathbf{H}\,] = [\mathbf{H}, \mathbf{FALSE}]$$

and

$$[\mathbf{TRUE}, \mathbf{1}\,] = [\,\mathbf{1}, \mathbf{FALSE}].$$

Likewise for the transposes:

$$[\,\dashv, \mathbf{H}\,] = [\,\mathbf{H}, \vdash\,]$$

and

$$[\,\vdash, \mathbf{1}\,] = [\,\mathbf{1}, \dashv\,].$$

The averages of the logical momentum, however, are stable under the converse transposition of the commutator:

$$\langle q\,[\,\bot, \mathbf{H}\,]\,q\rangle = \langle q\,[\,\vdash, \mathbf{H}\,]\,q\rangle$$
$$\langle q\,[\,\top, \mathbf{H}\,]\,q\rangle = \langle q\,[\,\dashv, \mathbf{H}\,]\,q\rangle$$

and

$$\langle q\,[\,\bot, \mathbf{1}\,]\,q\rangle = \langle\,[\,\vdash, \mathbf{1}\,]\,q\rangle$$
$$\langle q\,[\,\top, \mathbf{1}\,]\,q\rangle = \langle\,[\,\dashv, \mathbf{1}\,]\,q\rangle$$

ZERO-WEIGHT OPERATORS

Definition. We define the weight or mass $\mathbf{m(L)}$ of a matrix-logical operator **L** as an algebraic sum of the matrix elements

$$\mathbf{m(L)} = \Sigma a_{ij}.$$

The mass can be positive, negative or zero. Naturally the vacuum, represented by the empty operator **NONE**, comprising zero elements only, has zero mass,

$$\mathbf{m(0)} = 0.$$

There are other zero-weight operators, such as ▲ , σ and **H**, altogether **18** operators. Significantly, the square root polarization of **NONE**, which is zero-weight, gives rise to **M**, also zero-weight, $\sqrt{\mathbf{NONE}} = \mathbf{M}$, the only instance where the mass value of the operators is conserved with perturbation of the

vacuum. Although the fundamental way to obtain **M** is by square rooting **NONE**, it is also possible to obtain the momentum as the product of the synergy operators with zero-weight operators:

$$H\blacktriangle = M, \qquad H\sigma = M,$$

similarly

$$\blacktriangle 1 = M, \qquad \sigma 1 = M.$$

The operator sum of zero-weight nibbles is the zero-weight momentum:

$$\blacktriangle + \sigma = M.$$

Theorem

A matrix-logical string with an arbitrary sequence of **TRUE** and **FALSE** nibbles conserves **M**, whether the interaction is (a) multiplicative or (b) supercommutator:

(a)

$$M \bullet \perp \bullet T \bullet \perp \bullet \perp \bullet \perp \bullet T \bullet \perp \bullet T \bullet T \bullet \ldots = M$$

(b)

$$\{M, \perp \bullet T \bullet \perp \bullet \perp \bullet \perp \bullet T \bullet \perp \bullet T \bullet T \bullet \ldots\} = M$$

Proof

Any finite product of **TRUE** and **FALSE** nibbles is determined by the first entry

$$\prod_{n=1}^{n} L_n \equiv L_1.$$

L_1 is either $\perp$ or T, and

$$M \bullet \perp = M, \qquad M \bullet T = M,$$

which proves (a).

Observing that

$$\perp \bullet M = 0, \qquad T \bullet M = 0,$$

we derive the commutators

$$[M, \perp] = M, \qquad [M, T] = M,$$

and anticommutators

$$\{M, \perp\} = M, \qquad \{M, T\} = M,$$

Thus the supercommutators

$$\{M, \perp\} = M, \qquad \{M, T\} = M,$$

conserve **M**, which proves (b).

If no other nibbles except **TRUE** and **FALSE** participate in the thought process, the left logical momentum will be indefinitely preserved, but the right logical momentum reduces to a vacuum:

$$M \bullet \text{string} = M, \qquad \text{string} \bullet M = 0.$$

COGNIZERS

Cogito ergo sum.

René Descartes

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STRETCHING NATURE'S LAWS

We know ourselves as thoughts and ideas and not as neurons and brains. The nature of a symbol is very different from the nature of the object it stands for. A debate whether the mind is phenomenal or physical has brought us nowhere, simply because the mind and the brain are the two aspects of one undivided whole. There are various aspects of this undifferentiated reality. It is possible to localize the brain in space and in time but the thought process exists only in time. If cognizing phenomena are interpreted in temporal terms only, how then can the 'spatial' brain and the 'temporal' mind interact? One external solution is offered by relativity theory, which taught us that there are no events in space only, and no events in time only. Then both the brain, an object, and the mind, a symbol, can be treated as elements of undivided spacetime. Another intrinsic solution, which we have chosen, is topological. Cognizing is a topological effect and topology figures in the background of all the structures related to consciousness which we investigate in this study. In the topological framework, it is possible to reformulate the problem: the mind knows itself as a topology and not as a geometry.

The laws of topology connect logical thoughts to the brain. The brain in turn is tied to physical degrees of freedom, and therefore to the laws of physics. As a result consciousness mirrors in its theoretical structure the structure of physical reality. One may choose first to investigate reality and then seek a corresponding theory of consciousness. One can also first construct a theory of consciousness and then verify its correspondence to physical law. Both ways are legitimate, as long as both are taken as parts of one unified whole.

Cognizers are universal natural mathematicians, and in a sense consciousness is pure mathematics. When degraded to the level of communication, it takes the form of a language, losing in the process most of its distinctive features. However, the higher macroscopic level of cognitive topology is constructed from the quantum primitives of the lower level, which possess nontrivial and fundamental properties deserving attention in their own right. The important question is to determine how the logic of the higher level is translated into the quantum code of the lower level and vice versa. Understanding how discrete quanta merge into the continuum of consciousness and how the logical content is distributed among quantized carriers must allow

us also to gain greater insight into how the quantum level is converted into the classical level and back. In technical terms, we are seeking a fundamental equation one side of which must be extended in infinite-dimensional Hilbert space while the other side is confined to the finite logical interval:

$$\int_{H} (\text{Quantum field})\,dx = \int_{\Omega} (\text{Logical field})\,dq.$$

If such an equation, linking the physical brain with its logical content, could be explicitly formulated, it would relate the quanta and information in a way quite unlike anything previously encountered in physics. Ultimately it should give us a fundamental theory of the cognizing mechanism which can be experimentally tested. This is not an easy problem to solve and these are not easy questions to come to grips with. Clearly, one expects such a theory to be inherently relativistic or Poincaré-covariant, hence not dependent on particular frames of reference or fields. Logic must enter physics at the fundamental level, and even may entail an explanation of quantum mechanics as a derived theory.

To advance the argument we must bring again into focus the key idea of duality, which with regard to consciousness entails a controlled conversion between the Hilbert space of quantum field theory and the corresponding matrix logical space. For too long cognitive neuroscience has looked for a deep principle where there is none. Neither classical theory alone nor the quantum concept alone explains consciousness. Cognitive dynamics must consequently embrace both sides of the classical-quantum divide, linking the problem of consciousness to the fundamental question of classical-quantum interface.

CLASSICAL-QUANTUM INTERFACE

We may have to use a collection of different theories, that are valid in different regions, but which agree on the overlaps.

Stephen Hawking

Since the early days of quantum mechanics the elusive border between macrophysics and microphysics has been the subject of much controversy and intensive theoretical and experimental study. Thus far neither classical nor quantum mechanics has offered a completely satisfactory account as to why and by what means at this strange border the coherencies of complex probability amplitudes must abruptly give way to the ordinary probabilities of Hamiltonian dynamics and commuting quantities must be replaced by noncommuting ones. It is an indisputable fact that in crossing over the

quantum-classical divide the wavefunction formalism has to be replaced by a completely different set of classical equations. But shouldn't we pass through a certain mesoscopic state during this process where both descriptions momentarily coincide? Shouldn't there be a particular interface region where decoherence is already present but not yet strongly enough to fully enforce a macroscopic classical limit? The formalism of matrix logic gives an affirmative answer to this question and illuminates the fact that a complete theory of consciousness must combine logic and quantum mechanics. Technically speaking, the brain must be able to accept and to process quantum and classical information while in the quantum and classical regimes respectively.

We have suggested here that crossing through the quantum-classical divide constitutes the essence of the duality connecting logic and quantum mechanics. It makes much sense to put this result into a wider perspective, since many interesting applications become available in various theoretical and experimental setups. These include quantum field theory, mesoscopic systems, the semiquantization problem, quantum chaos and quantum computing.

The concept of duality has been in the physicist's lexicon since the late nineteenth century when Maxwell discovered the duality of electric and magnetic fields. In recent years duality has gained new prominence in connection with supersymmetric fields and the quark confinement problem. In string theory dualities promise to accomplish a long-sought superunification. The duality of matrix logic and the phase space of quantum mechanics indicate that the extension of the duality approach to from the purely physical scene into the domain of information is mutually beneficial to both enterprises. Information is physical, physics is logical. An observer (measurement) and to some even the observer's mind are important ingredients of physical theory. However, information symmetries, with few exceptions, have remained largely unexplored by modern physics, even though the information properties of matter and fields figure prominently in quantum and relativistic physics, providing impetus even for probing the quantum structure of information itself.

The possibility for the transfer of quantum degrees of freedom to the domain of classical information suggests that coherent superpositions are not exclusively microphysical (quantum-mechanical) and can, in fact, be made compatible with the macrophysical laws of composition in informational phase space. Mesoscopic Schrödinger cats, which possess both macroscopic and microscopic features, can provide a testing ground, in this way not only shading light on the controversies surrounding the fuzzy boundary between macro- and microworlds, but also fostering our understanding of the dual relation between quantum mechanics and logic which we discussed.

While classical dynamics is characterized by commuting coordinates and momenta, matrix logic comprises both commuting and noncommuting elements. The classical degrees of freedom, correlated with a quantum ones, congruently exhibit quantum behaviour through coherent superpositions, phase

inversion and entanglement. It must be emphasised that this quantization is explicitly dictated by fundamentals, needed to retain the consistency of the theory. Reversible classical-quantum transitions are theoretically tractable in the brain and can be experimentally explored. At the classical-quantum interface the quantum and classical degrees of freedom can be treated within one general formalism. These findings are relevant for analysing the conditions under which a measurement on a quantum system can be reversed.

QUANTUM HYPOTHESIS OF THE BRAIN

For a quantum theorist the brain is a part of the physical world and since the world is quantum-mechanical, so should the brain be at bottom. The extraordinary success of quantum mechanics leaves no doubt in anyone's mind about its validity. We have grown accustomed to, if not indoctrinated by, the all-powerful dogma of the wavefunction. It is viewed as a primary concept of quantum physics and by some even of physics in general, which would then include the physics of the thinking brain. Since its inception in the 1920s much effort has been invested in understanding the meaning of wavefunction. The majority of physicists believe that wavefunction is the foundation for resolving fundamental physical problems. The question of concern to us is not whether quantum mechanics is complete or incomplete in encompassing all of reality but whether it complete or incomplete in encompassing the reality of logic.

Many quantum concepts seem absurd when related to the classical realm of experience. When the linear superposition of quantum mechanics is extrapolated to the macrolevel, we are confronted with counterintuitive cat states. This clash between common sense and the prediction of quantum theory gives rise to the question whether logic is incorrect or wavefunction is not universally inapplicable. To retain the validity of quantum theory some suggest that quantum decoherence is responsible for the absence in the macroworld of the Schrödinger's cats. Decoherence results from an irreversible coupling of the quantum system to the macroframe. The off-diagonal elements in the density matrix are consequently cancelled, making information on the system classically interpretable. Quantum coherence, distinguished by the nonzero off-diagonal elements in the density matrix, in contrast, makes a classical interpretation impossible. This approach has advantages, but leaves unresolved the question whether quantum mechanics can provide a complete description of reality.

After the development of quantum mechanics many of us were caught up in the excitement and the belief that quantum theory might also explain the mystery of the mind. Striking similarities found between the thought process and the general quantum process gave rise to the quantum hypothesis of the brain functions which claims that consciousness reflects quantum-mechanical aspects of the matter of which our brains are made [Ref 73]. Such distinctly quantum-mechanical features as indeterminism, spontaneous transitions,

interference and tunnelling effects are equally well applicable to quantum processes as to the brain. Logical process appears to be to the general thought process what the classical limit is to the general quantum process. However, balancing the books at the end of the day we must admit that the quantum concept of the brain has fallen short of our expectations. A legitimate concern of the opponents of the quantum model of the brain is that it has failed to formulate meaningful predictions that could either vindicate or disprove the quantum approach.

Do we really need the quantum hypothesis to understand the brain? Since the birth of quantum mechanics many physicists, including myself, have thought of consciousness as being quantum-mechanical. At issue is a nontrivial question: can the laws of the logical brain be formulated without reference to wavefunction? The idea that there might be a reality which is not necessarily described by wavefunction has been rejected since Albert Einstein lost in the famous debate with Niels Bohr. It is also a general consensus that coherent superpositions, which lie at the heart of quantum mechanics, necessarily require the formalism of wavefunction. An unexpected bombshell, showing that this commonly held view is incorrect, exploded in matrix logic: the superposed mode of the thought process can be adequately accommodated without reference to wavefunction.

To get to the bottom of things we must consider **two fundamental functions of probability**: quantum probability amplitude, which is **complex-valued**, and tensor probability, which is **real-valued**:

$$|\psi\rangle \quad \text{and} \quad |\zeta\rangle$$

For two-state systems we then have two different rules of normalization. A quantum system, such as a spin-1/2, obeys the quadratic rule for complex amplitudes:

$$|\alpha|^2 + |\beta|^2 = 1.$$

A classical two-state system obeys the linear rule of normalization:

$$\bar{q} + q = 1$$

where $\bar{q}$ and q are ordinary probabilities. While in quantum mechanics, one deduces probabilities from probability amplitudes, in matrix theory they are explicit, making calculations unnecessary. In this scheme quantum and logical phenomena differ by the criteria whether the quadratic or nonquadratic rule of normalization applies. Why would nature choose two different rules of normalization? Perhaps there are even other rules we are not aware of.

The question which naturally presents itself is: what is the relationship between these two probability functions we are considering? Which function, if any, should be regarded as primary or fundamental? By answering these questions we can better understand the relation between cognitive logic and the Hilbert space of quantum theory. In a sense quantum physics is a theory of complex probability amplitudes. Since

$$\psi^*\psi = p$$

one can define ψ as the **complex square root** of probability [Ref 96]:

$$\psi = \sqrt[i]{p}.$$

and choose the logical function $|\text{Ç}>$ to be a primary concept. But the components of $|\text{Ç}>$ itself can be obtained as the inner products which reverses the priorities, giving reasons to consider ψ more fundamental:

$$|\text{Ç}> = \begin{pmatrix} <\psi|\psi> \\ <\varphi|\varphi> \end{pmatrix}.$$

Serious issues emerge in this framework. Thus far quantum mechanics has been exclusively concerned with the implication

$$\psi \rightarrow p.$$

In the theory of cognizers we are concerned with the converse implication

$$p \rightarrow \psi,$$

or more generally with the symmetry that exchanges truth-values and complex amplitudes

$$q \leftrightarrow \psi.$$

Such symmetry becomes instrumental if we accept the idea that the wavefunction of the cognitive brain can be altered by the faculty of the mind. Inadvertently one reduces the problem of the thinking brain to the fundamental physical problem of the quantum-classical interface. The symmetry which exchanges H and Ç must connect quantum states to the continuum. A transition from the quantum to the classical level explains how discrete states merge into the continuum of consciousness. A close analogy may be found in a motion picture where the effect of motion results from presenting to the eye fixed images, each slightly different from the other. The stream of consciousness is in this sense a quantum illusion, similar to a motion picture, collecting the quantized inputs, from the outside and from within, into a continuous cognitive motion. The matter is complicated by the fact that the actual mechanism of consciousness is inaccessible to consciousness. The film is dark between the frames. Consciousness has no access to anything in-between the quantum levels because there is nothing in-between.

Considering quantum states and corresponding logical statements, our goal is to determine the interface at which the states and statements merge into cognitive *state-ments*. Irrespective of our success in achieving this goal, matrix theory reveals that uncertainty and coherent superposition in logic are macrophysical and can be adequately dealt without wavefunction. Matrix logic mixes classical and quantum theories, macrophysics and microphysics, in an unusual way. It is quantum-mechanical in form but classical in essence. Because logical operations are represented by definite integer matrices, one may expect that that every Boolean state is taken to another Boolean state and

not to a superposition of states. However, our intuition is grossly at fault here. The interim logical states often disobey the classical law of probability normalization, giving rise to coherent superpositions. As a result matrix logic allows new types of intelligent processing with unique and more powerful features which are unattainable if the brain is to be controlled exclusively by the classical laws.

We have suggested here that quantum mechanics can be derived from logic by complex square rooting. It may be hard to accept such an abstract foundation of physical theory. Few physicists would like to see consciousness dictating the laws of physics and quantum mechanics cannot simply be argued away because of an axiom. There must be unique predictions stemming from this matrix logical approach which one is able to test in a physical laboratory or in the 'cognitive laboratory' of the brain. In quantum mechanics, wavefunction and unitary evolution are the underlying notions. To abandon such an effective and successful theory in favour of another, one must have very serious reasons indeed. One such reason is the deviation from purely unitary evolution in the operations of the brain, where linearity has to be given up. Whereas the macroscopic Schrödinger cats remain the elusive and frustrating goal of quantum experimentalists, in the brain coherent superpositions abound and easily available on request, demonstrating that in the logical brain the essence of quantum principles is unraveled, perhaps even more clearly then in quantum physics itself. Thirdly, and most importantly, noncommuting matrix logical coordinates provide the effective formalism for the third quantization, closing a major gap between the quantum formalism and spacetime.

The dynamical equations of a physical system concern the possible states to which the system may evolve. The dynamical equations of a cognitive system concern the possible statements the system may yield. Physicists are very reluctant to accept dependence of a physical state on a cognitive statement. In spite of much evidence to the contrary, it is tacitly assumed that these are independent. The advantage of matrix logic, easing the psychological barrier, is that it is a theory in which the statements are at the same time the states of the system, becoming dependent and intertwined in a fundamental way. The duality principle, relating logical statements and physical states, casts new light on the problem of the connection between the brain and phenomenal experience. To achieve a scientific understanding of consciousness it is not enough to gather information about the physical states of the brain. There also can be no full understanding of the mind which relies solely on the logical machinery of manipulating cognitive statements. The fundamental theory must embrace the states and the statements in one integral whole. Because the physical basis of the brain at core is quantum-mechanical, it is compelling to think that the cognizing effect goes with its roots in the quantum domain, which would then make consciousness a derivative of the quantum. But the possibility of obtaining wavefunction as the complex square root of the logical function, entails an explanation of quantum mechanics as a derived concept. On

completely different grounds matrix logic reopens the debate which began with Einstein-Podolsky-Rosen's seminal paper of 1934. Wavefunction does not provide a complete description of reality, not only due to quantum nonlocality and inseparability, as revealed by the Bell theorem. An even greater challenge comes from the study of topological consciousness, which is essentially nonlocal and singular. The thinking brain delivers a major blow to the existing theory of physics. Will it lead us to a more general theory?

SINGULARIZATION THEOREM

When we cross from the classical to the quantum realm the commuting quantities are replaced by noncommuting ones. At the quantum-classical boundary decoherence is already present but not yet strongly enough to fully enforce a macroscopic classical limit. We find ourselves in a limbo where neither quantum nor classical formalisms apply and a new formalism is needed. In order to see that matrix logic offers such a unified formalism, we must tackle the fundamental question of comparison.

We are not always aware of the fact, but our understanding of the world comes from comparison, from comparing things or concepts. The measurements in a physical laboratory, as well as our ideas, values and even feelings are determined when we compare them, explicitly or implicitly, with the existing, right or wrong, standard, a rule, a dogma or a doctrine. Comparison, however, is not a trivial operation, although some of us would very much want it to be so. It is necessary to consider three different kinds of comparison: classical, quantum and mesoscopic. Classical comparison can be described simply as a direct subtraction of the comparing quantities:

$$A - B,$$

which distinguishes it from quantum-mechanical comparison, which is the Lie commutator

$$[A, B] = AB - BA.$$

Making use of this criterion one conjectures that at the mesoscopic boundary, where the classical and quantum realms meet, the two fundamental rules of comparison must merge, becoming indistinguishable. In mathematical terms this entails that at the classical-quantum interface the equality:

$$A - B = [A, B]$$

must hold. Taking A and B to be logical operators, the problem of identifying the mesoscopic interface is then turned into the problem of finding the suitable operators satisfying the above equality. We have determined that in matrix logic such operators are the knowledge operators **TRUE** and **FALSE**:

$$FALSE - TRUE = [FALSE, TRUE].$$

The fundamental role of these logical operators for the classical-quantum interface is explicit in the singularization theorem, to be proven next. According to the theorem the changeover between the logical momentum and

the quantum spin operator produces no observable effects. Thought tells the brain how to spin. Spin tells the brain how to think.

Theorem

The action of the logical momentum **M** in cognitive space $\mathcal{C}$ equals the action of the quantum spin operator σ in Hilbert space H:

$$\langle q|M|q\rangle = \langle\psi|\sigma|\psi\rangle$$

where

$$\mathbf{M} = [\mathbf{FALSE}, \mathbf{TRUE}]$$

Proof (I)

Expressing **M** as the sum of two operators, we can write:

$$\langle q|M|q\rangle = \langle q|\sigma + \blacktriangle|q\rangle = \langle q|\sigma|q\rangle + \langle q|\blacktriangle|q\rangle$$

or explicitly

$$\langle q|\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}|q\rangle = \langle q|\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}|q\rangle$$

$$= \langle q|\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}|q\rangle + \langle q|\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}|q\rangle.$$

Since

$$\langle q|\blacktriangle|q\rangle = 0$$

deduce

$$\langle q|M|q\rangle = \langle q|\sigma|q\rangle$$

Due to normalization:

$$\langle q|\sigma|q\rangle = \bar{q} - q$$

But

$$\langle\psi|\sigma|\psi\rangle = \psi^*(\uparrow)\psi(\uparrow) - \psi^*(\downarrow)\psi(\downarrow) = \bar{q} - q$$

where

$$\bar{q} = \psi^*(\uparrow)\psi(\uparrow)$$

and

$$q = \psi^*(\downarrow)\psi(\downarrow)$$

are the probabilities of states of a two-state quantum system, for example, of spin-up and spin-down respectively. Hence

$$\langle q|M|q\rangle = \langle\psi|\sigma|\psi\rangle$$

□

We have determined that the logical computation in $\mathcal{C}$ is in no way different in outcome from the quantum-mechanical computation in H:

$$\langle q|\,\sigma\,|q\rangle = \langle\psi|\sigma|\psi\rangle$$

Both sides in the equation compute the difference of *up* and *down* probabilities: on the lefthand side it is

$$\bar{q} - q = p(|0\rangle) - p(|1\rangle)$$

and on the righthand side it is

$$\langle\psi|\sigma|\psi\rangle = \psi^*(\uparrow)\psi(\uparrow) - \psi^*(\downarrow)\psi(\downarrow) = p(|0\rangle) - p(|1\rangle) = \bar{q} - q$$

Moving backwards and forwards between the logical space and the Hilbert space does not alter the output value of the bracket, which remains invariant.

Proof (II)

Since a scalar is unaltered by transposition, we have the equality of the **IMPLY** and the **IF** atomic brackets, transposes of each other:

$$\langle q|\rightarrow|q\rangle = \langle q|\leftarrow|q\rangle$$

or

$$\langle q|\,\mathbf{IMPLY}\,|q\rangle = \langle q|\,\mathbf{IF}\,|q\rangle\,.$$

Writing the operators explicitly as matrices we have:

$$\langle q|\begin{pmatrix}1&1\\0&1\end{pmatrix}|q\rangle \;=\; \langle q|\begin{pmatrix}1&0\\1&1\end{pmatrix}|q\rangle$$

The **IMPLY** and **IF**, can each be written as the sum of the two operators

$$\langle q|\,\mathbf{FALSE + AND}\,|q\rangle = \langle q|\,\mathbf{NOR + TRUE}\,|q\rangle$$

which is

$$\langle q|\,\theta\mathbf{TRUE + AND}\,|q\rangle = \langle q|\,\theta\mathbf{AND + TRUE}\,|q\rangle$$

Reshuffling the operators:

$$\langle q|\theta\mathbf{TRUE - TRUE}\,|q\rangle = \langle q|\theta\mathbf{AND - AND}|q\rangle$$

which is

$$\langle q|\begin{pmatrix}1&1\\-1&-1\end{pmatrix}|q\rangle = \langle q|\begin{pmatrix}1&0\\0&-1\end{pmatrix}|q\rangle$$

or

$$\langle q|\,\mathbf{M}\,|q\rangle = \langle q|\,\sigma\,|q\rangle$$

□

The operators on both sides of the equation is the difference of an operator and its unitary image:

$$\mathbf{M} = \theta\mathbf{TRUE - TRUE}$$

$$\sigma = \theta\mathbf{AND - AND}$$

Proof (III)

A proof of the singularization theorem can also be obtained as a consequence of the *secret* metric which establishes the equivalence of linear and pseudo-Euclidian logical intervals. The average of the logical momentum evaluates to

$$\langle M\rangle = \bar{q} - q.$$

The average of spin operator evaluates to

$$\langle\sigma\rangle = (\bar{q})^2 - q^2 .$$

But due to the normalization $\bar{q} + q = 1$,

$$(\bar{q})^2 - q^2 = (\bar{q} - q)(\bar{q} + q) = \bar{q} - q$$

and

$$\langle q| M |q\rangle = \langle q| \sigma |q\rangle$$

□

The equivalence of the pseudo-Euclidian interval $\bar{q}^2 - q^2$ and the linear interval $\bar{q} - q$ indicates a possible link between logic and relativity.

Theorem

The averages of the spin operator σ and the signed differentiation operator $-\frac{d}{dq}$ are equal:

$$\langle q| -\frac{d}{dq} |q\rangle = \langle\psi| \sigma |\psi\rangle$$

Proof

Since

$$\langle q| M |q\rangle = \langle\psi| \sigma |\psi\rangle$$

and

$$\langle q| M |q\rangle = \langle q| -\frac{d}{dq} |q\rangle,$$

deduce

$$\langle q| -\frac{d}{dq} |q\rangle = \langle\psi| \sigma |\psi\rangle$$

□

The term 'singularization theorem' comes from the fact that with conversion from the quantum to the logical realm the nonsingular operator σ is replaced by the singular M. This process is accompanied by the loss of distinct eigenvalues. Whereas the unitary spin operator σ possesses the complete set of orthogonal eigenstates:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

and the corresponding pair of eigenvalues

$$\lambda_{1,2} = +1, -1,$$

M is singular with the degenerate eigenvalue $\lambda = 0$ and the eigenstate which is a coherent superposition of the spin eigenstates:

$$|S_-\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \end{pmatrix}.$$

In identifying the conditions of the quantum-classical interface, the singularization theorem suggests that quantum-mechanical properties may be given to the mind directly. According to this hypothesis the thinking machinery of the brain is a macroscopic window into the quantum realm, directly revealing the quantum laws. Does consciousness provide the long-sought key to the strange world of quantum mechanics? The quantum theory of matter is based on the observation that elementary particles, such as electrons and protons, have wave properties. The wave-particle dualism is not observed in the behaviour of a macroscopic amount of material. In macrobodies the incoherent contributions of the large number of constituent particles obscure their wave nature. But in Bose-Einstein condensates, for example, each particle or atom is precisely in phase with every other. Quantum phases extend across the condensate and can be observed with the naked eye. Due to coherence the microscopic becomes macroscopic. Likewise, wavefunction and matrix-logical function refer to different processes, microscopic and macroscopic respectively. But according to the singularization theorem there is a fundamental interface where both quantum and classical solutions coincide, and the microscopic becomes macroscopic or vice versa. Consequently, the quantum elements can be incorporated into classical cognitive dynamics, while the logical elements can take part in a quantum evolution. This substitution or exchange of quantum and classical solutions is an important element of the theory of the brain presented in this monograph. Whenever a matrix-logical string contains the momentum product $\langle q|M|q\rangle$, it can be replaced by the quantum product $\langle\psi|\sigma|\psi\rangle$:

$$\ldots \langle x|A|y\rangle\langle q|M|q\rangle\langle z|B|s\rangle \ldots = \ldots \langle x|A|y\rangle\langle\psi|\,\sigma\,|\psi\rangle\langle z|B|s\rangle \ldots$$

The entanglement between the $\langle\psi|\sigma|\psi\rangle$ and the classical rest of the string in effect creates a cat state, a string comprising valid propositions where one might be a negation of the other. Matrix logic is naturally converted to the quantum-mechanical mode and the coupling of classical and quantum degrees of freedom is the fundamental feature of cognitive logic. The superposed states are strongly correlated with classical states, forcing a logical process to a definite result in which all nonclassical fluctuations are completely eliminated. Matrix-logical strings yield a Boolean output but logical states have a freedom to cross over from the macrophysical to the nonclassical regime which is fuzzy and quantized. This situation is similar to a macrobody which as a whole is deterministic and classical, but is made up of elements which are essentially quantum-mechanical and quantized.

Making use of the quantum-classical interface a quantum state can induce a classical state which we register cognitively, while a cognitive state is substituted by a spin state which is dealt with quantum-mechanically by the brain. Through quantum substitution the thought process has access to underlying quantum probabilities, and conversely, the quantum level can project itself onto the cognitive level. In a more abstract sense, the quantum

level acts as the differentiation operator on classical states, while the machinery of logical thinking, like a natural Stern-Gerlach apparatus, acts selectively on spins, giving them a particular orientation.
The singularization theorem is relevant for the quantum biochemistry of the brain where the large dimensionality of the system requires approximation, and for quantum computing, which requires controllable and reversable quantum-classical transitions. On a mesoscopic scale, where the classical and quantum realms overlap, the quantum and classical degrees of freedom can be treated within the same formalism, suggesting that in cognitive systems, under appropriate conditions, quantum decoherence can be reversed.

THE BRAIN-MIND INTERFACE AS A QUANTUM-CLASSICAL INTERFACE

The brain can register thought but not the physical processes underlying it. Laboratory measurements can give readings on physical quantities but cannot detect cognitive quantities, which are non-Hermitian. This is a new conceptual framework in which the brain-mind problem can be meaningfully analyzed. The notion of non-Hermitian observables brings new clarity to the question of cognizers. As we cross from the quantum realm to the realm of logic, the Hermitian and non-Hermitian quantities exchange their roles: what was observable quantum-mechanically becomes unobservable cognitively and vice versa, what was accessible to the mind becomes inaccessible to a physical instrument, however sophisticated it is or might one day be. Apparently, this is not a technological but a fundamental limitation.

This proposal passes many consistency tests. The singularization theorem is one of them, explicitly identifying the connection between non-Hermitian cognitive logic and Hermitian quantum mechanics. As we change over from the quantum side to the logical side of the equation, singularization of the operator is taking place: the nonsingular σ is replaced by the singular logical momentum $\mathbf{M}$. One can easily invert a spin back to identity, but a thought cannot actually turn backwards. Logical momentum is singular. What is significant is that, besides singularity, the operators in the fundamental equation

$$\langle q|M|q\rangle = \langle\psi|\sigma|\psi\rangle$$

also differ in another important respect which has direct bearing on our analysis. On the righthand side we have the Hermitian operator

$$\sigma^{+} = \sigma, \qquad \det \sigma = -1,$$

while on the lefthand side the non-Hermitian operator

$$\mathbf{M}^{+} \neq \mathbf{M}, \qquad \det \mathbf{M} = 0.$$

Even though $\mathbf{M}$ and σ are two fundamentally different entities, the equality holds, providing for a quantum-classical interface. In the logical momentum

the off-diagonal elements are undetectable, because the positive coherence is cancelled by precisely the same amount of a negative one. Put another way, **scaling down the normed probability components by squaring produces no change in their difference.** But the identified quantum-classical interface is also a Hermitian-non-Hermitian link, connecting quantum observables and 'nonobservable' logic, hence it also naturally emerges in the role of the brain-mind interface.

The singularization theorem reveals that the quantum-classical interface provides a solution to the problem of the brain-mind interface, delivering a major blow to the quantum hypothesis of the brain. The notion of observables is basic to physics where the quantities which we measure, such as position or momentum, must be represented by a Hermitian operator. But while quantum-mechanical momentum is a Hermitian quantity, logical momentum is not. Logical momentum, therefore, is not a quantum-mechanical observable and cognitive states cannot be observable quantum-mechanically.

The truth-value which we have been accustomed to thinking of as a scalar is in fact a nontrivial non-Hermitian quantity, which must be raised to the rank of an observable, measurable by the intelligent brain but not quantum-mechanically. To connect physics and the mind we must to start thinking quantum-mechanically, that is, to realise that the scalar Boolean true 1 and false 0 are the eigenvalues of the corresponding **TRUE** and **FALSE** operators which are singular (bi)antisymmetric and non-Hermitian. The scalar true 1 and false 0 are the logical quantum numbers associated with corresponding eigenstates of these operators. Even though matrix logic cannot be considered entirely quantum-mechanical, this introduces the quantum principle at the very foundation of logical theory. The mixed quantum-classical nature of matrix logic manifests itself in the fact that the eigenstates of **TRUE** and **FALSE** comprise both the classical and nonclassical superposed bits:

TRUE:

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \quad \lambda_1 = 1$$

and

$$|S_-\rangle = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \quad \lambda_2 = 0$$

FALSE:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \quad \lambda_1 = 1$$

and

$$|-S_-\rangle = \begin{pmatrix} -1 \\ 1 \end{pmatrix}, \quad \lambda_2 = 0$$

Theorem

The comparison of the **TRUE** and **FALSE** logical operators is the momentum operator:

$$\mathbf{FALSE - TRUE = M},$$

the comparison of their diagonalized forms is the spin operator:

$$\mathbf{FALSE}_{diag} - \mathbf{TRUE}_{diag} = \sigma$$

Proof

With the aid of the implications we diagonalize **TRUE** and **FALSE** by the similarity transformation:

$$\mathbf{IF} \cdot \mathbf{TRUE} \cdot \mathbf{IF}^{-1} = \mathbf{TRUE}_{diag} = \mathbf{AND}$$

which is

$$\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \mathbf{TRUE}_{diag}$$

and

$$\mathbf{IMPLY} \cdot \mathbf{FALSE} \cdot \mathbf{IMPLY}^{-1} = \mathbf{FALSE}_{diag} = \mathbf{NOR}$$

which is

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \mathbf{FALSE}_{diag}$$

But

$$\mathbf{NOR} - \mathbf{AND} = \mathbf{FALSE}_{diag} - \mathbf{TRUE}_{diag} = \sigma$$

Note:

Three fundamental operators in matrix logic, the antisymmetric time $\blacktriangledown$, the antipersymmetric spin σ and the (bi)-antisymmetric momentum **M**, satisfy the antiunitarity

$$\theta \mathbf{L} = - \mathbf{L},$$

indicating that the orthogonal rotation of time, spin and momentum changes the direction of time, the orientation of spin and logical momentum to its opposite.

COGNITIVE DELTA

The brain is neither classical nor quantum but both, indicating the existence of a whole variety of computational paradigms, not just classical or quantum. The continuous and discrete processes are intrinsically intertwined in cognitive operations. Neither classical logical truth-function alone nor wavefunction alone fully accounts for the thinking processes. Naturally, a theory capable of describing consciousness adequately must rely on a formalism which can consistently join macrophysical certainty and quantum indeterminism. The purpose of this section is to demonstrate that the Dirac delta function provides new insight into this problem.

Before we turn to technicalities, we give an example of perceptual quantization, which helps to understand that cognizing is quantizing, and also a sensitive high-resolution effect. Look at the picture below, focusing on the line in the middle and try to cross your eyes, until the two chaotic arrays of lines are fused. It is easy to obtain such a fusion in a psychophysical laboratory, but you can do without the equipment after some effort.

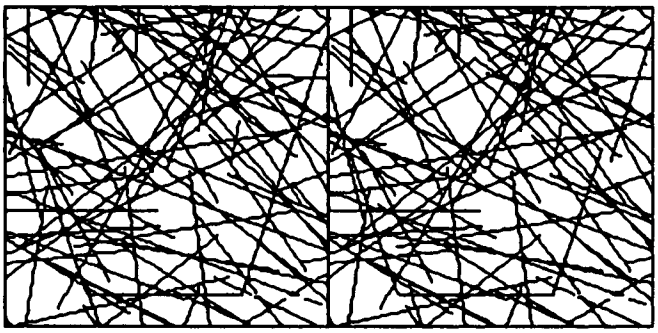


Fig. 22
Cognitive quantum effect.

When the two arrays of lines are fused by your brain, an abrupt change will occur. The figure of a pentagon jumps out of the flat picture into **3-D** and stays there, provided you exercise the necessary voluntary control. What your eyes do in **3-D**, the mind does in **4-D** and possibly in higher matrix dimensions. The cognitive elements are extended **n**-dimensional objects which consciousness controls and manipulates through topological transformations.

The cognizing effect is based on comparison and self-comparison, with classical and quantum mechanisms interacting in a coherent fashion. One meaningful way to deal with these interactions is by tracing over the classical-quantum tensor product

$$\mathrm{Tr}|q\rangle\langle\psi| = \langle q|\psi\rangle.$$

This computation is fundamental and takes a centre-stage in our analysis. There is, however, another way to treat the problem of the thought mechanism in the context of the field theory. With the aid of the singular delta function, the integration of classical and quantum logical elements takes literally the form of actual integration.

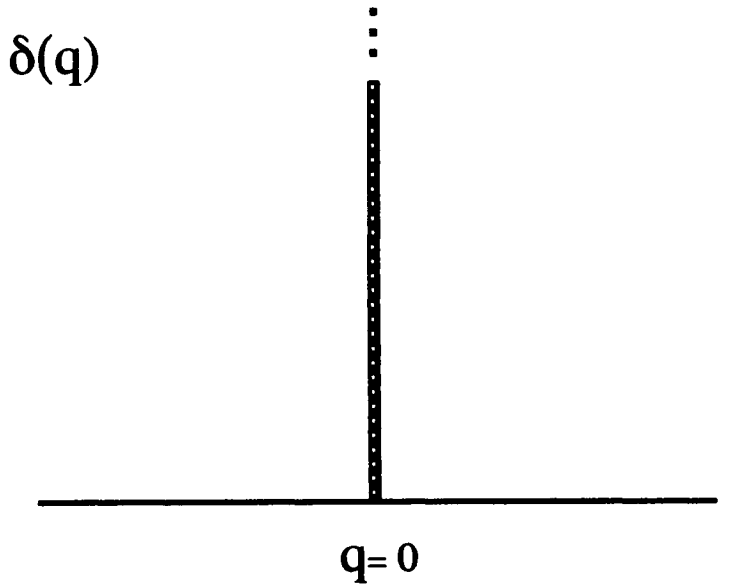
The delta function is not really a function but a distribution. It is defined not through the values of the argument as an ordinary function but by the rules of integration of the product of the delta function with a continuous function:

$$\int_{-\infty}^{+\infty} \mathbf{L}(q)\delta(q-\xi)dq = \mathbf{L}(\xi)$$

Under the integral we have a continuous function L(q) and a singular delta, which enables us to deal simultaneously with classical and quantum processes. The easiest way to understand the delta function is to consider a unit phase volume

$$v = \varepsilon \frac{1}{\varepsilon} = 1$$

With $\varepsilon \Rightarrow 0$, the volume converges to an infinitesimally narrow, infinitely high singular spike of the unit area. In this respect the Dirac delta function $\delta(q)$ is an extension of the Kronecker delta δ_{ij} into the infinite. The singular spike of the unit area is centred at $q = 0$, being zero at all other points:



We also introduce a logical complement of the delta function

$$\overline{\delta(q)} = \begin{cases} 0 & \text{for} \quad q = 0 \\ 1 & \text{for} \quad q \neq 0 \end{cases}$$

which is a negative mirror image of Dirac's delta $\delta(q)$, that is, an infinitesimally narrow puncture of continuous space at q=0 where the function collapses to zero, while at all other points it is the infinite unity:

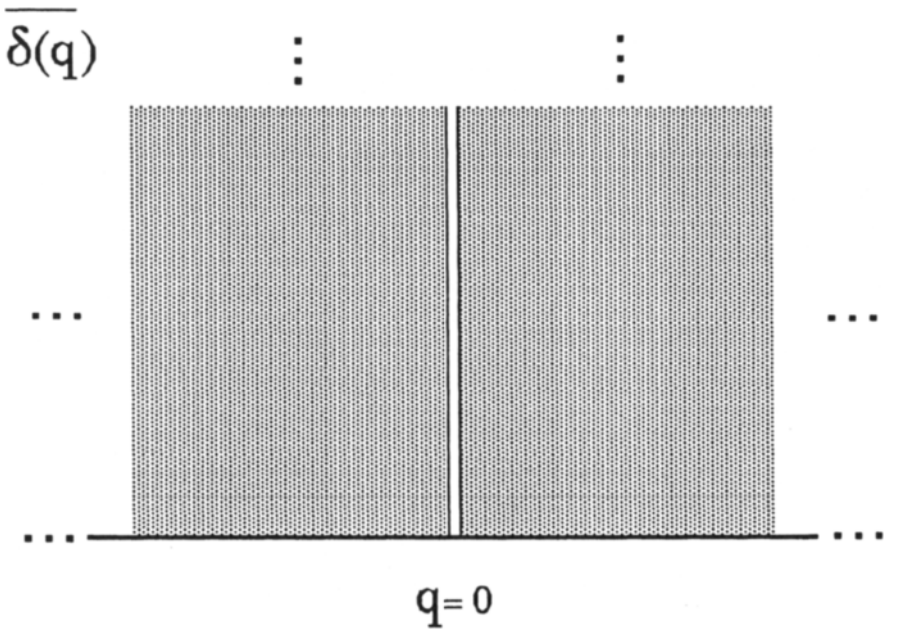


Fig. 23
The complement of the cognitive delta.

The cognitive delta can be compared with a photon emitted into an empty space, and its complement with a neutrino traversing a space filled with matter. In some sense, thoughts for brain is what neutrinos for universe. Although a quantum theorist may use the delta function in both instances, the neutrino traversing a filled space is very different from a photon traversing a vacuum.

With the formalism of the cognitive delta function we intend to answer several critical questions. One of them is how the classical behaviour of the brain can be controlled and monitored from the quantum level. In quantum mechanics we are typically concerned with the influence of macroscopic systems, a measuring device, on a microscopic system. The reverse problem, how a microscopic system influences a macrophysical system, is commonly ignored, primarily because the formalism of quantum theory has little to offer in this respect. However, quantum chaos theory has taught us that in systems with complex organization, an insignificant microscopic change in the input variable can cause a huge alteration of the output.

Let us characterize a cognitive logical state by a truth-functional volume which is the product of its false and true components:

$$V(|q\rangle) = \text{verum} \bullet \text{falsum}$$

This volume must not be thought of as having any significance in real space, but as a label varying continuously in fuzzy logic and constant in Boolean logic. Restricting ourselves only to discrete logical states, we have

$$V(|q>) = \begin{cases} 0 & \text{for} \quad |q> = |x> \\ 1 & \text{for} \quad |q> = |s_{\pm}> \end{cases}$$

The cognitive delta describes the instant jump between the classical volume which is 0 and the quantized volume which is 1:

$$|x> \Leftrightarrow |s_{\pm}>$$

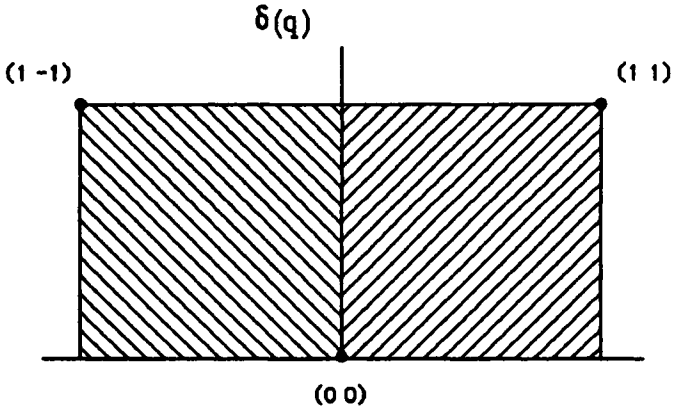


Fig. 24
Two spikes with unit volumes corresponding
to the singularity points (1 1) and (1 -1).

We associate Boolean states with the zero volume, and superposed states with the unit volume. The thought process proceeds as an alternating sequence of **bangs** and **crunches**, with a creation of the unit universes (which in the complex plane are the eigenstates of time) followed by their collapse to origin (zero). We now can remodel this singular behaviour in terms of the delta function which also explodes from zero value into an infinite unit spike. Cognitive states are the singularity spikes of the Dirac delta. This burst is a discontinuity converts a Boolean state into a superposed state.
As we mentioned, the delta function is defined by the rules of integration with continuous functions that can be scalar, vector or tensor. In this fundamental picture the cognitive operations integrate a continuous convex with the isolated logical points. Considering the integral of the product of the delta function and the logical momentum we get:

$$\int_{-1}^{+1} \mathbf{M}(q)\delta(q-\xi)dq = \mathbf{M}(\xi) = \begin{cases} 1 & \text{if} \quad \xi = 0 \\ -1 & \text{if} \quad \xi = 1 \end{cases}$$

where the delta function acts as a singular switch, determining the orientation of the logical momentum:

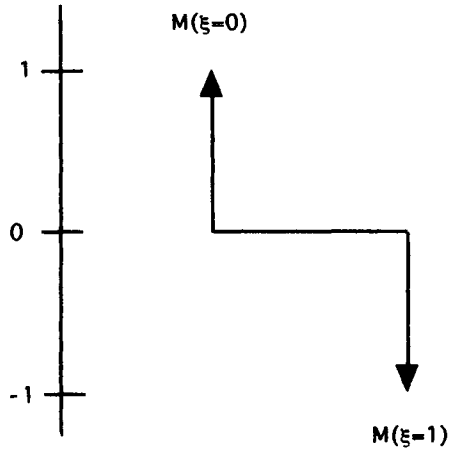


Fig. 25
The orientation of the logical momentum
is controlled by the cognitive delta.

The role of the cognitive delta in the thought process is to monitor continuous logical functions and derandomize them to the discrete values. In this sense the quantum level exerts a control over the classical domain.
We will make use of some properties of the delta function. Under the integration sign the delta function is even, $\delta(-q) = \delta(q)$, and

$$\int_0^q \delta(q)dq = \begin{cases} 1/2 & \text{if} \quad q > 0 \\ -1/2 & \text{if} \quad q < 0 \end{cases}$$

The derivative $\delta'(q)$ is odd, and

$$\int L(q)\delta'(q)dq = -L'(0)$$

Clearly

$$q\delta(q) = 0$$

which for a logical variable implies

$$\bar{q}\delta(q) = (1-q)\delta(q) = \delta(q)$$

But

$$q\delta'(q) = -\delta(q)$$

and from the last two equalities one derives

$$\bar{q}\delta(q) = -q\delta'(q)$$

and

$$\frac{\bar{q}}{q} = -\frac{\delta'(q)}{\delta(q)}$$

Theorem

Self-reference disables the logical delta function.

Proof

Consider the universal logical constants "true from anything"

$$<p|\rightarrow|1> \equiv 1$$

and "anything from false"

$$<0|\rightarrow|q> \equiv 1$$

The integration with the delta gives

$$\int <p|\rightarrow|q>\delta(q-1)dq = 1, \qquad \int <p|\rightarrow|q>\delta(p)dp = 1.$$

Then

$$\delta(q-1)dq = \delta(p)dp$$

and formally

$$\frac{dq}{dp} = \frac{\delta(p)}{\delta(q-1)}$$

In a condition of self-reference, like the Liar paradox, $|p>$ and $|q>$ are negations of each other. We therefore can rewrite the equation as

$$\frac{dq}{d\bar{q}} = \frac{\delta(\bar{q})}{\delta(q-1)}$$

The left hand side gives us

$$\frac{dq}{d\bar{q}} = \frac{dq}{-dq} = -\frac{dq}{dq} = -1$$

which entails

$$\frac{\delta(\bar{q})}{\delta(q-1)} = \frac{\delta(1-q)}{\delta(q-1)} = -1$$

and

$$\delta(1-q) = -\delta(q-1)$$

But since the delta function is even

$$\delta(1-q) = \delta(q-1)$$

the result will hold if and only if

$$\delta(q) \equiv 0$$

□

This result reveals the fundamental role of the delta function for consciousness. Self-reference disables the delta function, which in turn makes cognizing impossible, as we all know through our inability to join together the two mutually excluding outcomes such as the two statements of the Liar.

The logical complement of the cognitive delta is of paramount importance for logical inference. An inference codon is the complement of the logical vacuum

$$\overline{\alpha - \alpha}$$

Coupling the vacuum to $\delta(q)$ amounts to the addition of the logical nibble $+\alpha$ and the antinibble $-\alpha$:

$$\delta(q) = \alpha + \delta(q) - \alpha$$

Confining this compound under the complement, one obtains a quantized configuration of the inference codons

$$\overline{\alpha + \delta(q) - \alpha}$$

Substitution of the spike into a codon yields a contradiction, while the absence of a spike gives a tautology

$$\overline{\alpha + \delta(q) - \alpha} = \begin{cases} \text{Contradiction} & \text{for} \quad \delta(q) = 1 \\ \text{Tautology} & \text{for} \quad \delta(q) = 0 \end{cases}$$

THE DELTA THEOREM

In the beginning of this section we conjectured that the totality of the logical level must be related to the totality of the quantum processes in the brain:

$$\int_H (\text{Quantum field})\,dx = \int_\Omega (\text{Logical field})\,dq.$$

Making use of the cognitive delta function, it will now be our goal to determine the explicit form of this equation, connecting the logical mind and the quantum brain.

Theorem

The half-infinity integral of the delta function is universally equal to the matrix-logical truth integral:

$$\int_0^\infty \delta(x)dx \equiv \int_0^1 <p|q> d\mu$$

Proof

The delta side of the equation is associated with a many-body system of the quantum elements of the brain, and because the delta function is even,

$$\int_0^\infty \delta(x)dx \equiv \frac{1}{2}$$

This is evident in the Fourier representation of the delta:

$$\frac{1}{2\pi} \int_0^\infty dx \int_{-\infty}^{+\infty} e^{-i\omega x} d\omega \equiv \frac{1}{2}$$

To prove the theorem, we must prove that the integrals on the logical right side of the equation is also universally 1/2, independently of the variable of integration:

$$\int_0^1 <p|q>\,dq \equiv \int_0^1 <p|q>\,dp$$

The logical integrals over the unit interval must give 1/2, which is indeed the case:

$$\int_0^1 <p|q>\,dq = \int_0^1 (\overline{pq} + pq)dq = \int_0^1 (1 - p - q + 2pq)dq$$

$$= (q - pq - \frac{q^2}{2} + pq^2)\Big|_0^1 = (1 - p - \frac{1}{2} + p) = \frac{1}{2}.$$

We have chosen q as the variable of integration. Because the other variable p vanishes, the integral is independent of p. But since q and p in the integrand are fully symmetric, the same would apply to the variable q, if we chose p as as the integration variable. Then q vanishes and the integral again yields 1/2.

□

The cognitive delta theorem connects the Hilbert space of quantum theory and the matrix space of logic in a fundamental way. It allows us to convert naturally between the quantum and classical realms, that is between the brain, viewed as a many-particle system, and the logical system of consciousness. The imbedding of discrete spectra in a continuum (binding the continuum to the discrete eigenvalue) must lead to resonance states with nontrivial properties. When a quantum unit is embedded in a continuum with perturbation, in addition to the real poles, the continuum develops a complex pole. The resulting non-Hermitian term leads to oscillations which the brain observes as the logical momentum, not dependent on the truth-value q or false-value $\bar{q}$ alone but on their difference $\bar{q}$ - q. The momentum operator induces a quantization of states which undergo a nonunitary quantum jump due to the presence of delta function in the logical propagator. Another way of saying that cognizing essentially includes a singular element (quantized) is to say that the brain generates states of coherent superposition.

LOWERING THE ENERGY BARRIER: THE TUNNELING OF LOGICAL OPERATORS

The delta theorem allows one to convert naturally from the quantum to classical domain, that is from a many-particle system, such as the brain, to a logical system such as consciousness. It follows that systems which exhibit consciousness are necessarily many-particle complex systems. A system comprising just few a particles cannot realize consciousness. The logical integration is similar to the normalization of the wavefunction, where the integration is carried out over the complete Hilbert space of the problem:

$$\int_{-\infty}^{+\infty} <\psi^* | \psi > dx = \int_{-\infty}^{+\infty} |\psi|^2 dx = 1.$$

The truth integral is close in spirit to the path integral in quantum field theory, although it is determined by logical order rather than by time order. The path integral for a moving quantum particle represents the transition amplitude as a coherent sum over all possible histories. Likewise, the truth integral represents the integral logical charge of the affirmation function:

$$\int_0^1 <p|q> dq = \int_0^1 <p|\mathbf{YES}|q> dq$$

but also, implicitly, of the negation function:

$$\int_0^1 <p|\mathbf{NOT}|q> dq = 1 - \int_0^1 <p|\mathbf{YES}|q> dq = 1 - \frac{1}{2} = \frac{1}{2}.$$

The independence of the truth integral on the variable of integration requires a system of strictly unitary operators, **YES** and **NOT**. Then and only then does the cancellation of the adjoint variable occur that eliminates all potential divergences, yielding an integer finite Boolean answer. To arrest the divergence of the path integral we require the correction terms which take into account virtual interactions of a particle with vacuum. The corrections of the truth integral concern logical renormalizations of **YES** and **NOT**. In the Boolean deterministic limit the transmultiplication law applies and the integrand in the truth integral would be written:

$$<x|y> = <x|\rightarrow|y><x|\leftarrow|y> = <x|\rightarrow\blacklozenge\leftarrow|y>.$$

However, generally the operators cannot cross over the bracket barriers and merge into a Hadamard product. The transmultiplication law, as it stands, does not apply in fuzzy logic:

$$\int_0^1 <p|q> dq \neq \int_0^1 <p|\rightarrow|q><p|\leftarrow|q> dq$$

However, it is possible to recover the equality, introducing a correction term (ct):

$$\int_0^1 <p|q> dq = \int_0^1 dq(<p|\rightarrow|q><p|\leftarrow|q> + ct)$$

where

$$ct = - <p|\overline{\rightarrow}|q><p|\overline{\leftarrow}|q> = - <p|a^*|q><p|a|q>$$

is the signed product of the annihilation and creation brackets.
When the perturbation effect is small and negligible, in the expansions for logical operator waves

$$e^{-L} = I - L + ...$$

higher-order terms can be neglected. But for a^* and a they are simply nonexistent, and for the chosen renormalization the truth integral can be reduced to the exact expression in terms of a^* and a only:

$$\int_0^1 <p|q>\,dq = \int_0^1 dq(<p|e^{a^*}|q><p|e^{a}|q> - <p|a^*|q><p|a|q>).$$

We have considered the basic case, but renormalization of the truth integral can take a very complex form, for example:

$$<p|q> = <p|\rightarrow|q><p|\leftarrow|q> - <p|a^*|q><p|a|q>$$

$$= <p|\{a,\vee\}|q><p|\{\vee,a^*\}|q> - <p|[\perp, \leftarrow]|q><p|[\top, \rightarrow]|q>$$

$$= <p|\{a,\vee\}|q><p|\{\vee,a^*\}|q> -$$

$$<p|[\perp,\{a^*,\vee\}]|q><p|[\top,\{a,\vee\}]|q>$$

$$= <p|\{[\top, \rightarrow],\vee\}|q><p|\{\vee,[\perp, \leftarrow]\}|q> -$$

$$<p|[\perp,\{[\perp, \leftarrow],\vee\}]|q><p|[\top,\{[\top, \rightarrow],\vee\}]|q>$$

$$= <p|\{[\uparrow \downarrow, \rightarrow],\vee\}|q><p|\{\vee,[\wedge\vee, \leftarrow]\}|q> -$$

$$<p|[\wedge\vee,\{[\wedge\vee, \leftarrow],\vee\}]|q><p|[\uparrow \downarrow,\{[\uparrow \downarrow, \rightarrow],\vee\}]|q>$$

$$= .....$$

The correction terms cast new light on the mechanism underlying intuition, which is unclear. It happens that we access new knowledge by direct spontaneous induction and through nonlogical means somehow learn from within. It is almost as though the system tunnels or jumps over a potential barrier. The transmultiplication theorem gives some clues. In matrix logic the brackets are not simply the punctuation, but are the potential barrier, confining the operator to a designated place in an atomic bracket. In the matrix string

$$<x|L_1|y><x|L_2|y> ...$$

the operators L_1 and L_2 act only on the logical states in the immediate vicinity, while the operator in the neighbouring atomic products is totally out of reach for the operator's action. However, a situation may arise when the brackets becomes 'transparent" to an operator, and the operator may tunnel to another bracket, just as a quantum particle crossing the potential barrier into a forbidden region. This effect can occur if and only if the pair of logical spaces are coherently synchronized (correlated). Then the operators can enter into a direct interaction:

$$\langle x|L_1|y\rangle\langle x|L_2|y\rangle = \langle x|L_1 \blacklozenge L_2|y\rangle,$$

where $L_1 \blacklozenge L_2$ denotes the Hadamard product of L_1 and L_2.
If the states are not Boolean, the correction term must be subtracted, creating an entanglement. The correction term of a term is the complement of the term, which exists if the operator sum L_1+L_2 does not contain zero matrix elements.

The rule governing the fusion of logical operators is normal ordering. We put the true bit 1 in correspondence to the spin-up $\uparrow$, and the false bit 0 to the spin-down $\downarrow$. Then a logical operator can be realised as spin-spin interaction, with 1 nested into $\downarrow\uparrow$ and 0 into $\uparrow\downarrow$. Each matrix element in a operator is a coupling of two spins, in total 8 spins per operator. The implication operators in such an encoding scheme become

$$\rightarrow \; = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} \downarrow\uparrow & \downarrow\uparrow \\ \uparrow\downarrow & \downarrow\uparrow \end{pmatrix}, \qquad \leftarrow \; = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} \downarrow\uparrow & \uparrow\downarrow \\ \downarrow\uparrow & \downarrow\uparrow \end{pmatrix}.$$

Transmultiplication fusion begins with the juxtaposition:

$$\rightarrow \blacklozenge \leftarrow \; = \begin{pmatrix} \downarrow\uparrow & \downarrow\uparrow \\ \uparrow\downarrow & \downarrow\uparrow \end{pmatrix} \blacklozenge \begin{pmatrix} \downarrow\uparrow & \uparrow\downarrow \\ \downarrow\uparrow & \downarrow\uparrow \end{pmatrix} = \begin{pmatrix} \downarrow\uparrow\downarrow\uparrow & \downarrow\uparrow\uparrow\downarrow \\ \uparrow\downarrow\downarrow\uparrow & \downarrow\uparrow\downarrow\uparrow \end{pmatrix}$$

For comparison consider the Hadamard product of **AND** and **OR**:

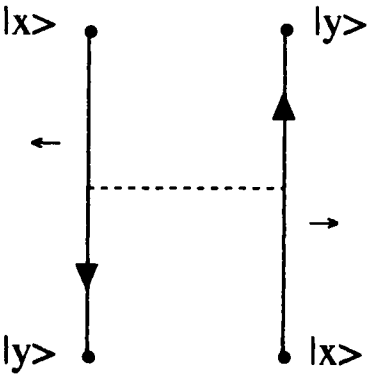
$$\wedge \blacklozenge \vee \; = \begin{pmatrix} \uparrow\downarrow & \uparrow\downarrow \\ \uparrow\downarrow & \downarrow\uparrow \end{pmatrix} \blacklozenge \begin{pmatrix} \uparrow\downarrow & \downarrow\uparrow \\ \downarrow\uparrow & \downarrow\uparrow \end{pmatrix} = \begin{pmatrix} \uparrow\downarrow\uparrow\downarrow & \uparrow\downarrow\downarrow\uparrow \\ \uparrow\downarrow\downarrow\uparrow & \downarrow\uparrow\downarrow\uparrow \end{pmatrix}$$

The evaluation of matrix elements in the juxtaposition is reduced to the evaluation of the normal product of the spin-up and spin-down operators. Since in normal ordering nonzero contributions come only from the alternating sequences

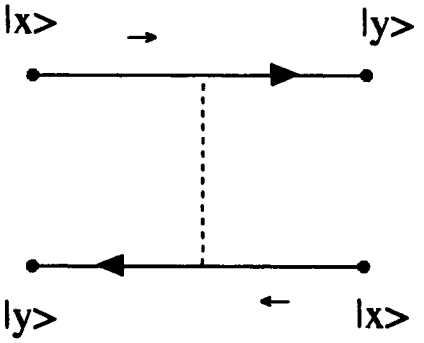
$$\downarrow\uparrow\downarrow = \downarrow \quad \text{and} \quad \uparrow\downarrow\uparrow = \uparrow,$$

any term containing the repetition sequence $\downarrow\downarrow$ or $\uparrow\uparrow$ is necessarily false and vanishes. This effectively reduces to zero the off-diagonal elements in both of the Hadamard products considered above. Irrespective of which operator enters the interaction bus first, the Hadamard product is commutative; on completion of the interaction the proper operator is emitted into communication channel.

A cognitive diagram corresponding to the strictly Boolean Hadamard product can be drawn as follows



where the dashed line corresponds to the interaction of operators. Since we are concerned merely with an output of the diagram, nothing prevents us from conceiving an equivalent but different diagram where a virtual interaction takes a centre-stage just like the creation of the electron-positron pair from a photon:



Since the unitary operations **YES** and **NOT** do not generate superposed states, this would imply absence of consciousness for the 'yes-no'-only logical system. Indeed, with **YES** and **NOT** one cannot form a functionally complete logic. A logical system realized in such an incomplete basis is in a sense only partially conscious. But as soon as we add the cognitive delta to the system, the creation of coherent states becomes possible and the cognizing mechanism can be enacted.

ENERGY FUNCTION

There is a fundamental connection between the problem of quantum measurement and the problem of the brain.The attempt to make quantum theory into a complete theory of physics runs into a major stumbling block: the unitary evolution of the Schrödinger equation does not contain anywhere within itself the mechanism which can cause the collapse of the

wavefunction. Consequently, the quantum equations require an agent external to quantum mechanics to perform the measurement. If consciousness collapses the wavefunction, where does the energy for this process comes from? Because the cognitive operations of the brain do not involve irreversible macroamplification, we have attempted to explain the cognizing mechanism as resulting from the acts of self-measurement [78]. **The physics of the brain is a physics of self-detection** where the brain 'observes' its own states, in contrast to the division into measuring device and measured system in canonical quantum theory. Exploring this idea, one remains within the framework of canonical quantum mechanics but the equation of motion for the density matrix is modified. In the equation

$$\frac{d\rho}{dt} + [H_x + H_y + H_{xy}, \rho] = 0$$

we retain only the interaction Hamiltonian H_{xy}, stripping the Hamiltonian of the remaining terms H_x and H_y associated with the measuring and measured systems respectively. Then the equation of motion takes the form:

$$\frac{d\rho}{dt} + [H_{xy}, \rho] = 0.$$

Although this approach was conceptually very useful for understanding some aspects of the cognizing mechanism, it did not establish the explicit form of the 'cognitive' Hamiltonian, and the physics of a problem cannot be clarified before the Hamiltonian is found. By and large the formalism of quantum mechanics fails to accommodate the elusive phenomena of cognizing, not surprisingly giving rise to the idea that a modification of the quantum concept might be necessary to accommodate consciousness. But if one should not force the model of the brain into the procrustean bed of quantum theory, which covariant theory is more suitable to account for the problem of cognitive energy?

The notion of energy is one of the central notions of physics. Energy exists in different forms: mechanical, thermodynamical, chemical, nuclear, rest mass energy, etc. Physicists have determined the exact transition coefficients for converting one form of energy into another. In fact the most celebrated equation of physics

$$E = mc^2$$

quantifies the conversion of mass into energy. As physics has grown more and more complex, much of the original intuition behind the concept of energy has been forgotten and is often taken for granted. Have we discovered all forms of energy? Although we all may have some vague intuitive ideas about the mental energy of the logical mind, it is a challenging task to define it in precise quantitative terms. This goal can be achieved by introducing the notion of (topo)logical or simply logical energy, needed to analyse mental dynamics and interactions in the brain-mind system. The idea of topological charges dispels the cloud of mystery which surrounds the phenomena of consciousness, in particular the question how cognitive actions can induce changes in the physical states of the brain.

It is necessary to make a distinction between logical truth-value charges and logical energy charges. In the classical limit truth-values are determined by the Boolean idempotency law:

$$x^2 = x \quad \Rightarrow \quad x = 0, 1$$

while the energy values obey the parallel-serial duality:

$$\varepsilon^2 = 2\varepsilon \quad \Rightarrow \quad \varepsilon = 0, 2.$$

The logical quantum numbers 0 and 2 are the eigenvalues of the logical energy operators **H** and **1**, which puts logical analysis in direct contact with Hamiltonian physics.

There are wider implications stemming from this approach. The logical evolution operator links the energy operators to the unbroken topological phase described by the Hadamard operators:

$$<q|.H|q> \pm <q|NOT|q> \; = \begin{cases} <q|1|q> \\ <q|H|q> \end{cases}$$

The unbroken phase is a Hadamard code which interacting with itself yields the 2 matrix

$$\widetilde{H}(\cdot)H(\cdot) = 2.$$

This fundamental property is preserved in the canonical remnants of the unbroken topological phase, the energy operators **H** and **1**, which too are nontrivial 2's. We have determined that topological energy characterises the amount of half-twists accumulated or released by a nonorientable manifold

$$T = 2\pi(n + \frac{1}{2}),$$

which puts it in direct and fundamental relation to quantum energy, identified by the quantum-mechanical Hamiltonian

$$H = \hbar T = 2\pi\hbar(n + \frac{1}{2}).$$

Comparing both equation, it is readily seen that the action quantum $\hbar$ can assume the role of as a fundamental transition coefficient

$$H = \hbar T,$$

suggesting an unusual new definition for $\hbar$, as the ratio of quantum and topological energies contained in a quantum information system:

$$\hbar = \frac{H}{T}.$$

It is important and must be emphasized that according to our scheme only systems with closed topology may accumulate and carry logical energy, such as the vacuum loop in quantum field theory, the atomic orbits, the rings in organic chemistry or in DNA or protein biomolecules which are curled into a closed structure. Having the action quantum $\hbar$ as a transition coefficient would preclude one from ascending to the macrolevel, for example of neurons wrapped into a closed circuit. When a closed loop is cut open, topological energy can be released in a way similar to the release of energy from a curled

rubber band. One prediction concerning topological energy is that the brain accepts it in quanta of 2. When acted upon by the synergy operators, the logical momentum either duplicates or reduces to zero:

$$\mathbf{HM = 2M}$$
$$\mathbf{1M = 0.}$$

Theorem

Logical synergies satisfy the equality for matrix exponents:

$$e^{A+B} = e^{A}e^{B}$$

which, as opposed to a number exponent, holds only on condition that A and B commute: AB = BA. Since

$$[\,\mathbf{1}, \mathbf{H}\,] = 0$$

it follows that

$$e^{1+H} = e^{1}e^{H}$$

which, neglecting higher-order terms, reduces to

$$e^{2YES} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 2 \end{pmatrix} = 3YES$$

$$e^{2YES} = 3YES.$$

We now consider several theorems, linking topological (logical) and quantum (physical) energies in the relativistic framework.

Definitions:

The formalism of matrix theory gives rise to a concept of logical energy which can be studied quantum-mechanically. In logical terms the energy average <E> is a product of the relative truth values:

$$\begin{aligned} <p|H|q> &= \bar{p}\bar{q} + pq - p\bar{q} - \bar{p}q = 1 - 2q - 2p + 4pq \\ &= (1-2q) - 2p(1-2q) = (1-2q)(1-2p) \\ &= (1-q-q)(1-p-p) = (\bar{q}-q)(\bar{p}-p). \end{aligned}$$

The logical energy is a quadratic function of the pseudo-Euclidian logical interval, reduced to the linear interval

$$<E> = ((\bar{q})^2 - q^2)^2 = (\bar{q} - q)^2$$

It is calculated by tracing the product of the density matrix Ω and the logical **Lagrangian** operator **H = YES - NOT**:

$$\begin{aligned} <E> = \mathbf{Tr}\,\Omega\mathbf{H} &= \mathrm{Tr}\left(\begin{pmatrix} (\bar{q})^2 & \bar{q}q \\ q\bar{q} & q^2 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\right) \\ &= (\bar{q})^2 - \bar{q}q - q\bar{q} + q^2 \\ &= (\bar{q})^2 - 2\bar{q}q + q^2 \\ &= (\bar{q} - q)^2 \end{aligned}$$

The energy function can be treated classically as a continuous function

$$\langle E \rangle = 1 - 4q + 4q^2$$

or quantum-mechanically as an operator whose exponentiation is the evolution operator **NOT**:

$$\mathbf{N} = e^{-\mathbf{H}}$$

Naturally **H** is the minus logarithm of the unitary negation operator

$$\mathbf{H} = -\ln \mathbf{NOT},$$

which is a logical evolution operator conserving the scalar product

$$\langle q|\mathbf{N}\mathbf{N}|q\rangle = \langle q|q\rangle$$

just as the quantum evolution operator U conserves the scalar product in Hilbert space

$$\langle \psi|U^*U|\psi\rangle = \langle \psi|\psi\rangle$$

The energy function in continuous logic is a differentiable function that runs through the unit interval

$$\langle E \rangle \in [0, 1].$$

If we restrict ourselves to the classical discrete Boolean logic, the energy is an isoenergetic function for all possible states of the system, be they true or false:

$$\mathbf{H}(|x\rangle) = \mathbf{H}(|\bar{x}\rangle$$

Hence, Boolean energy is a constant for which we employ a special notation

$$\mathbf{B} = \langle x|\mathbf{H}|x\rangle \equiv 1$$

In the nonclassical domain the logical energy is quantized into levels and its values should be obtained quantum-mechanically as the eigenvalues of the corresponding eigenstates

$$\mathbf{H}|q\rangle = \lambda|q\rangle, \qquad \lambda_{1,2} = 0, 2.$$

The eigenvalue $\lambda_1 = 0$ correspond to the eigenstate $|s_+\rangle$, and another eigenvalue $\lambda = 2$ to the eigenstate $|s_-\rangle$:

$$\mathbf{H}|s_+\rangle = 0|s_+\rangle, \qquad \mathbf{H}|s_-\rangle = 2|s_-\rangle.$$

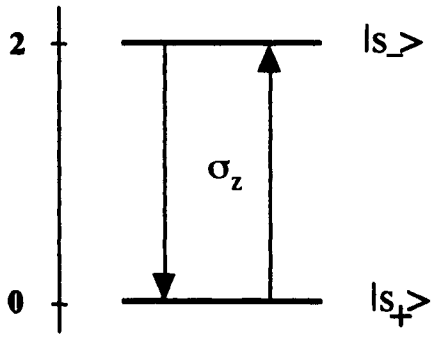


Fig. 26
Discrete levels of the logical energy function.
The spin operator swaps two eigenstates.

Since the system of eigenstates is complete and orthonormal

$$\langle S_+ \mid S_- \rangle = \delta(S_+ - S_-),$$

the energy operator can be diagonalized by the Hadamard codes:

$$\mathbf{H_{diag}} = .HH.H^{-1} = \begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}^{-1}$$

$$\mathbf{H_{diag}} = H.HH.^{-1} = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}^{-1}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

where

$$\mathbf{H_{diag}} = \begin{pmatrix} 0 & 0 \\ 0 & 2 \end{pmatrix} = 2\mathbf{AND}$$

A fundamental diagonalization of the energy operators is obtained by the time exponentiation:

$$\overset{\blacktriangle}{\mathbf{H}} = \mathbf{H_{diag}}, \quad \overset{\blacktriangledown}{\mathbf{1}} = \mathbf{1_{diag}}.$$

The motivation for expressing the **Lagrangian H = -ln NOT**, and selecting **NOT** as the evolution operator and not other unitary operators, like ▼ or σ, is clear from the following. We can dismiss σ as the evolution operator, since the corresponding Hamiltonian is **2AND**, i.e. the diagonalized form of **H**, which we have already derived

$$\mathbf{E} = e^{-2\mathbf{AND}}$$

and

$$(\langle q | e^{-2\mathbf{AND}} | q \rangle)^2 = \langle q | \mathbf{H} | q \rangle$$

The time operator ▼ is unitary

$$\langle q \blacktriangle\blacktriangledown | q \rangle = \langle q | q \rangle,$$

and can also be used as the logical evolution operator. However, the corresponding Hamiltonian is the Hadamard operator .H,

$$\blacktriangledown = e^{-.\mathbf{H}}$$

and the energy associated with .H is contained in <q|H|q>, making .H, in this respect, redundant:

Proof:

(a)

$$\langle q | \mathbf{H} | q \rangle = \langle q | q \rangle - \overline{\langle q | q \rangle}$$

(b)

$$\begin{aligned} \langle q | .\mathbf{H} | q \rangle &= 1 - 2q\bar{q} = 1 - 2q(1-q) \\ &= 1 - 2q + 2q^2 = (1-q)^2 + q^2 \\ &= \bar{q}^2 + q^2 = \langle q | q \rangle \end{aligned}$$

(c)

$$\langle .\mathbf{H} \rangle = \langle \mathbf{H} \rangle - \langle \mathbf{NOT} \rangle$$

It is now clear that we can determine all essential properties of the logical energy function by focusing only on the **Lagrangian H** and on the equations of motion in which it is involved.

THE MOMENTUM THEOREM

Phenomenological observations indicate that logical momentum results from the discrepancy between the physical fact and cognitive effect. Our goal now is to prove that the difference between logical energy $E_{logical}$ and quantum energy $E_{physical}$ gives logical momentum:

$$< E_{logical} > - < E_{physical} > = <M>$$

Specifically what we require is to prove the following relation

$$H - \frac{H}{\varepsilon_0} = M$$

where

H = YES - NOT is a logical **Lagrangian**

$H = \hbar\upsilon\,(a^*a + \frac{1}{2})$ is a quantum-mechanical Hamiltonian

$\varepsilon_0 = \frac{\hbar\upsilon}{2}$ is the energy of the vacuum zero oscillations

To obtain the proof of the relation, we take as a point of departure the logical momentum, expressed in the optimal spinor σ_z-code:

$$M = H\sigma_z$$

With the aid of the annihilation and creation operators, we get:

$$H\sigma_z = H[a, a^*] = H(aa^* - a^*a),$$

which in view of

$$\{a, a^*\} = aa^* + a^*a = I$$

reduces to

$$H(aa^* - a^*a) = H(I - a^*a - a^*a) = H - 2Ha^*a.$$

Making use of the quantum-mechanical Hamiltonian H we derive

$$H - 2Ha^*a = H - 2H\,(\frac{H}{\hbar\upsilon} - \frac{1}{2}) = H - \frac{2HH}{\hbar\upsilon} + H = 2H - \frac{2HH}{\hbar\upsilon}$$

Transferring factor 2 into the denominator and recalling that $2H = H^2$ this reduces to

$$2H - \frac{2HH}{\hbar\upsilon} = H^2 - \frac{HH}{\varepsilon_0} = H(H - \frac{H}{\varepsilon_0})$$

The equation for the momentum operator is now written

$$M = H(H - \frac{H}{\varepsilon_0}).$$

Comparing with the initial equation, this will be true on the condition that

$$\sigma_z = H - \frac{H}{\varepsilon_0}$$

Hence

$$< H > - < \frac{H}{\varepsilon_0} > = < \sigma_z >$$

But in view of the singularization theorem

$$< \sigma_z > = < M >,$$

we can write

$$< H > - < \frac{H}{\varepsilon_0} > = < M >$$

which is the discrepancy between the logical and quantum energies:

$$< E_{logical} > - < E_{physical} > = < M >$$

□

The term of the physical energy in our equation is the number of quanta acquired into or lost by the system

$$\frac{H}{\varepsilon_0} = \frac{n\varepsilon_0}{\varepsilon_0} = n, \qquad n \geq 0$$

and for the normalized system

$$<\psi|n|\psi> = n.$$

Creation and annihilation of quanta are two competing processes, described by the equations

$$a^*|n> = (n + 1)\,|n>$$

$$a|n> = (n - 1)\,|n>$$

The operator a^* increases the number of quanta by one, the operator a decreases the number of quanta by one. It is easily shown that

$$a^*|n> = \sqrt{n+1}\,|n+1>$$

$$a|n> = \sqrt{n}\,|n-1>$$

Then applying n times the operator a^* to the empty vacuum state $|0>$ yields the population

$$|n> = \frac{(a^*)^n}{\sqrt{n!}}|0>$$

Depending on the balance of creation and annihilation of quanta, the logical momentum can be positive

$$+ <M> = \bar{q} - q \qquad \text{if} \quad <H> > n$$

negative

$$- <M> = q - \bar{q} \qquad \text{if} \quad <H> < n$$

or zero

$$<M> = 0$$

if the system is in the state of equilibrium $\Delta E = 0$, implying also that

$$\bar{q} = q.$$

Particular interest attaches to the vacuum degeneracy $n = 0$ when the average of energy coincides with the modulo average of the momentum

$$\langle H \rangle = | \langle M \rangle |.$$

This decoupling from the physical component of energy is attainable in conditions of discrete Boolean logic, subject to the involution

$$(\bar{x} - x)^2 = |\bar{x} - x|$$

Indeed one expects the deterministic limit to occur when quantum number n is very large, $n \rightarrow \infty$, and not when $n \rightarrow 0$. This counterintuitive situation underlies the fundamental importance of classical binary logic which displays almost total independence on physical quantum entropy. "Almost" reflects the fact that modulo brackets are needed in the involution relation.

Our goal was to explain the mechanism of the generation of the logical momentum which, as we determined, arises due to discrepancies in (topo)logical and quantum-mechanical energies. In pursuing this goal we did away with the spin operator σ_z. However the result

$$\langle H \rangle - \langle \frac{H}{\varepsilon_0} \rangle = \langle \sigma_z \rangle$$

or

$$\langle H \rangle = \langle n \rangle + \langle \sigma_z \rangle$$

is significant in its own right, indicating that both the orbital momentum and spin contribute to the logical **Lagrangian**. In nonrelativistic approximation the Hamiltonian does not contain the spin operators and the wavefunction of the system ψ can be written as the product of the coordinate function φ, depending only on position variables and spinor function χ, depending only on spinor variables

$$\psi = \varphi\chi$$

or as a linear combination of such products. In view of the equality

$$\langle M \rangle = \langle \sigma_z \rangle,$$

and the nonequality

$$\langle H \rangle \leq \langle M \rangle,$$

if the spinor contribution to the logical **Lagrangian H** is positive, the nonspinor contribution must be negative

$$\langle \frac{H}{\varepsilon_0} \rangle < 0$$

and vice versa.

These results provide an important framework for experimental verification of the theory. Even though the logical momentum is non-Hermitian, and therefore is not subject to a physical, but only a cognitive reading, both physical and logical energies are represented by Hermitian operators, and are hence in principle can be experimentally measured. Logical eigenvalues must be embedded in a discrete physical spectrum. The differential form of logical momentum applies when the thought process is in continuous mode, which is not generally the case. For the discrete Boolean logic or when logical thought is quantized, the matrix form of the momentum operator

applies. Determining the value of logical energy, we can determine logical momentum, which are related quadratically:

$$<\text{ENERGY}> = <\text{M}>^2.$$

LOGICAL MOMENTUM AND THE KLEIN-GORDON EQUATION

Thought is a reality without reality, a Nicolas Bourbaki of cognitive science, which exists because it does not actually exist. Until now we have treated this intractability of the mental in the context of newly defined non-Hermitian observables. An even more important aspects emerge in the quantum-relativistic treatment of the logical momentum. Here we rely on the fact that any logical operator can be extended physically in the Pauli spinor base as a product of inversion, rotation and phase shift [Ref 91].
The Hamiltonian H, satisfying the Klein-Gordon equation

$$\frac{\hbar^2}{c^2}\frac{\partial\psi^2}{\partial t^2} = [\hbar^2\nabla^2 - m^2c^2]\psi$$

is given by

$$H = (\sigma_z + i\sigma_y)\frac{p^2}{2m} + mc^2\sigma_z$$

Exploring the fact that

$$\frac{dx}{dt} = \frac{1}{i\hbar}[x, H]$$

we get

$$\frac{dx}{dt} = (\sigma_z + i\sigma_y)\frac{p}{m}.$$

But

$$\sigma_z + i\sigma_y = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$$

is the logical momentum which has zero eigenvalues. In relativistic theory the eigenvalues of an operator, in our case the operator of velocity, do not always

correspond to possible results of measurement, which provides new clues to intractability of thought.

LOGICAL ISOSPIN

In the isospin theory the proton and neutron are treated as the two states of one particle, the nucleon, which nuclear interaction does not distinguish. Proton $|p\rangle$ and neutron $|n\rangle$ carry respectively the electric charges 1 and 0. We now introduce a model of a logical isospin in order to obtain the logical energy charges 0 and 2. The two states of the logical isospin are superposed states $|S_-\rangle$ and $|S_+\rangle$. Similar to the nucleon states, the superposed states are orthogonal

$$\langle p|n\rangle = 0 \qquad \text{and} \qquad \langle S_+|S_-\rangle = 0.$$

But in place of the unit closure relation for the nucleon:

$$\sum_{i=1}^{2} |x_i\rangle\langle x_i| = |0\rangle\langle 0| + |1\rangle\langle 1| = \mathbf{I}$$

the closure relation for logical isospin is the 2 matrix:

$$\sum_{i=1}^{2} |S_i\rangle\langle S_i| = |S_+\rangle\langle S_+| + |S_-\rangle\langle S_-| = 2\mathbf{I}$$

which is in direct relation to the Hadamard identities:

$$.H.\tilde{H} = 2\mathbf{I}, \qquad {}^{\cdot}H^{\cdot}\tilde{H} = 2\mathbf{I}.$$

The tensor products of the two orthogonal states of the nucleon are the charge creation and annihilation operators:

$$|p\rangle\langle n| = a^*, \qquad |n\rangle\langle p| = a,$$

$$a^*|n\rangle = |p\rangle, \qquad a|p\rangle = |n\rangle$$

As with the logical isospin we derive the nilpotent logical momentum operator:

$$|S_-\rangle\langle S_+| = M$$

and with the aid of the singularization theorem, can identify the spin operator, realizing supersymmetrically the dual role of the creation and annihilation of the superposed states:

$$\sigma_z|S_-\rangle = |S_+\rangle, \qquad \sigma_z|S_+\rangle = |S_-\rangle.$$

The pair of orthogonal states of the logical isospin can be extracted from an arbitrary normed state with the aid of the **edge** operators **M** and **1**: acting on any normed state creates a superposed state

$$\mathbf{M}|q\rangle = |S_-\rangle, \qquad \mathbf{1}|q\rangle = |S_+\rangle,$$

When the operators are iterated, the first nullifies and the second duplicates

$$\mathbf{M}^2|q\rangle = \begin{pmatrix}0\\0\end{pmatrix}, \qquad \mathbf{1}^2|x\rangle = \begin{pmatrix}2\\2\end{pmatrix}.$$

If spin intervenes,

$$\mathbf{1}\sigma_z\mathbf{1}|q\rangle = 0|S_+\rangle, \quad \mathbf{M}\sigma_z\mathbf{M}|q\rangle = 2|S_-\rangle.$$

The inner product of the superposed states is nil:

$$\langle S_+|\, S_-\rangle = 0$$

while the inner self-product is 2:

$$\langle S_+|\, S_+\rangle = \langle\, S_-|\, S_-\rangle = 2$$

In comparison with the nucleon, where the fundamental alternating triads are transposes

$$a^*\bullet a\bullet a^* = a^*, \qquad a\bullet a^*\bullet a = a,$$

for the logical isospin both triads result in the empty operator

$$\mathbf{1}\bullet\mathbf{M}\bullet\mathbf{1} = \mathbf{0}, \qquad \mathbf{M}\bullet\mathbf{1}\bullet\mathbf{M} = \mathbf{0}.$$

There is a fundamental connection between topological energy and spin. From the point of view of supersymmetry it is intriguing to speculate that the properties of logical isospin may somehow be connected to the properties of the graviton which carries spin 2.

Theorem

In the atomic product the spin operator σ_z is the fixed matrix point of the logical **Lagrangian H**:

$$\langle q|\mathbf{H}\sigma_z|q\rangle = \langle q|\sigma_z|q\rangle.$$

Proof

According to the singularization theorem

$$\langle q|\,\mathbf{M}\,|q\rangle = \langle q|\sigma_z|q\rangle.$$

According to the logical equation of motion

$$\mathbf{M}|q\rangle = \mathbf{H}\sigma_z|q\rangle,$$

which closing the bracket yields

$$\langle q|\mathbf{M}|q\rangle = \langle q|\mathbf{H}\sigma_z|q\rangle.$$

Hence

$$\langle q|\mathbf{H}\sigma_z|q\rangle = \langle q|\sigma_z|q\rangle$$

□

Collateral:

The average $\langle H \rangle$ of the logical energy $H = M\sigma_z$ is the product of averages of the logical momentum $\langle M \rangle$ and spin $\langle \sigma_z \rangle$:

$$\langle q|H|q \rangle = \langle q|M|q \rangle \langle q|\sigma_z|q \rangle$$

which entails that the average of the product $\langle M\sigma_z \rangle$ is the product of averages:

$$\langle q|M\sigma_z|q \rangle = \langle q|M|q \rangle \langle q|\sigma_z|q \rangle$$

Proof

The energy average is the squared average of the momentum:

$$\langle q|H|q \rangle = \langle q|M|q \rangle^2$$

But

$$\langle q|M|q \rangle = \langle q|\sigma_z|q \rangle,$$

Hence

$$\langle q|H|q \rangle = \langle q|M|q \rangle \langle q|\sigma_z|q \rangle$$

□

TRUTH-DEPENDENT LAGRANGIAN

The time parameter does not figure explicitly in logical equations but is expressed via the dynamical variables of truth-value or false-value. Can logical operators be made explicitly dependent on such values? In quantum mechanics the energy operator may be constant or vary, leading to the two different, time-independent and time-dependent, equations of motion. The logical Hamiltonian can also be expressed in dynamical terms as a matrix function of truth-value.

Theorem

The product of the momentum M, the density matrix Ω and the spin σ operators is the truth-dependent logical **Lagrangian**

$$\langle q|H|q \rangle = \langle q|M\Omega\sigma|q \rangle$$

where

$$M\Omega\sigma = \begin{pmatrix} \bar{q} & -q \\ -\bar{q} & q \end{pmatrix}$$

Proof

(1) Computational:

$$\langle q|\begin{pmatrix} \bar{q} & -q \\ -\bar{q} & q \end{pmatrix}|q\rangle = ((\bar{q})^2 - q\bar{q} \quad -q\bar{q} + q^2)\begin{pmatrix} \bar{q} \\ q \end{pmatrix}$$

$$= (\bar{q}(\bar{q} - q) \quad -q(\bar{q} - q)\begin{pmatrix} \bar{q} \\ q \end{pmatrix}$$

$$= (\bar{q})^2(\bar{q} - q) - q^2(\bar{q} - q)$$

$$= (\bar{q} - q)((\bar{q})^2 - q^2)$$

$$= (\bar{q} - q)(\bar{q} - q)$$

$$= (\bar{q} - q)^2$$

(2) Deductive:

The energy and momentum are related quadratically:

$$\langle q|\mathbf{H}|q\rangle = \langle q|\mathbf{M}|q\rangle^2.$$

According to the singularization theorem:

$$\langle q|\mathbf{M}|q\rangle = \langle q|\sigma|q\rangle$$

Then

$$\langle q|\mathbf{H}|q\rangle = \langle q|\mathbf{M}|q\rangle\langle q|\sigma|q\rangle$$

where the outer product $|q\rangle\langle q|$ is the logical density matrix Ω which proves the theorem:

$$\langle q|\mathbf{H}|q\rangle = \langle q|\mathbf{M}\Omega\sigma|q\rangle.$$

□

It must be emphasized that

$$\mathbf{H} \neq \mathbf{M}\Omega\sigma$$

and the truth-independent and truth-dependent **Lagrangians** are equal only in averages:

$$\mathbf{Tr}\,\Omega\mathbf{H} = \mathbf{Tr}\,\Omega\begin{pmatrix} \bar{q} & -q \\ -\bar{q} & q \end{pmatrix}$$

but

$$\mathbf{H} \neq \mathbf{M}\Omega\sigma.$$

THE MISSING ENERGY LEVEL

The fact that the spectra of logical energy are solved to the two eigenvalues 0 and 2, with the energy level 1 conspicuously missing, provides critical data for experimental verification of the matrix theory of the brain. One possible way to interpret this result is in terms of chemical bonding. It is a commonly held view that nuclei do not participate in life processes, and that only the electronic structure around them changes when biological life occurs. This is a faulty position. When the hydrogen atom is stripped of its electron, it

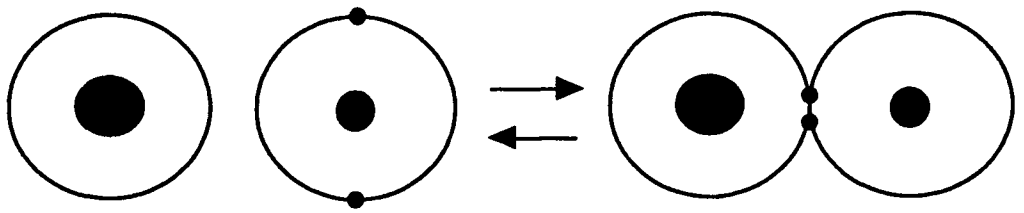
turns into a proton, which is an elementary nucleus, which plays an important role in biochemistry. The elementary chemical building blocks of the brain are the hydrogen **H**, and the isospin invariant atoms of phosphorus **P**, oxygen **O**, nitrogen **N**, and carbon **C**, with a particular system of valence electrons determining their chemical properties.

Fig. 27
The valence electrons of the elementary building blocks of the brain.

There are two principal ways in which the bonding of atoms occurs: ionic or electrovalent and covalent bonding. Ionic bonding is characterized by electron transfer, when one of the atoms loses one or more electrons, while the other atom gains one or more electrons. In covalent bonding, electrons are not transferred but are shared by two atoms. Excluding those atoms where the first shell is the only one present and two electrons form a stable structure, an atom generally reacts through the loss or gain of electrons in such a way as to secure an octet of electrons in its outer shell. For example, if we follow the octet principle, when oxygen undergoes an ionic reaction, each oxygen atom with electron configuration $1s^2 2s^2 2p^4$ accepts two electrons and attains a noble gas, the neon, configuration $1s^2 2s^2 2p^6$.

When two nonmetals react, a covalent compound is produced. The characteristic of this bond is that electrons are shared by atoms. Since nonmetals have high ionization potentials, they do not lose electrons in chemical reactions. Rather, a successful reaction between two atoms requires that both of the atoms have at least a slight attraction for additional electrons. Each of the atoms supplies one electron to the covalent bond which consists of a pair of electrons. For example, two atoms of hydrogen combine to form a molecule H-H, where the dash depicts a covalent bond. While the electrons must be considered as belonging to the molecule as a whole, each hydrogen atom can now be thought of as having the noble gas configuration of helium; this consideration is based on the premise that both shared electrons contribute to the stable configuration of each hydrogen atom. Covalent bonding includes a special type of covalent bond in which both of the shared electrons come from but one of the bonded atoms, the coordinate covalent bond. For example, for the coordinate phosphorus to oxygen bond, both of the bonding electrons are supplied by the phosphorus atom. Another example of the coordinate bond is the nitrogen to oxygen bond. The valence electrons of the nitrogen atom N can form a coordinate bond with O, which has one empty $2p$-orbit and can act as a donor. Instead of the dash an arrow is used to designate a coordinate

bond, pointing away from the atom that supplies the electron pair for the bond (the donor) and towards the atom that shares the electrons (the acceptor) but does not contribute them for bond formation: $N \rightarrow O$.
The concept of a coordinate bond is useful; however, undue importance should not be attached to this distinction. All electrons are alike, no matter what their source. What is important for us is the connection one can naturally make between the coordinate bond and the logical quantum numbers of synergy operators: $\lambda = 0$ and $\lambda = 2$. The first corresponds to the absence of a bond, and the second to the presence of a bond, requiring two electrons for its formation. This scheme adequately accommodates the loss of one energy level between the ground state and the occupied state. The system exists in two distinct eigenstates which agree with covalent coordinate bonding:



For the **Lagrangian H** a changeover from symmetric to antisymmetric eigenstate swaps the synergy logical number 0 for 2:

$S = 0$ - eigenstate $|S_+\rangle$

$S = 2$ - eigenstate $|S_-\rangle$

For the **Hamiltonian 1** the same changeover swaps 2 for 0:

$S = 2$ - eigenstate $|S_+\rangle$

$S = 0$ - eigenstate $|S_-\rangle$

Consequently, there are two ways to monitor dynamically coordinate bond formation and dissolution. One way is by changing the symmetry of the eigenstate, and the second is by changing the logical synergy operator.
Living systems excel in molecular recognition mechanisms. For example, smell receptors have the capability of reading out chemical structure of a chemical agent, giving a sensation of a particular smell. Alteration of electronic structure can induce conformal changes in protein molecules. A molecule will begin to behave 'intelligently' if molecular recognition is not addressed to another molecule but to itself, and the coordinate bond's dynamics can provide a basis for such a self-interpreting mechanism.
A promising candidate is the neuronal membrane which is a protein-lipid bilayer, with hydrophilic (water-attracting) groups facing outwards and hydrophobic (water-repelling) groups facing inwards. Proteins are linear strings of amino acids that fold into a complex three-dimensional shape. The ability of both structural and enzymatic proteins to recognize specific molecular patterns depends on topology, namely, shape complementarity that

is reminiscent of a lock-key mechanism. The switching action on proteins takes the form of making or breaking a covalent or coordinate bond. A sufficiently large macromolecule can be reversibly transformed from open linear topology to a closed topology by means of coordinate bond. As far as the physical chemistry is concerned nothing out of the extraordinary happens when the two ends of the macromolecule are connected by a bond. There is however an exact fit to logical energy, since two electrons have to be donated to form a connection, and the population is zero, no electrons, if there is no connection or when the connection is broken. The system *measures* itself with the electron-free end by 'testing' the presence of the pair of electrons at the other end. This process can deliver to the system information about itself which is essential for the cognizing mechanism.

BRAIN STATES ARE NONCLASSICAL

We have determined that the eigenstates of the logical energy operators are coherent superpositions which are essentially nonclassical. This fact has important repercussions. Quantum unitary evolution ends in the act of measurement, inducing the change from *quantum to classical*:

$$\hbar \Rightarrow 0, \quad \rho^2 \Rightarrow \rho$$

There is nothing in the formalism of physical theory which accommodates the inverse transformation from *classical to quantum*:

$$0 \Rightarrow \hbar, \quad \rho \Rightarrow \rho^2$$

In marked contrast, matrix logic naturally describes both the conversion from *classical to quantum* and from *quantum to classical*. When a classical state evolves into a quantized state this is closely analogous to the capture of a particle by an atom. The energy spectra of a free particle is continuous and to an extent can be treated classically. But as soon as the particle is confined to an atomic structure, the change from classical to quantum-mechanical description becomes unavoidable. The energy of the particle in the atom can assume only discrete values corresponding to eigenstates of the system and must be represented by an operator. The dynamical matrix logic describes a similar process. The brain acts as if it were a 'macroscopic atom', taking in classical logical states and quantizing them into levels. Bilateral topology closes into unilateral nonorientable topology. The topological current, propagating along

a logical loop, is similar to an electron spinning on atomic orbit, only now the 'atom' involves macroscopic numbers of elements, be they electrons, atoms or neurons.

A fundamental thinking unit is a logical unit which can reside in the classical pure states

$$|1> = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad \text{or} \quad |0> = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

in which complete information about the states is available, or the mixed states with a deficit of information,

$$|q> = \begin{pmatrix} \bar{q} \\ q \end{pmatrix}$$

where

$$\bar{q} + q = 1.$$

An arbitrary classical state is a superposition of the pure states:

$$q \cdot |1> + (1-q) \cdot |0>.$$

If we use the probabilistic interpretation of states, the physical operations, usually defined only for classical states, are naturally extended to quantized states when we find the unit to be in the states

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

which are **True and False** and **Neither True Nor False** states, uninterpretable in terms of classical probability.

To advance this argument further let us consider the behaviour of an atomic device of matrix logic $<x|L|y>$ and take the argument states $|x>$, $|y>$ to be the normed states, subject to the laws of classical probability. Due to the associativity of matrix multiplication, the evaluation of the product $<x|L|y>$ can be accomplished in a different order. Without loss of generality let us consider particular atomic products, where one computation is strictly Boolean, while the other gives rise to the states of coherent superposition:

$$<0|\mathbf{OR}|1> = \begin{cases} <1|1> = 1 \\ <0|S_+> = 1 \end{cases} \qquad <0|\mathbf{IF}^{-1}|0> = \begin{cases} <0|0> = 1 \\ <0|S_-> = 1 \end{cases}$$

Even though the states $|x>$ and $|y>$ were taken to be the classical states, the status of $|y>$ changes dramatically under the impact of the operator. What is particularly surprising is that the classical and nonclassical reductions, although internally different, give the identical output. In this manner matrix theory reveals the presence of nonclassical elements in logic, presumed to be

fully deterministic and classical. With the exception of the unitary **YES** and **NOT** and the singular **TRUE** and **FALSE** all matrix logical operators may evolve a Boolean state into a coherent superposition or to collapse it to the origin, the empty state. Because any sufficiently complex thought process involves such superposed undetermined states, this suggests that the corresponding brain states must be considered nonclassical.

The conclusion that logical operations comprise hidden quantum elements has far-reaching implications. Till now it was tacitly assumed that a logical deduction is deterministic throughout. But classical logic in fact conceals within itself, and can make use of, coherent superposition, an essentially nonclassical feature. Moreover, a thinking systems has the option of choosing between the classical and the quantum modes of operation. In classical theory, the entire thought process - the formulation of a problem, the evaluation and the solution - resides in the classical domain where the coherent superpositions that are characteristic of quantum theory do not occur. But in matrix theory any nontrivial logical problem propels the cognitive system into quantized states, even though the brain's final destination is back in the realm of classical logic.

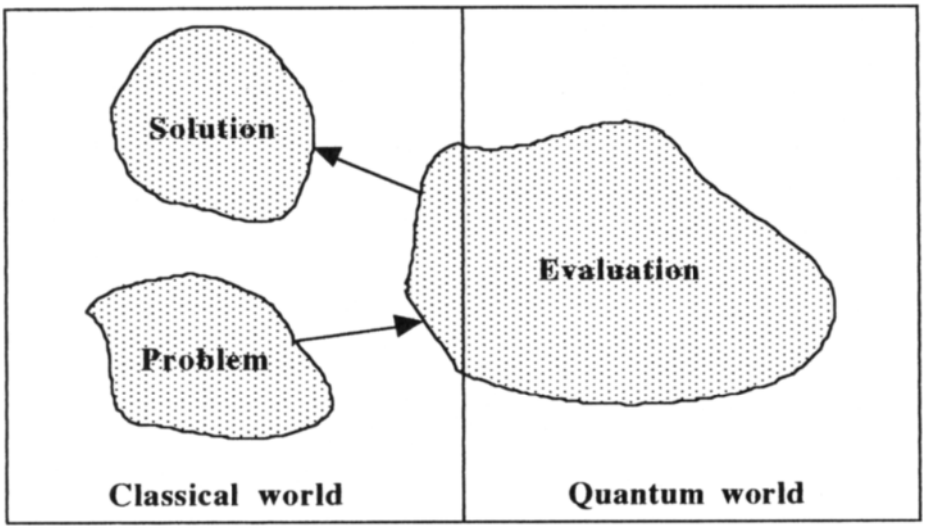


Fig. 28
Classical and quantum components of logical thinking.

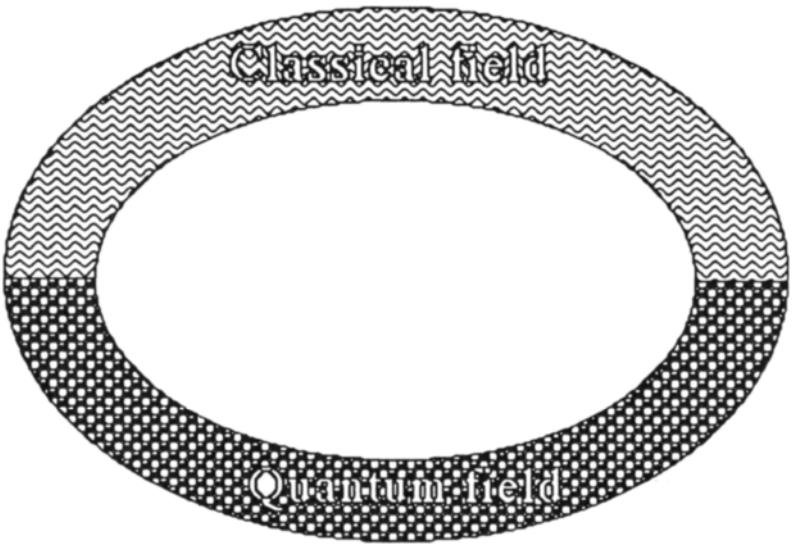
In classical logic, quantum superpositions cannot occur. It is one of the important findings of matrix logic that the nonclassical states are proved to be generally unavoidable. The nonconvex structure of the thought process, arising due to the presence of superposed and empty states, is a fundamental aspect of cognition.

DENKTORS

The thinking brain is a macroscopic system but essentially includes nonclassical elements. Like a spinning coin that is both head and tail until it falls, cognitive elements exist in coherent superposition of the orthogonal true and false states [Ref 86]. Differentiated thought is driven into the eigenstates, where the initial probability normalization is temporarily disrupted. Only when a problem is solved, does the brain reverts to classical rules which exclude coherent superpositions. Because the physics behind such superposition is inherently quantum-mechanical, there are deep and compelling reasons to view the brain states as nonclassical. Matrix logic has led us to the conclusion that the cognizing mechanism cannot be put into the straightjacket of classical physics. But does that automatically imply that the mode of operation of the brain is essentially and fundamentally quantum-mechanical in nature? Are the brain states genuinely quantum-mechanical? How can we explain the fact that a macrosystem displays nonclassical features that by all accounts appear to be quantized? An answer to this question is suggested by the concept of self-measurement [Ref 78]. **The physics of thought is a physics of self-measurement**, in which we regard the brain as an universal measurement device with a closed topology which prevents a decay into a linear protocol, thus creating an unique information phase which is neither classical nor quantum, or, alternatively, is classical and quantum at the same time.

This neither-classical-nor-quantum model has serious implications. In a physical laboratory the measurement scheme involves two distinct elements: a measuring system and a measured system. But a cognizer is a self-observing entity where the clear-cut distinction between the measured and the measuring does not applies. Self-measurement is neither classical nor quantum but both. As opposed to destructive measurements, self-measurement is nondestructive, not necessarily involving irreversible macroamplification [Ref 78]. Quantum information in the brain is distilled without decoherence, and the off-diagonal elements in the density matrix are preserved, just like in quantum mechanical interaction-free measurements [Ref 19], introduced in quantum mechanics twenty years after they were explored in the quantum theory of the brain.
When quantum mechanics makes a prediction about the future states of a physical system, it has no bearing on the current state of the system. But when a cognizing system anticipates its future behaviour, the prediction itself becomes an integral part of and interferes with the current data. Cause and effect appear to be inseparably looped and can no longer be treated in canonical linear fashion. In topologically closed cognizing systems, where the input is identified with the output and cause with effect, the fundamental structure of causal relations breaks down. The system falls into an information phase, a singularity, where the **strong** causal rules are replaced by the **weak** cognitive rules. As much as consciousness is the result of natural evolution, it is the net result of physical devolution. In a closed topology the question which part measures and which part is measured loses its meaning. Instead of the

parts we can speak about classical and quantum fields closed on each other, thinking of the former as waves and of the latter as particles:



Through an understanding of the interaction of classical and quantum fields in a closed topology, we can gain insight into the cognizing mechanism, which is the interaction of a field with itself. Measurement degenerates into interaction. The quantum field acts on itself via the classical field. The classical field acts on itself via the quantum field. There is no pure classical field. There is no pure quantum field. Only a theory which can freely alternate between the classical and quantum modes can be applied. But what is this unknown general theory? Physics is jigsaw puzzle with two pieces, classical and quantum mechanics, each having its own distinct mathematical language. One may decide to describe the quantum field by wavefunctions and the classical field by deterministic functions. But such scheme comprising two distinct fields is only an abstraction. In the fundamental thinking unit the classical and quantum fields are tightly coupled, as a result inheriting properties of each other. How can we describe a total system to which neither classical nor quantum language applies, a system which is quantum-mechanical in form but classical in essence? Apparently, we would fail if we chose to treat the information phase quantum-mechanically, because its quantum section cannot be disentangled from its classical section. By the same token we cannot treat it entirely macrophysically, because the classical section cannot be disentangled from the quantum one. One cannot describe the information phase either with the aid of the Schrödinger equation of motion alone or with the classical Hamilton equations alone. A quantum-classical system which is **open** can be studied in

parts, one part at a time. The Schrödinger equation would describe the quantum section and the Hamiltonian equations the classical section. But we *cannot* separate the sections of the information phase of consciousness without destroying it, because it is a **closed** topology which constitutes the information phase! Since the wavefunction is not suitable for the description of the classical section and macrophysical functions are not suitable for the description of the quantum section, we need a new formalism, which should take the middle ground between the classical and the quantum.

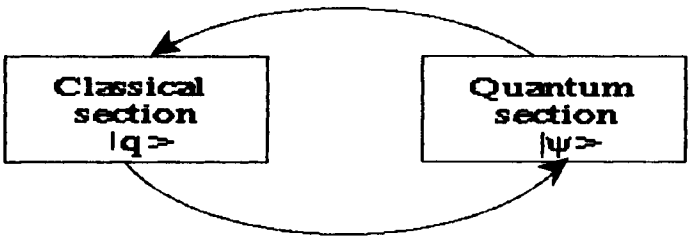


Fig. 29
The information phase comprises the classical and quantum sections, joined into a closed topology.

When we consider a measurement process in quantum physics we are typically concerned with the impact of a macroscopic device on a measured quantum system, a particle, which is forced into a particular eigenstate. The question how the particle influences the macrosystem is generally disregarded. A classical device is simply a 'heavy' notebook for recording the results of the measurement of a 'light' quantum particle. What is important is that the measuring device must be an example of chaos: it is an unstable system in which a small change in the input variables causes a large change in the output. This aspect takes a centre-stage in a self-detecting system of consciousness which is quantum in form but classical in essence. In the brain the classical and the quantum processes exist side by side, conditioning each other in an intricate fashion. Because the description of the information phase cannot be obtained either classically or in terms of wavefunction, we consequently introduce a new construct, the **denktor function**, where the 'denktor' is a German-English composite term standing for "thinking vector" [Ref 93].

Definition. A denktor is a template function, denoted by the empty space in brackets, | >, which exists in the two fundamental forms: real probability $|p>$ or complex quantum probability $|\psi>$, these being freely exchangeable.

The functions of physics are mathematical *statements* concerning the states of a physical system. The states of the brain are physical *states* concerning cognitive statements. What distinguishes a denktor function is that it is a state and a statement at the same time, a *state-ment*. The states of the brain are simultaneously statements of the brain. In the classical section the denktor's template is inserted with the probability, and in the quantum section with wavefunction. Because the classical and quantum sections are merely abstractions, we have introduced them simply for the purpose of finer analysis, and because we simply *don't know* at which section consciousness is at a particular instant, the classical and quantum descriptions must be in a strong correlation which allow us to move freely between the states and the statements. Bohr and Einstein's opposing points of view on wavefunction are incompatible but may not necessarily be mutually exclusive. Cognitive reality joins the description of reality and knowledge about reality. As proved by the singularization theorem, systems exist which yield the same result whether computed quantum-mechanically or classically:

$$\langle q|M|q\rangle = \langle\psi|\sigma|\psi\rangle.$$

The cognitive system carries out analysis of itself, self-comparison, and the same relative truth-value is computed on both sides of the equation. The theorem does not exactly provide a dynamical description, which we need. Theoretical analysis has determined that, in full analogy to wave and matrix quantum mechanics, the logical momentum exists not only in the form of a traceless singular matrix but also in the form of a signed differential operator:

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}|q\rangle = M|q\rangle = -\frac{d}{dq}|q\rangle.$$

Introducing the differential form of the momentum into the singularization theorem

$$\langle q|-\frac{d}{dq}|q\rangle = \langle\psi|\sigma|\psi\rangle,$$

then expelling the differentiation operator in front:

$$\langle q|-\frac{d}{dq}|q\rangle = -\frac{d}{dq}\langle IF\rangle,$$

we derive the mixed quantum-logical dynamical equation:

$$-\frac{d\langle IF\rangle}{dq} = \langle\psi|\sigma|\psi\rangle$$

where one side is classical and the other is quantum-mechanical.

This result shows how in the self-measuring system the classical section, associated with the transitive thought, and the quantum section, associated with spin, can be dynamically joined together into a unique system which registers its own state. The elements of the system alternate between unitary and classical evolution. A quantum state on the righthand side changes over to classical on the lefthand side. The process is synchronized so that both the classical and quantum sections give identical readings. The existence of the manifolds, where the quantum and classical computations yield equivalent

results, illuminates the fact that a fundamental theory of the brain must join macrologic and micrologic into one unified theory.

Operator-operand uncertainty

As discussed earlier, classical physics makes use of numbers and the quantum physics of operators. But the information phase is neither a scalar nor an operator, because topological closure prevents us from deciding uniquely between the two. This operator-operand uncertainty is similar to the uncertainty principle in quantum mechanics, although now it concerns not the canonical phase coordinates but the descriptions themselves, classical and quantum. The operator-operand uncertainty distinguishes the autoproducts of matrix logic [Ref 88], but also the atomic products where, without impediment, one changes between the operator and scalar formats, for example, for conjunction:

$$p \wedge q \Leftrightarrow \langle p | \wedge | q \rangle$$

The brain can accept both the numbers and operators and freely changes between them. Any attempt to force consciousness into a particular format is equivalent to cancellation of consciousness. Then the closed topology breaks up, the information phase is destroyed and the states become either classical or quantum.

UNDERSTANDING THE INFORMATION PHASE

The theory of information has been always considered a purely theoretical field, detached from physics. Because information did not refer explicitly to physics, it has for a long time been incorrectly assumed that the foundations of information theory, and consciousness for that matter, are purely abstract. Being accustomed to the classical concept of information we are not used to thinking of cognizing in physical terms. We look on the thought process as being made up of theoretical, mental or logical operations, even though it is clear that thinking is elicited by specific interactions in the brain matter. But today we conceive of information as something real, as real as space and energy. If there is a new information phase, shouldn't the thinking brain present new physical effects? A physicist may claim with great confidence that there is nothing fundamentally new in the brain which physics does not know. This is a very typical and great blunder, because consciousness is precisely the new extraordinary effect which the 'known physics' was not able to accommodate until now.

We have discussed the quantum phase of matter, described by complex probability amplitudes, and the classical phase of matter, described by actual probabilities. In neither of these phases is information a decisive factor. But the thinking brain is a distinct information phase which cannot be considered **either** classical **or** quantum, requiring for its description denktors, which in a

perplexing way make use of both complex amplitudes and probabilities. Despite early suggestions that the cognitive brain represent a distinctly new phase, it was not until the development of matrix logic that the theoretical foundations of this phase were laid down and properly formalized. This formalization is achieved with denktors which have multiple faces. We know about the existence of various phases of matter and fields, like gas, liquid, solid state, plasma, the Bose condensates and others. The transition from a certain phase to another is an abrupt rupture with the past. When a gas turns into a liquid or the liquid into a solid, the system undergoes a rapid catastrophic change. All these changes, however, concern the Noether charges of the system. What is special for the information phase and what distinguishes it from other phases is its dependence on topological charges. It is a **topological** phase as opposed to the various **geometrical** or Noether phases of physics, as one may choose to call them. In our scheme the information phase occurs with the transition of the orientable topology into the nonorientable one. Like any phase transition the information phase is a fundamental catastrophe. Its formation is accompanied by an abrupt change affecting **all** elements of the system simultaneously, just like global waves extending across a Bose condensate or a superconductor, described by a single wavefunction. When we twist and glue the edges of a bilateral manifold, a transition to the information phase occurs. Significantly, the information phase seems to be free of relativistic constraints, 'tachionic'. All spacetime points of the manifold, including the absolutely remote ones, instantly and simultaneously will turn into a unilateral topology. This is similar to the EPR paradox, where an absolutely remote particle instantly 'knows' what orientation of spin it must choose when we procure the spin measurement on the adjoint particle.

We now determine a fundamental connection between the information phase and the unbroken topological phase $H(\cdot)$ which is a Klein bottle or connected sum of two projective planes. The classical and quantum sections of denktors are designed self-referentially and it is the Klein bottle which enables us to convert an arbitrary classical state to a quantized state of coherent superposition, or vice versa. We have the orthogonal classical basis $\{|0\rangle, |1\rangle\}$ and the orthogonal quantum basis $\{|s_+\rangle, |s_-\rangle\}$:

$$\langle 0|1\rangle = 0, \qquad \langle s_+|s_-\rangle = 0,$$

which mutually expand into each other:

$$|0\rangle = \tfrac{1}{2}(|s_+\rangle + |s_-\rangle), \qquad |1\rangle = \tfrac{1}{2}(|s_+\rangle - |s_-\rangle)$$

explicitly:

$$\begin{pmatrix}1\\0\end{pmatrix} = \tfrac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \tfrac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}, \qquad \begin{pmatrix}0\\1\end{pmatrix} = \tfrac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} - \tfrac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}.$$

and

$$|s_-\rangle = |0\rangle - |1\rangle, \qquad |s_+\rangle = |0\rangle + |1\rangle$$

explicitly:

$$\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} + \begin{pmatrix}0\\1\end{pmatrix}, \qquad \begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}0\\1\end{pmatrix}.$$

The classical states are the equally weighted signed superposed states, which in turn are equally weighted signed Boolean states. The classical states are normalized, but we are not demanding the normalization of the superposed states, which, of course, is easy to achieve with the quantum factor $1/\sqrt{2}$. In general, the state of the denktor will be a linear combination of states, and the self-measurement projecting a particular state, diagnoses which of the 4 possible states actually occurs. Describing the unbroken topological phase by the Hadamardians, we choose H. to mediate between the two orthogonal eigenstate bases. We proceed by inserting the Klein bottle at the border between the classical and quantum sections. Suppose the denktor is impregnated with a classical value and we want to bring it to a quantized state. Applying the Hadamard transform, we get

$$H.|0\rangle = |S_+\rangle, \qquad H.|1\rangle = |S_-\rangle,$$

In the phase space of true and false coordinates, the state $|0\rangle$ was rotated 45^0 clockwise and expanded by $\sqrt{2}$. Likewise, the state $|1\rangle$ underwent the same metamorphosis but after being flipped. Explicitly,

$$\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}, \qquad \begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}.$$

In reverse, we can extend the classical states into quantum states by acting on them with the same, but halved, Klein bottle:

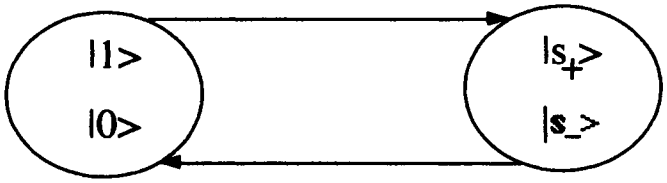
$$1/2H.|S_+\rangle = |0\rangle, \qquad 1/2H.|S_-\rangle = |1\rangle.$$

This are 45^0 contraclockwise rotations, one with the flip, of the quantized eigenstate basis. Explicitly,

$$\frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix}, \qquad \frac{1}{2}\begin{pmatrix}1&1\\1&-1\end{pmatrix}\begin{pmatrix}1\\-1\end{pmatrix} = \begin{pmatrix}0\\1\end{pmatrix}.$$

The system could be made unitary, hence, quantum-mechanical if the bottle is not halved but divided by $\sqrt{2}$. We observe that we can change the basis for each denktor by applying the Klein bottle. Then phase reversal in the basis $\{|0\rangle, |1\rangle\}$ induces the orthogonal rotations, left or right, in the stress rotated basis $\{|S_+\rangle, |S_-\rangle\}$, just as phase reversal in quantum basis induces orthogonal rotations, left or right, of the Boolean basis. It will therefore be sufficient if our system is able to diagnose phase reversal in either basis. If we then apply the Klein bottle to each of the basis states, the states in the adjoint basis swap over. Hence, in the rotated basis, as in the original basis, we can diagnose the phase reversal by checking whether the swap of states occurred or not. The phase reversal of $|S_-\rangle$ in the quantized basis orthogonally rotates the adjoint basis. For the Boolean basis this swaps the basis states, which is the amplitude reversal. Therefore, the phase flips of $|S_-\rangle$ in the quantized basis are the amplitude flips in the rotated basis. The key point is that in the information phase shift in the quantum section can nondestructively monitor the truth status

of the classical section. Through topological transformations the fundamental thinking unit can freely change the eigenstate basis states, restoring the denktor to an intended value, classical or quantum. Classical and quantum states are functionally complete sets of eigenstates spanning each other:



In matrix-logical terms the classical states |0> and |1> are the eigenstates of **AND** and the quantized states |S-> and |S+> are the eigenstates of **NOT**. It is known that the logical basis **{AND, NOT}** is functionally complete; hence our system of classical and quantum eigenstates constitutes a functionally complete system. Thought moves freely across the classical-quantum divide: classical states project themselves onto quantum, and quantum onto classical.

$$|S_-\rangle = |0\rangle - |1\rangle, \qquad |S_+\rangle = |0\rangle + |1\rangle$$

$$|0\rangle = \tfrac{1}{2}(|S_+\rangle + |S_-\rangle), \qquad |1\rangle = \tfrac{1}{2}(|S_+\rangle - |S_-\rangle)$$

This system is complete and self-referential. Most importantly, there are operators which can perform the rotation of one orthogonal basis to the other orthogonal basis. We know that logical differentiation converts classical states into coherent superposition:

$$\mathbf{[FALSE, TRUE]}|x\rangle = |S_-\rangle, \qquad \mathbf{\{FALSE, TRUE\}}|x\rangle = |S_+\rangle$$

which is

$$\mathbf{M}|x\rangle = |S_-\rangle, \qquad \mathbf{1}|x\rangle = |S_+\rangle$$

Unfortunately, the differentiation operators are singular: det **M**=0, det **1**=0, and the reverse transition from quantized states to classical is not possible. The factor $\frac{1}{2}$ which appears in quantum to classical transitions suggests that we use the Hadamard code, which is nonsingular:

$$H(\cdot)\tilde{H}(\cdot) = 2\mathbf{I}, \qquad H(\cdot)^{-1} = \frac{\tilde{H}(\cdot)}{2}$$

It is then possible to achieve reversible classical to quantum conversion:

$$H.\,|0\rangle = |S_+\rangle, \qquad H.\,|1\rangle = |S_-\rangle$$

and quantum to classical conversion:

$$\tfrac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}|S_+\rangle = |0\rangle \qquad \tfrac{1}{2}\begin{pmatrix}1 & 1\\ 1 & -1\end{pmatrix}|S_-\rangle = |1\rangle$$

Likewise for the bosonic coordinates:

$$.H\,|0\rangle = |S_-\rangle, \qquad .H\,|1\rangle = |S_+\rangle$$

and

$$\tfrac{1}{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}|S_-\rangle = |0\rangle \qquad \tfrac{1}{2}\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}|S_+\rangle = |1\rangle$$

The essence of the information phase lies in the reversible transformation between the classical $\{|0\rangle, |1\rangle\}$ and the superposed basis $\{|S_-\rangle, |S_+\rangle\}$. It is significant that the quantization of classical states and the recovery of classical states from the quantum states are both nonunitary processes, as the norm of the states is not conserved but expanded or contracted by the factor $\sqrt{2}$:

$$\||S_\pm\rangle\| = \sqrt{2}\ \||x\rangle\|.$$

If the Klein bottle is constructed with quantum elements, we get a stable topological cat state which does not decohere. What was thought of as a baffling paradox of quantum mechanics is a new information phase of matter, realized in the brain. No doubt it is an unique phase, but how unique it is? One can imagine other possible realizations. If one could create a container of neutrinos, isolated from all other particles and charges, such a container would attain a nonorientable information phase.

The topologically closed system of consciousness behaves very much as if it were a macroscopic nucleus with quantized energy levels. When the topological current of a particular thought is dispatched, it is analogous to a decay process: the particle which is initially in a metastable bounded state tunnels out into a continuum of states. A particle crosses over from the quantized to the continuous regime. In this scheme consciousness is associated with an operator with quantized eigenstates, while the thoughts must be treated classically as particles escaping from the energy well of the unilateral topology. The leakage through the barrier enables the particle to escape from the well by quantum tunnelling. Though local logic might be classical, when the topological phase is treated in its totality, whether interrupted by the decay or not, what is normally hidden at the unconscious level tunnels to the macroquantum level of consciousness. Then the unilateral thought process shows its capacity to tunnel between the minima of the effective topological potential. Moreover we can create such macroscopic quantum tunnelling at will, as for example, in the Liar paradox.

Topological coupling of classical and quantum degrees of freedom is a fundamental feature of the information phase. The tensor product of quantized states gives the momentum operator

$$|S_-\rangle\langle S_+| = \mathbf{M},$$

which, acting on classical states, quantizes them

$$\mathbf{M}|q\rangle = |S_-\rangle,$$

naturally converting logic to the quantum-mechanical mode. The momentum operator is differentiation. Superposed states are formed as a result of logical differentiation, but also by the 'ordinary' action of a matrix logical operator on classical states: $\mathbf{L}\,|x\rangle = |S_\pm\rangle$. If the inverse $\mathbf{L}^{-1}$ is available, the reverse quantum to classical conversion is attainable. But there are other means in matrix logic, for example, with a halving factor one can convert a coherent superposition into a classical normed state:

$$\frac{1}{2}|S_+> = |q>.$$

These facts indicate the importance of investigating the mechanism through which the logical brain can exert control over the classical-quantum interface. Provided quantized and classical states are strongly correlated, a thought process should collapse in the end to a definite result in which all nonclassical fluctuations are completely eliminated. This is misleadingly similar to observation in quantum mechanics when a quantum system reaches an eigenstate. Before turning to the heart of the matter, it is necessary to recognize that the information phase cannot be simply deduced from quantum laws alone. The quantum-mechanical states are characterized by complex amplitudes, which, due to interference, allow the state vector to point in different directions at once. A particle can simultaneously be in different states, having different identities at once, being everywhere and nowhere. But this ambiguity is lost at the macroscopic level.

The formalism of quantum mechanics dictates that a microphysical system can exist in coherent superposition, and using an internal degree of freedom it is possible to drive a quantum particle into a superposed state. This differs fundamentally from a classical to quantum superposition when two distinct macrophysical states form a cat state. The end result of both procedures, however, is the same and both must be taken into account. A self-detecting system resides in states of coherent superposition which are essential for the cognizing mechanism. The singularity of a cognitive state entails that it can be extended in a classical or and quantized basis, and in the topologically closed system we cannot distinguish between them. We simply do not know at which section we are at a particular instant. The singularization theorem provides for invariancy, if we change over both the acting operators, spin and momentum, and the spaces, Hilbert space and matrix logical phase space. Theoretically this entails that we know in which section we are measuring. But since in fact we do not, it is advantageous to keep an acting operator intact, but to change over from a density matrix to a logical density matrix. The condition of invariance then

$$\mathbf{Tr}\Omega\mathbf{L} = \mathbf{Tr}\rho\mathbf{L}$$

where $\Omega = |q><q|$ is a matrix logical density and ρ is a quantum density matrix $\rho = |\psi><\psi|$. Does a logical operator $\mathbf{L}$ satisfying this equality exist? Setting the wavefunction $|\psi>$ to the coherent state

$$|\psi> = \frac{1}{\sqrt{2}}|S_\pm>.$$

we determine that the corresponding density matrices are the halved synergies

$$\rho = \frac{1}{2}|S_{\pm}><S_{\pm}| = \begin{cases} \frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \\ \\ \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \end{cases}$$

Due to the closed topology self-comparison must yield the value zero, implying in a sense that time stops in the system. Then besides the vacuum, the only nontrivial vanishing trace, satisfying the classical-quantum interface are with the time operators. A value which is observed in the classical section of the self-detecting system must also be observed in the quantum section. We calculate:

$$\mathrm{Tr}\frac{1}{2}\begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \cdot \mathrm{TIME} = 0, \qquad \mathrm{Tr}\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \cdot \mathrm{TIME} = 0.$$

The respective classical and quantum traces are equal,

$$\mathrm{Tr}\Omega \cdot \mathrm{TIME} = \mathrm{Tr}\rho \cdot \mathrm{TIME}$$

and given only the result of the tracing, it is not possible to determine from which section it came , classical or quantum.

We have investigated various aspects of the conversion between classical states and quantized states which is the fundamental feature of the thinking brain:

BRAIN

CLASSICAL STATES $\Leftrightarrow$ QUANTUM STATES

Between the quantum and classical theories there lies a poorly understood fuzzy field of mesoscopic physics, which does not fully follow either classical or quantum laws,

$\hbar = 0$	$\hbar \sim 0$	$\hbar \neq 0$
Macrophysics	Mesoscopics	Microphysics

Understanding how the brain exersises control at the over classical-quantum border should shed light on the field of mesoscopic physics and the question of the quantization of the classical system. Finding the answer to the question of how big Schrödinger cats can become and how far can quantum-classical boundary in the brain be pushed up requires control over individual atoms and molecules which is not easy to achieve. Breeding various kinds of Schrödinger *kittens* made of a handful of particles will enable us to learn more about the properties of information phase of consciousness.

QUANTIZATION OF THE CLASSICAL SYSTEMS

To some, quantum mechanics is a familiar tool for solving problems in the microworld, to others it is a sort of magic. In the jungle of modern formalisms we often lose track of those simple yet fundamental ideas which led to the creation of quantum theory. It is wise from time to time to return to those initial concepts and reasoning which led Planck, Einstein, Schrödinger, Heisenberg and others to lay down the foundations of quantum theory. The behaviour of the microworld is notoriously strange. Both in classical and in quantum mechanics there are orthogonal state spaces. However, while in classical theory this fact entails nothing extraordinary, in quantum mechanics we can take a pair of orthogonal states and construct a state which is a coherent superposition of both. Because coherent superposition has great significance for the quantization of a classical system, prior to going into the subject of such great complexity it might be useful to recall some basic quantum ideas. Classical fields evolve without coherent superpositions and entanglement. It is generally believed that classical fields when coupled to a quantum fields do not inherit quantum properties. This point of view has been shaken by quantum effects displayed at the macrolevel through such phenomena as superconductivity, the flux quantization effect and by mesoscopic Schrödinger cats, realized on ion traps. One must distinguish between macroquantum phenomena originating in the superposition of a large number of microscopic variables and those displayed by a single isolated macroscopic degree of freedom. Irreversibility and decoherence, the distinct features of quantum to classical coupling, are replaced by reversible logic in complex biosystems comprising a very large number of interacting elements [Ref 72, 78]. The brain is a system of billions of densely packed neurons, and the large dimensionality requires approximation, where different classical trajectories are in correlation with asymptotically distinct quantum evolutions. The cognitive brain is governed by the quantum laws at the bottom and by the deterministic laws of logic at the top. What is important here is that information becomes an integral and fundamental part of the description which cannot be ignored. The Hilbert space of quantum theory and the phase space of classical information must be consistently connected. Although we are naturally motivated to study the brain in a quantum-theoretic framework, my purpose is not to address the question what information can be extracted from a quantum system, but conversely to study how much of quantum mechanics can be 'stored' in classical logic and to understand from *first principles* how the domain of classical information merges with the quantum domain.

Just like classical fields, classical logic seems on surface to be evolving without quantization and coherent superpositions. Coherent superpositions are commonly viewed as being essentially microphysical and without occurrences in the macroworld. The issue, however, cannot be considered fully settled to

the satisfaction of all, as is indicated by the various attempts to re-address quantum problems classically. For example, in stochastic electrodynamics, random electromagnetic fluctuations generate perturbations which can mimic certain quantum phenomena while retaining a purely classical basis. Whereas in the quantum view $\hbar$ is a unit of quantization, in the stochastic electrodynamics view it is merely a measure of field strength. Exactly to what extent a classical technique can accommodate and reproduce the results of quantum mechanics remains an open question. Here matrix logic makes an important contribution. Developing the possibility of the transfer of quantum degrees of freedom to the domain of classical logic, it suggests that coherent superpositions are not exclusively microphysical and can, in fact, be made compatible with the macrophysical laws of composition in the logical brain. The quantization theorem, proven early, reaches out into deep aspects of physical theory. Because information bits can reside not only in the orthogonal Boolean states but commonly form coherencies, logical inferences involve not only discretization but also, and more importantly, quantization, indicating that consciousness, even though definite in the input and output states, is nonclassical at bottom. Coherent logical superpositions strike down the Laplacian model of the brain, indicating that the cognitive mechanism cannot be explained purely classically.

It must be emphasised that the denktor function is not a semiclassical function, where one can make use of a density matrix involving both classical probabilities and quantum amplitudes. A denktor can disobey classical probability normalization, which makes it uninterpretable classically. It also does not conform to quadratic probability normalization, which then makes quantum probability amplitudes unsuitable. Logic enters physics at the fundamental level, but does not appear to be inherently quantum-mechanical. Consciousness would not work unless it had freedom to cross back and forth between the nonclassical regime, which is fuzzy and quantized, and the macrophysical Boolean regime, which is deterministic throughout. This situation is similar to a macrobody which as a whole behaves classically, but is composed of basic quantum elements. What that means in quantum-theoretic terms is that with the crossover from the classical to the quantized regime, matrix degrees of freedom can no longer be treated as classical variables but must be represented by operators.

Matrix logic comprises both commuting and noncommuting elements. As we turn from a description of logic in terms of scalar functions to a description in terms of operators, the classical degrees of freedom, strongly correlated with quantum ones, congruently exhibit quantum behaviour through coherent superpositions, phase reversal and entanglement. This quantization is explicitly dictated by fundamentals needed to retain the consistency of the theory as a whole. In the information phase the microscopic and macroscopic realms overlap, and the quantum and classical degrees of freedom can be treated within one general formalism. These findings are relevant for analysing the conditions under which a measurement on a quantum system can

be reversed. Reversible classical-quantum nondestructive measurements are tractable in matrix logic and will lend themselves to experimental exploration.

The quantization theorem also determines that classical and quantum states can be treated jointly within the general formalism of matrix theory. There exists a special nonorientable manifold where macrologic merges naturally with quantum mechanics. The ability of nonlinear logical systems to convert Boolean states to the quantized coherent mode allows us to utilize the scheme of matrix logic for the quantization of classical systems in general. Because classical and quantum information exists side by side in the brain, influencing each other in an intricate fashion, we can, without impediment, choose between the classical treatment of information in terms of Poisson brackets or unitary evolution, that is, between the Hamilton equations or the Heisenberg equation of motion. Useful examples can be found in the transition to chaos in classical systems. When the momentum transfer occurs from a modulated standing wave of near-resonant light to a sample of ultracold atoms, the phase space goes from global stability to chaos, that is, from a classical regime to quantum evolution.

Bosonization

The information aspect of matter and fields figures prominently in quantum and relativistic theories and provides an impetus for probing the quantum structure of information itself. A great deal of quantum logic can be stored in matrix logic, which connects the classical and quantum levels in an unusual way and sheds new light on the phenomenon of consciousness. There is hope that mesoscopic systems, which possess both macroscopic and microscopic features, can provide a testing ground for the study of the fuzzy boundary between the macro- and microworlds. Further down the road, the gauge symmetries in logic must be explored for a deeper understanding of the information phase.

Initially the focus of quantum-theoretic description of the brain was on fermions. The matrix logical Kronecker strings were put in correspondence to a fermionic lattice with translational symmetry [Ref 88]:

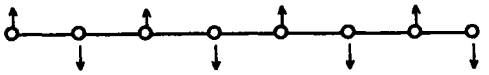
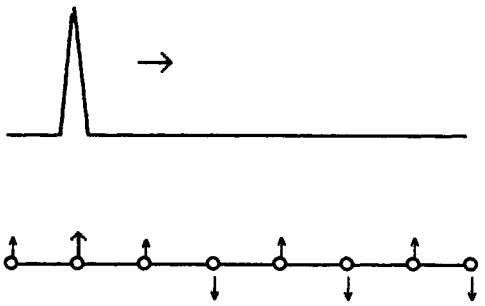


Fig. 30
Lattice of coupled spins.

To avoid difficulties with boundary effects, we considered a lattice of infinite extent. Spins are coupled locally, each spin with the neighbouring spin in its immediate vicinity, which allows us to monitor the alternation of spins by the Pauli exclusion rule. When we flip a spin at a given point it induces a perturbation spinor wave propagating along the lattice **L**:



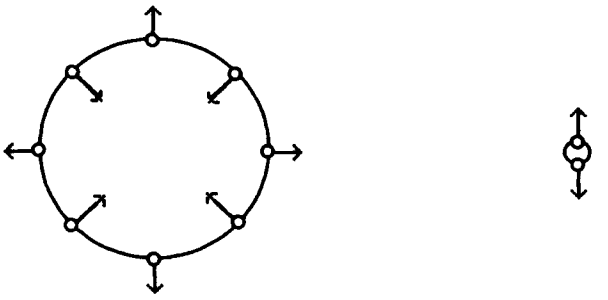
Transfer of spin inversion from initial to final position executes the unitary transformation of the lattice:

$$< \bar{x}_l \,|L|\, y_n> = <x_l \,|\theta L|\, \bar{y}_n>.$$

The singularization theorem tells us that the changeover between the logical momentum and the quantum spin produces no observable effects on a fermionic lattice. If instead of changing the operator, we keep it intact but change to the Hilbert space of wavefunction, the quantum expectation appears to be in no way different from the corresponding classical expectation:

$$<q|\, \sigma_z |q> = <\psi^*|\, \sigma_z |\psi>.$$

This is an important statement, but in the brain we are interested not only in the impact of a **quantum** operator on a classical or quantum state. We are also concerned with the impact of a **classical** operator on a quantized state. To avoid boundary effects, we considered a lattice of **infinite** extent. We also can avoid the difficulties if we take a **finite** lattice but one which is closed on itself:



Bosonization, such as the Cooper pairing of electrons in superconductors or in the magnetic monopole in strongly interacting non-Abelian Yang-Mills models, should occur if the closed lattice is 'compactified' to the minimal lattice with two spins.

If the field is bosonic, then the invariance on the classical-quantum interface extends to the logical momentum:

$$\langle q|\, \mathbf{M}\, |q\rangle = \langle \psi^*|\, \mathbf{M}\, |\psi\rangle$$

$$= \psi^*(\alpha)\, \psi(\alpha) - \psi^*(\beta)\, \psi(\beta) + \psi^*(\alpha)\, \psi(\beta) - \psi^*(\beta)\, \psi(\alpha)$$

$$= \psi^*(\alpha)\, \psi(\alpha) - \psi^*(\beta)\, \psi(\beta) + \psi^*(\beta)\, \psi(\alpha) - \psi^*(\beta)\, \psi(\alpha)$$

$$= \psi^*(\alpha)\, \psi(\alpha) - \psi^*(\beta)\, \psi(\beta).$$

Consequently, for a symmetric wavefunction, either available or engineered through bosonization, the action of the *quantum* operator on the *classical* field for all practical purposes equals the action of the *classical* operator on the *quantum* field:

$$\langle q|\, \sigma_z |q\rangle = \langle \psi^*|\, \mathbf{M}\, |\psi\rangle.$$

The logical operator acts *top-down* on quantum states in exactly the same way as the quantum operator acts *bottom-up* on classical states.

The idea of the macroquantum nature of cognitive information finds indirect support if we analyze the eigenspectra of logical operators. Then a process similar to the condensation of Cooper pairs in superconductors can be one of the plausible causes of logical bosonization. Fermi particles can accommodate the logical **AND**, with the eigenvalues 0 and 1, and **NOT**, with the eigenvalues -1 and 1, but the synergy operators, with the eigenvalues 0 and 2, where logical energy is either zero or contains two quanta, make a strong case for bosons.

The operator representation is fundamental for the quantization of classical logic. In addition to classical **and** quantum phenomena, such quantization points to the existence of a distinct class of **neither** classical **nor** quantum systems, with reversible coupling which does not involve irreversible macroamplification. The key element of the description of such a '**neither-nor**' system is that the expectation values of quantum operators are calculated classically, without reference to the wavefunction. A real-valued coherent superposition violates both the linear and the quadratic rules of normalization. Since the quadratic rule is a consequence of the superposition principle, the information phase of the cognitive brain provides the first and only example of violation of the superposition principle, the underlying principle of quantum mechanics. Irrespective of whether this violation gives new clues to the fundamental question of consciousness, it enables one to derive the classical *continuum* from the *discretum* through a limiting process. But in this valid exercise we must not lose sight of the no less important issue of restoring the *discretum* from the *continuum*. While construction of a quantum observable from a classical observable may or may not lead to a self-adjoint operator, dequantization must end up with Hermitian representations.

Coming back to the question whether and to what extent the quantum level dictates the intelligent behaviour of the brain, as the result of our investigation the answer we must give to the quantum hypothesis of the brain appears, on one hand, to be definitely negative: nowhere does the need arise for the machinery of a complex wavefunction in the logical operations of the brain, be they Boolean, mixed or superposed. To accommodate logical thought we have to abandon quadratic quantum normalization, but hoping instead to employ the classical probability normalization universally, we fail. The truth evolution of cognitive states commonly leads to coherencies incompatible with macrophysical laws. This indicates that the cognitive mechanism cannot be put into the straitjacket of either classical or quantum theory. And this is not pure mathematical conjecture. The actual experimental conditions can be set up which enable us to 'observe' cognitive interferences. In perception we can create an apparent motion of perfectly stable visual percepts. At the higher level of logical thinking the coherent superposition of the **True** and **False** can be obtained in self-reference discourse, exemplified by the Liar paradox which actualizes the normally hidden denktors in coherent superposition. Introspection tells us that the very problem of consciousness we are trying to solve is a coherent superposition.

The fact that logical states are nonclassical does not automatically imply they are quantum-mechanical. Cognitive logic complies neither with the quantum laws of probability amplitudes nor with the laws of classical probability. This situation is further aggravated by the fact that cognitive observables are not self-adjoint. A physical input to be *understood* by the mind has to be converted to a non-Hermitian format. The cognitive operation in the brain emerges as a fundamental process which begins precisely where the standard physics of Hermitian observables is no longer applicable because Hermitian and non-Hermitian quantities exchange their roles. This point of view differs fundamentally from existing approaches to the brain, which start from either the classical or the quantum-mechanical premiss.

The idea that abstract concepts are non-Hermitian quantities has more than just a philosophical theoretical appeal. The physics of the thinking brain sets up a precedent which has no clear-cut analogs in either classical or in quantum mechanics. That the thought process is quantized but without complex probability amplitudes suggests that a modification of quantum theory is necessary to accommodate consciousness. Consciousness is a fundamental process creating the conditions under which a logical state is guided to a chosen point in matrix space while a dual quantum state converges to a corresponding point in Hilbert space. The cognitive states display a number of typically quantum properties, yet at the same time it is essentially deterministic and macrophysical in character. If logic is an indication, and the present author believes it is, then the physics of the brain cannot be fully understood on the basis of either classical or quantum mechanics alone. Matrix consciousness transforms logic into a fundamental science and must explain wavefunction as a derivative notion.

THE S-MATRIX AND THE SYNAPTIC MATRIX

In the prevailing concept of the brain, as a network of neurons separated by synapses, the neuron is viewed as a fundamental element. One constructs a theory which explains the functioning of the brain in terms of bioelectrical neuron firing which triggers the flow of Ca^+ and K^+ ions across the membrane. An individual neuron is the recipient of information from other neurons. A bioelectric spike of depolarization reaching the synapse causes the synaptic vesicles, filled with molecules of the excitatory and inhibitory mediators or neurotransmitters, to discharge their content. If excitatory potentials result in postsynaptic membrane depolarization of **10** to **20** mV, this could be enough to generate an impulse in the next neuron that would travel down to the next synapse. Being constrained by the threshold conditions, spike potentials are satisfactorily described by classical physics, if only because an impulse propagates along a predetermined path. The flux of the mediator is semiclassical, with quantum-mechanical laws coming into play, allowing coherent superpositions to enter the picture. Most importantly, such an arrangement allows for an interchange of quantum-mechanical and classical modes of operation, just as is required by matrix operator logic. Such a quantum-classical alternating medium is suitable for self-measurement, which is essential for consciousness. Identification of the first neuron in a sequence with the last one creates a closed topology, where each element functions simultaneously as a generator of mesoscopic states, through the discharge of a mediator into the synaptic cleft, and as a macrodetector, through depolarization and the spike potential.

We are getting used to thinking about neural transmission as communication between different neurons. But communication between different synapses can tell us even more about the actual physics of the brain. A switch from describing the brain as a network of neurons connected by synapses to describing it as a network of synapses connected by neurons is in some way similar to a switch between a Feynman diagram and a S-matrix diagram in quantum field theory. As opposed to the neuron, the states of a unit in such a synaptic network cannot be determined with certainty, but are associated with probabilities. This shifts the emphasis from a transmission of Boolean spikes between neurons to 'propagation of uncertainty' from one synapse to another. The totality of such a synaptic network can be analyzed in terms of 'quantum fields' interconnected by classical neurons operating almost macrophysically. In doing so, we turn the neuronal scheme inside-out: the fundamental element is not a neuron, but instead we consider a network of synapses connected by neurons. The emphasis is shifted from neurons to synapses. Just as with the S-matrix method we are not concerned with what happens inside a synapse but only with what comes in and what comes out. The S-matrix diagrams are very

different from the spacetime diagrams of field theory. The diagrams do not picture the detailed mechanism of interaction, but merely specify the initial and final particles. Instead of a explicit network of interactions in S-matrix theory, one simply draws a circle without specifying what goes on inside it. It is a more general symbolic representation of particle interactions. Although an S-matrix diagram contains much less information than a Feynman diagram, it avoids the difficulty of localizing the interaction of particles with finite size.

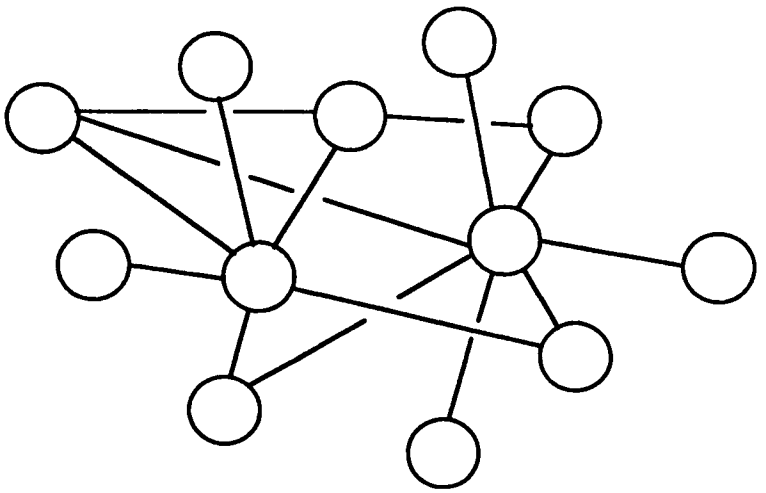


Fig. 31
Synaptic matrix is the network of synapses, denoted by circles, connected by neurons, denoted by links.

The neuronal matrix and the synaptic matrix can be treated as dual, functioning as an organized whole. The flux of excitatory and inhibitory mediators in synapses can accommodate coherent quantum superposition in the brain. Topological closure forms a self-detecting architecture where each element has a dual role as a measured system and a measuring system.The synaptic matrix network is made up of a huge number of molecules, astronomically greater than the number of neurons in the brain. For synaptic superpositions, quantum coherence decays much faster than the usual neuronal observables of the system, implying that the separation between entangled states is an extremely large number. The operations of the brain involve many billions of logical elements. The operator describing consciousness ought to be a very large matrix, that would strain one's computational resources to an

extreme. To understand how a simple nibble and a particular thought can be actualized in this complex universe we can use the methods of sparse matrices.

THE BRAIN'S SPARSE MATRIX

The theory of matrices is an area where logic and physics impinge on one another in a fundamental way. Cognitive output is given in scalar form, which hides the important features of the thought process, making us unaware of the mind's extra dimensions. Although there is much more clarity in scalars than in matrices, much information is lost when a matrix thought is confined to a one-dimensional output, just as a one-dimensional creature may never grasp what a square or a cube is. The realization that the complexity of consciousness is obliterated at the output led us to develop matrix logic, where the scalars are no longer elementary primitives but matrices. Matrix theory started a conceptual reform of logic which is still in progress. It is not to be excluded that in future we may not only describe logic with matrices but learn to think matricially, and to see a very different reality.

Matrix theory is at the heart of the solution methods used for handling many scientific and engineering problems. These methods have found their way into neuroscience, where we treat the brain as a matrix operator. Because the brain's matrix is of very large size, we face serious computational problems, some of which are simply intractable. To convey a sense of their complexity, consider matrix problems of a size reaching hundreds of millions to billions of entries (neurons) which must routinely solve logical problems using massively parallel quantum computations. Fortunately enough, computer-assisted imaging techniques reveal that during a particular cognitive activity the majority of neural elements are in a nonactive stand-by mode, and therefore would contribute zero elements to the brain's overall logical matrix. For example, a system consisting of one million rows, where each row involves on average a few nonzero elements, the resulting matrix will have several millions of nonzero entries. The thinking brain works globally, but typically a particular activity is localized, suggesting that we have a very large but sparse matrix. A sparse matrix is one with very few nonzero elements.

Large sparse eigenvalue problems arise in calculations of electronic structures. Suppose we need to compare two eigenstates with fuzzy values. Due to round-off or conversion to the quantized mode, the comparison may succeed when it should fail. The eigenstates may be separable when Boolean precision is allowed, but they would fail a test for orthogonality on fuzzy sets. This is why sparse matrices are useful, scaling down the huge problems to a manageable subspace. One efficient method for dealing with sparse problems is fractal concatenation. The original matrix is reordered to have block diagonal form. Each diagonal matrix is itself a block diagonal matrix. This reordering

is obtained by using a succession of Schur complement reductions associated with an independent reordering by blocks. Variants of this method are similar to multilevel methods and domain decomposition techniques. When the blocks become larger, the method is more akin to a domain decomposition approach. When they are very small, a multigrid approach then results. In solving very large systems this provides for flexibility, bridging the gap between the two efficient algorithms.

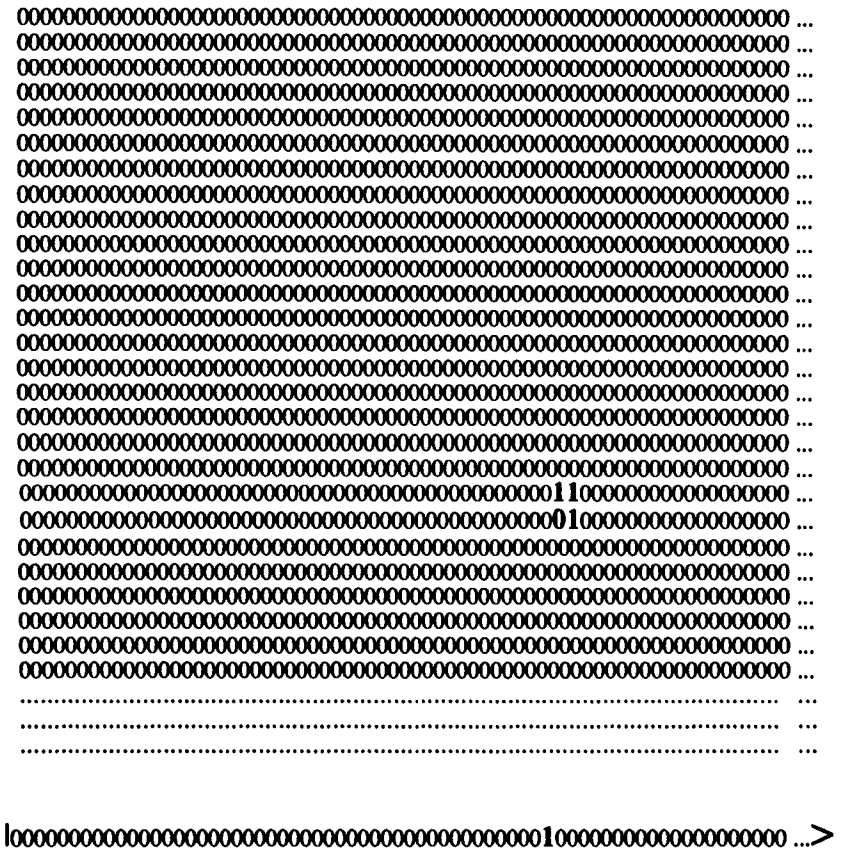


Fig. 32
A sparse matrix with a single operator impregnation
and the corresponding hyperBoolean basis eigenstate.

Sparse matrix problems require special techniques which avoid or reduce the storage of zero elements and work only with nonzero entries. An important problem is that of solving large sparse linear systems, where the main goal is to develop parallel iterative solution methods that are at the same time robust

and efficient. For small problems, direct solution methods like Gaussian elimination are usually preferred because of their reliability. However, direct methods are not feasible for large problems because of the very high memory requirements. It should be noted that memory rather than computational cost is the primary limitation of direct methods. Standard iterative methods usually consist of a conjugate gradient accelerator and a preconditioner. Often, the gradient accelerator is an optimal process which obtains the best iterate from a dynamically constructed subspace of approximations. Preconditioning is a process that helps the accelerator in some ways by transforming the original problem into a dual problem that is easier to solve. When a system is so transformed, the accelerator is likely to converge faster.

BETHE'S DOUBLE GROUP AND CONSCIOUSNESS

Consciousness is an information singularity, enabling a thinking system to observe itself by nondestructively keeping track of the logical operations. As self-reference takes effect, the emergent thoughts effectively isolate themselves from the outside universe. In systems with a nonorientable topology new effects appear: a dynamical system, which is decoupled from outside spacetime, can gain knowledge not just from outside but also from within. For too long brain science has looked to neurodynamics for a deep principle of consciousness where there is none. Mindware may be a thing in itself but it has to be physical and the quantum topology of spin enables us to connect it to fundamental principles of physics.

Theorem

Logical rotors of negation and affirmation are determined by rotation of spin toward the principal axis alignment:

$$\textbf{rot NOT} = \langle p| [\sigma_x, \sigma_z] |q\rangle$$

$$\textbf{rot YES} = \langle p| [\sigma_z, \sigma_x] |q\rangle.$$

Proof

Making use of the relation between comparison and spin operators,

$$\blacktriangle = i\sigma_y$$

and the commutation equality

$$[\sigma_i, \sigma_j] = 2i\,\sigma_k,$$

we derive:

$$\textbf{rot NOT} = 1 - 2q - (1 - 2p) = 2(p - q)$$

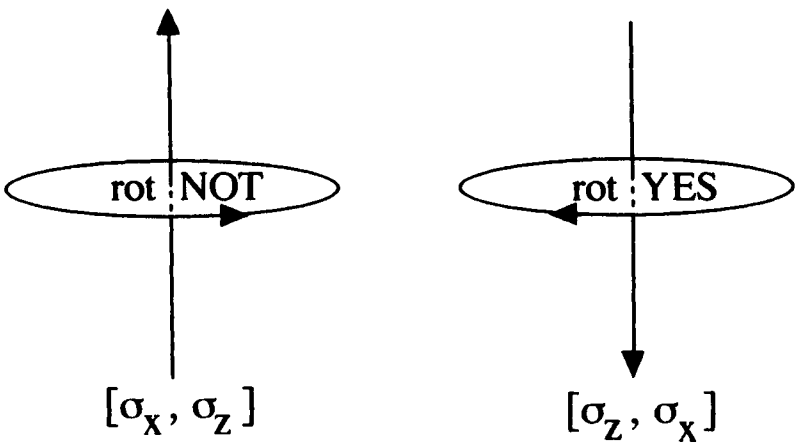
$$= \langle p| 2\blacktriangledown |q\rangle = \langle p| - 2i\sigma_y |q\rangle$$

$$= - \langle p| [\sigma_z, \sigma_x] |q\rangle = \langle p| [\sigma_x, \sigma_z] |q\rangle$$

and

$$\textbf{rot YES} = - \textbf{rot NOT} = \langle p| [\sigma_z, \sigma_x] |q\rangle.$$

The inversion of logical rotors is explicitly linked to the dynamical inversion of spin:



These equations connect the logical process and the physical brain directly. The lefthand side of the equations is logical and the righthand side is physical, joining information and quantum primitives into a single theory. The rotors of negation and affirmation, the two key logical operations, determine a quantum

information gain, acquired by unitarily rotating a spin towards the optimal axis alignment. It is almost as though the brain is to the mind what electricity is to magnetism. The divergence of cognitive information is zero, which entails that the field lines of the system are closed, creating a singularity. This closure is not a trivial loop but forms defects on the field line, the topological knots which cannot be smoothed out nor localized. In strong resemblance to closed magnetic field lines, mindware should be associated with a closed-on-itself topology, indicating that consciousness is a topological effect. The symmetry of the logical field equations is spoiled by the fact that we can observe electric charges and currents in the brain, the neuroconductor, but not logical charges. In the framework of two dual, geometrical and topological, theories of the brain we suggest that just as magnetic flux is excluded from a superconductor, the rotating spin expels logical current from a 'neuroconductor'. We obtain a fundamental definition of the thought process, elicited by the physical transformations in the brain matter. A spinor action in the brain generates a dual logical action, and a logical action gives rise to a corresponding quantum action. The two dynamical processes exist side by side and it is no longer possible to divide the system into a deductive part and an inductive part.

A distinct feature of spin symmetry, important for connecting the theory with experiment, is Bethe's double group identity

$$\psi(4\pi)= \psi(0).$$

Prior to the Stern-Gerlach experiment which identified a new quantum quantity, the spin, dogma held that any physical system under a 2π rotation should return to its initial state. Rotation on the angle 2π is the identity transformation, which should not change anything. A spinor, however, counterintuitively inverts the sign after 2π revolution:

$$\psi(2\pi)= -\psi(0).$$

Consequently, the spinor is not brought back to the original state under 2π rotation, but only after 4π rotation. Only then does the strange phase factor -1 cancel out, leading to no inconsistencies. Although the unitary spinor transformation

$$\psi' = e^{i\sigma \mathbf{n}\frac{\theta}{2}}\psi = (\cos\frac{\theta}{2} - i\sigma \mathbf{n}\sin\frac{\theta}{2})\psi$$

where $\mathbf{n}$ is the unit vector, offers two valid solutions, corresponding to the same Lorentz transformation, to avoid ambiguity we assume by convention that negative spin rotation is a *fiction*, and only positive rotation is real.

Bending the laws of mathematics to suit our needs is not the best way to proceed. But that is precisely what we do, showing little respect for actual facts. The unanswered question, however, remains: where does the other

solution go? Why should it vanish without any physical reasons, except for our apparent desire to get rid of it? And even if one could find such a reason, spinor behaviour would still remain ambiguous with the changeover from left to right. If the operator P denotes parity transformation or left-right inversion of a spinor, then P^2 is the identity transformation. In ordinary circumstances one calculates the eigenvalues of P by relying on the fact that the two-times inversion must be identity

$$P^2 = 1.$$

But such a parity transformation can also be treated as 2π rotation which for the spinor will change the sign to its opposite. Therefore we must consider two adjoint parity operators and a new additional set of eigenvalues. For a true spinor $P^2 = 1$, hence, the eigenvalues are $P = \pm 1$. For a pseudospinor $P^2 = -1$, hence, the eigenvalues are $P = \pm i$.

Since quantum mechanics gives a reading only on real or Hermitian quantities and the eigenvalues of the pseudospinor are non-Hermitian, they cannot be a part of a physical measurement, nor can they enter as observables into the covariant equations of motion. But in matrix logic the eigenvalues $\pm i$ are the fundamental eigenvalues of the time operator, and the physical mechanism of consciousness can naturally take advantage of the Bethe double-group identity. What is excluded from physical situations as a nonevent turns out to be an important cognitive quantity. This scheme goes as follows. The positive branch of the double group, which we normally consider in the abbreviated physical version of the theory, corresponds to the physical degrees of freedom. The negative branch should not be thrown out because of its inconvenient complex eigenvalue, but must be associated with the cognitive degrees of freedom. When a Hermitian part of spinor evolves from a given to the final quantum state, a non-Hermitian logical state concomitantly evolves according to the rules of double-group identity. The cancellation of the factor -1 in the spinor transformation does not occur, nor is it required in logic, where one does not conjugate amplitudes to get probabilities, and where the truth-values can be negative improbabilities. Both the probabilities and improbabilities are explicit and enter into logical descriptions directly.

Placing double-group identity at the foundation of the cognitive mechanism leads to a nontrivial conclusion. Consciousness cannot depend on logical states linearly. Consequently cognitive logic may violate the linear superposition principle and unitary evolution, the underlying principles of quantum mechanics. To show explicitly the important role of the Bethe double-group identity for the topology of consciousness, we must first find the knot function of a spinor explicitly. Since

$$ZIP\ |K_{\pm}(n)\rangle = |S_{\pm}\rangle$$

this identifies the integrand of the knot function:

$$|\mathbf{K}_{\pm}(n)> \; = \; \frac{1}{2\pi}\begin{pmatrix} \cos^2\theta \\ \pm\sin^2\theta \end{pmatrix}$$

We eliminate the topological defect and zip up the knot into a nonorientable topology, provided the integration limits agree with the Bethe double-group identity:

$$\int_0^{4\pi} |\mathrm{Knot}_{\pm}> d\theta \; = \; \frac{1}{2\pi}\int_0^{4\pi}\begin{pmatrix} \cos^2\theta \\ \pm\sin^2\theta \end{pmatrix} d\theta \; = \; |S_{\pm}>$$

where we explored the fact that

$$\int_0^{4\pi} \cos^2\theta \, d\theta \; = \; \int_0^{4\pi} \sin^2\theta \, d\theta \; = \; \frac{1}{2}\int_0^{4\pi} (\cos^2\theta + \sin^2\theta) d\theta \; = \; \frac{1}{2}\int_0^{4\pi} d\theta = 2\pi$$

Knot-free logical states can be treated as pure phases with the unit amplitude:

$$|q> \; = \; \begin{pmatrix} \cos^2\theta \\ \sin^2\theta \end{pmatrix}$$

and the circumference of the circle 2π. Multiplying all states of the brain by the same phase factor produces no observable physical manifestations. If a strictly coherent state is obtained, a knot can be formed with the circumference 4π which is twice the circumference of the circle. This is encoded in the knot's amplitude:

$$|\mathbf{K}_{\pm}(n)> \; = \; \frac{1}{2\pi}\begin{pmatrix} \cos^2\theta \\ \pm\sin^2\theta \end{pmatrix}$$

where $\pm$ accounts for the left and the right knots.

The spinor double-group identity is directly linked to the topological theory of the brain, identifying the integration limits for the ***ZIP*** operation which recovers the nonorientable state of coherent superposition $|S_{\pm}>$ from a knot. Consider the pure phase wave with the unit amplitude

$$e^{-i\theta} \; = \; \cos\theta - i\sin\theta$$

and the corresponding logical state

$$|q> \; = \; \begin{pmatrix} \cos^2\theta \\ \sin^2\theta \end{pmatrix}$$

The circumference of a knot is 4π, which is twice the circumference of the circle,

$$\cos^2\theta + \sin^2\theta = 1,$$

and because the knot is a 'defect', its components are entangled:

$$\frac{1}{\sqrt{2\pi}} e^{\pm i\theta} = \frac{1}{\sqrt{2\pi}} (\cos\theta \pm i \sin\theta)$$

where $\pm$ distinguishes between the left and the right knots. The corresponding logical state acquires a prefactor

$$|\mathbf{K}_{\pm}(n)> = \frac{1}{2\pi} \begin{pmatrix} \cos^2\theta \\ \pm \sin^2\theta \end{pmatrix}$$

Because spin is important in the relativistic framework, the fact that spin enters logic in a fundamental way gives one the feeling that the physics of the brain may eventually prove to be inherently relativistic and even contribute to further advancement of relativity theory.

Consciousness is a state of coherent superposition which has to be actively maintained. The role of spin is fundamental for consciousness because with its double group identity it offers a natural physical process in coherent superposition

$$\psi(2\pi) * \psi(0)$$

Coherent states can manifest themselves spontaneously but quickly decohere before it is possible to observe and make use of them. Basically, in the quantum universe there is not enough time for a Schrödinger *kitten* to mature into a Schrödinger cat. The essence of cognitive mechanics lies in extending the lifetime of the states of coherent superposition. Treating this extension of lifetime relativistically as a constant time dilation with the factor 2, one excludes time from the system. For a nonorientable topology of a spinor, the identity is not 2π but 4π. The speed of propagation of a topological charge on a spinor loop is then approximated by

$$v = \sqrt{\frac{3}{4}}\, c$$

where c is the fundamental velocity. This would require the basic code of the brain to operate at surprisingly high subrelativistic velocities, which obliges one to consider the microphysical basis of consciousness.

Logical operators are invariant under the combined operation of symmetry: the orthogonal transformation, transposition and converse transposition,

$$\theta \tilde{\tilde{L}} = L$$

When these operations are put in correspondence with time reversal, space inversion and charge conjugation respectively, one arrives at a matrix-logical analogue of the CPT-theorem [Ref 88]. The time reversal transposes cause and effect, and implication is conserved under any transformation in which the

swap of cause and effect is accompanied by the swap of true and false. This 2-dimensional model breaks down in an explicitly relativistic 4-dimensional formulation. In matrix logic the (2×2) logical operators act on 2-dimensional logical states. In reverse manner to the second quantization, where the wavefunction is turned into an operator acting on more general quantum states, we convert logical operators into logical states. The operators, whose role is to act on logical states, are now themselves transformed into logical states, to be acted upon by the relativistic (4×4) operators of the spatial inversion P, the time reversal T and the charge conjugation C:

$$P = \begin{pmatrix} -1 & & & 0 \\ & -1 & & \\ & & -1 & \\ 0 & & & 1 \end{pmatrix}, \quad T = \begin{pmatrix} 1 & & & 0 \\ & 1 & & \\ & & 1 & \\ 0 & & & -1 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & & & -1 \\ & 1 & & \\ & & -1 & \\ 1 & & & 0 \end{pmatrix},$$

where P and T are symmetric and C is antisymmetric

$$\tilde{C} = -C.$$

We change over from a (2×2) operator **L** to a 4-dimensional state $|L\rangle$, writing it as a bispinor [Ref 88]:

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \Rightarrow \begin{pmatrix} a \\ c \\ b \\ d \end{pmatrix}.$$

For example, the operators **TRUE** and **FALSE** are transformed into the following states:

$$\textbf{TRUE} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}, \qquad \textbf{FALSE} \Rightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}.$$

The 4-dimensional logical states are not invariant under the CPT operation which converts **TRUE** to **FALSE**:

$$CPT\ |\textbf{TRUE}\rangle = |\textbf{FALSE}\rangle.$$

It follows that additionally, the fourth operation is needed to preserve the logical bispinor state. It is readily calculated that this additional operation is the **Dirac** bispinor $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$ which is the 4-dimensional **NOT**:

$$\gamma_5 CPT\ |\textbf{TRUE}\rangle = |\textbf{TRUE}\rangle.$$

LOGICAL OSCILLATOR

Oscillatory behaviour is typical of many physical and biological systems. The brain is in fact a complex oscillator which we understand only vaguely. In spite of seemingly monotone and repetitive rhythms, it produces a diverse and constantly changing train of thoughts. We know about a large repertoire of psychological and physiological oscillations, some of which are reflected in much studied electroencephalographical activity (EEG). But does this oscillatory behaviour extend to the realm of cognitive logic? Can we, taking clue from a Hamiltonian harmonic oscillator, gain insight into the underlying code of the logical oscillator of the brain? In this section we give an affirmative answer to this question. A harmonic oscillator, which is one of the basic concepts both in classical and quantum physics, has a direct relevance to cognitive logic. SELF is a fixed point, but consciousness endlessly oscillates.

To describe the dynamics of the logical oscillator the first thing we need to do is to identify the Hamiltonian function of the system. In an unexpected turn of events we discover a direct link between the logical oscillator and a quantum oscillator. Given that the logical momentum and energy must be quadratically related

$$<\mathbf{H}> = <\mathbf{M}>^2,$$

one deduces the explicit form of the logical **Lagrangian H** which synergetically joins the affirmation and negation operators:

$$\mathbf{H} = \mathbf{YES} - \mathbf{NOT} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}.$$

and

$$<\mathbf{H}> = (\bar{q} - q)^2$$

whereas

$$<\mathbf{M}> = (\bar{q} - q).$$

This form of the **Lagrangian** agrees with the derivation of the **Lagrangian** as the second-order mixed derivative of the logical density matrix

$$\frac{d^2\Omega}{dpdq} = \mathbf{H}.$$

It also correctly identifies **NOT** as the logical evolution operator:

$$e^{-\mathbf{H}} = \mathbf{NOT}.$$

Taking into account both signs of the corresponding matrix dimension, we deduce the logical **Hamiltonian**

$$\mathbf{1} = \mathbf{YES} + \mathbf{NOT} = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix},$$

which is consistent with

$$e^{-1} = -\,\mathrm{NOT}.$$

The operators **H** and **1** are the logical synergies which commute

$$[\mathbf{H}, \mathbf{1}] = \mathbf{0},$$

hence share a common set of the eigenstates and the pair of logical quantum numbers

$$\lambda_{1,2} = 0, 2.$$

As bisymmetric operators the synergies commute with the logical evolution operator, hence are invariant under the unitary transformations:

$$\mathbf{NHN} = \mathbf{H} \quad \text{and} \quad \mathbf{N1N} = \mathbf{1}.$$

Another way to state the same fact is to say that the synergies are unaltered with a changeover from the Schrödinger to the Heisenberg logical picture:

$$\mathbf{H}_S = \mathbf{H}_H \quad \text{and} \quad \mathbf{1}_S = \mathbf{1}_H.$$

It must be emphasized that **N** shares eigenstates with **H** and **1** but there are no common logical quantum numbers, which for **N** are $\lambda_{1,2} = -1, 1$.

We now will determine that the quantum formalism provides a meaningful link to logic, indicating that the logical oscillator can be studied in quantum-mechanical fashion. Consider an integer digital form ($\hbar$=1) of a quantum oscillator subjected to a perturbation force [Ref 91]. In terms of the annihilation and creation operators its Hamiltonian is

$$H = \{a^*, a\} \pm (a^* + a),$$

where the sign "±" determines a congruent or negative perturbation.
Writing the Hamiltonian explicitly, we determine that the two forms of the quantum Hamiltonian are exactly the two fundamental logical energy operators:

$$H = \begin{cases} \{a^*, a\} + (a^* + a) = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \\ \{a^*, a\} - (a^* + a) = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}. \end{cases}$$

Because the logical **Hamiltonian 1** is the universally constant tautology, independent of the truth-value of logical variables:

$$\langle \mathbf{1} \rangle \equiv 1,$$

it is more suitable for our purposes to make use of the **Lagrangian H** which varies. In the Boolean limit, $q, \bar{q} \in E_2$, the average $\langle q|H|q\rangle = 1 - 4q + 4q^2$ is at the maximum

$$\langle \mathbf{H} \rangle = 1$$

and is at the minimum $\langle \mathbf{H} \rangle = 0$ in the undecided state

$$q = \bar{q} = \frac{1}{2}.$$

The 'undecided' extremal point of the logical energy average is shown in Fig.33.

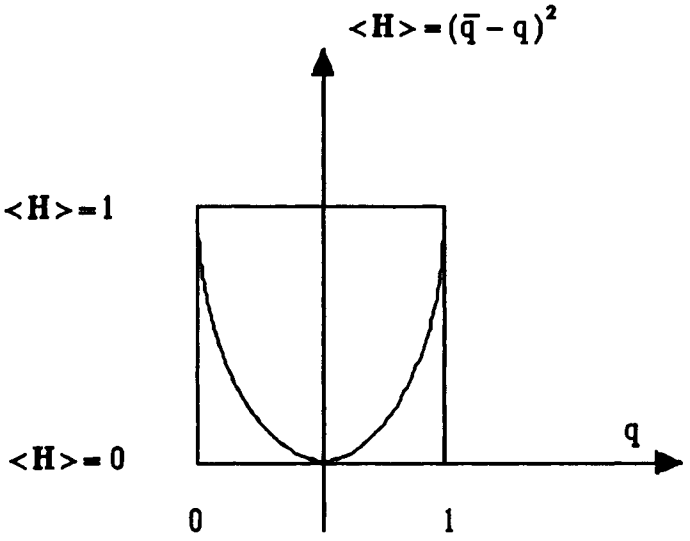


Fig 33
The average <q|H|q> as a function of truth-value.

We now turn our attention to the classical one-dimensional harmonic oscillator whose energy function is given by

$$H = \tfrac{1}{2}(p^2 + \omega^2 x^2)$$

where the momentum p and position x are the canonical phase coordinates. With the aid of the corresponding Hamiltonian equations:

$$\frac{dx}{dt} = \frac{\partial H}{\partial p} = p, \qquad \frac{dp}{dt} = -\frac{\partial H}{\partial q} = -\omega^2 q,$$

one derives the equation of motion for the harmonic oscillator

$$\frac{d^2x}{dt^2} = -\omega^2 q$$

To obtain the equation of motion for a logical oscillator, we adopt a similar approach and consider the canonical coordinates: the truth-value q and the logical momentum <M>. Making use of the master equations we derive:

$$\frac{dq}{dt} = \frac{\partial <H>}{\partial <M>} = \frac{\partial(\bar{q} - q)^2}{\partial(\bar{q} - q)} = 2(\bar{q} - q) = 2<M>$$

and

$$\frac{d<M>}{dt} = -\frac{\partial <H>}{\partial q} = -\frac{\partial(\bar{q}-q)^2}{\partial q} = 4-8q = 4<M>.$$

We determine that the rate of change of the logical momentum is twice the rate of change of the truth-value:

$$\frac{d<M>}{dt} = 2\frac{dq}{dt}.$$

Differentiating the first equation a second time, with the aid of the second, we derive the equation of motion for the logical oscillator which shows that logical oscillation is monitored by the octal logical momentum:

$$\frac{d^2q}{dt^2} = 2\frac{d<M>}{dt} = 8<M>.$$

This result unexpectedly provides support for the hypothesis of the intelligence code whose codons, to satisfy the requirement of self-duality of the code, involve **8** spins [Ref 91, 93]. Exploring the singularization theorem, the equation of motion for the logical oscillator takes the fundamental form

$$\frac{d^2q}{dt^2} = <8\sigma>$$

where the factor **8** can be interpreted as the Boolean tautology matrix index,

$$\sum_{i=1}^{16} L_i = 8 \cdot 1.$$

The dynamics of the logical oscillator is determined by the logical momentum

$$\frac{dq}{dt} = 2<M>.$$

It follows that in the undecided state $\bar{q} = q$ the logical momentum falls to zero,

$$\frac{dq}{dt} = 0$$

A nonzero logical momentum is needed for thoughts to occur, which might be intuitively clear, but now we have determined that fact from the formalism alone.

Making use of the master equation for the logical oscillator, it is possible to include the annihilation and creation operators in our formalism explicitly, which leads to the mixed differential equations:

$$\frac{dq}{dt} = 2\frac{d<a^*>}{dq}, \qquad \frac{dq}{dt} = 2\frac{d<a>}{dq}.$$

Likewise, with the aid of the differential form of the logical momentum, we can introduce the explicit dependence on truth-value:

$$\frac{dq}{dt} = 2<-\frac{d}{dq}> = -\frac{d}{dq}<q|q>$$

where we explore the fact that the derivative of the logical inner product is the average of the derivative, expanded by a factor of 2. The duplicating *genetic*

factor 2, constantly reappearing in the formalism of the logical oscillator, reflects fundamental properties of the intelligence code, suggesting the existence of a common biological composition rule. Change of the truth-value by $\pm\Delta q$ changes the logical momentum by $\pm 2\Delta q$. It is symptomatic, that the logical synergy operator duplicates logical momentum:

$$\mathbf{H}\,\mathbf{M} = 2\mathbf{M}.$$

Finally, excluding time entirely from the equations of motion, we obtain a purely logical time-independent description of the logical oscillator:

$$-\frac{d}{dq}<q|q> \;=\; 2<-\frac{d}{dq}>,$$

$$\frac{d<\mathbf{H}>}{dq} \;=\; 2\frac{d}{dq}<q|q>.$$

Subjected to dynamical perturbation, a logical oscillator will rotate between complementary falsum and verum states. In the limiting case these oscillations take the form of quantum jumps between two Boolean truth levels. Since the evolution operator is the operator sum of the annihilation and creation:

$$\mathbf{N} = a^* + a,$$

should a logical state be occupied (true), it will be annihilated into a false state, and should it be empty (false), then the true state will be created.

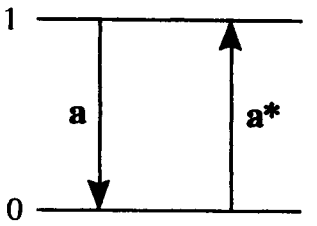


Fig. 34
Discrete truth levels of logical oscillator.

In the classical limit the oscillator is quantized into a pair of separable truth levels determined by the pair of logical quantum numbers 1 and 0 of the number operator:

$$\mathbf{AND} = a^*a.$$

If logical inference is self-referential, then along a closed orbit we may encounter quantized coherent superposition which mixes the mutually exclusive propositions. This is the case with the Liar paradox where periodic oscillations override the law of Excluded Middle, leading to absurdity and self-intersection. In the Liar we encounter a consecutive change between the two mutually exclusive propositions. However, what is a major paradox in

classical theory is a natural property of the logical oscillator evolving in a self-referential mode along a closed orbit.

In the theory of the logical oscillator which we have developed here time plays the role of a parameter, and not of a dynamical variable. We derived the truth-dependent equations of motion from which time is excluded altogether. However, to gain proper insight into the nature of cognizing systems we need to account for time in a fundamental way. To reintroduce time we need to raise it to the rank of an observable.

TIME AS CONSCIOUSNESS' DYNAMICAL VARIABLE

Matrix theory reveals a fundamental link between consciousness and time. The understanding of the nature of time, both in physics and in logic, has never been easy and the role of time remains controversial and unclear. Relativity theory unravels the fundamental link between motion and time and treats time on an equal footing with spatial degrees of freedom. The notion of a spacetime event is a basic concept of relativity, but in quantum mechanics it is defined approximately because quantum events are indeterminate and probabilistic. In quantum physics the way we treat time is fundamentally different from the way in which space is treated. As opposed to the position coordinate, time is not a dynamical variable. Many attempts by quantum theorists to raise time to the rank of an observable were unsuccessful. In quantum-mechanical equations time continues to enter as an external parameter, and in a laboratory we are usually concerned with determining the values of physical quantities at a certain instant of time. However, one may also ask for the instant of time at which an observable quantity yields a certain value. To answer this question time must be raised to the rank of a dynamical variable.

It must be clearly understood that the prominent status of time in relativity theory did not transform it into a dynamical variable either. To become an observable physical quantity, time must depend on the initial state of the system and on its dynamical evolution. More importantly, to incorporate time as a dynamical variable into the quantum formalism we must be able to

represent it by a Hermitian operator. But the attempts to identify a suitable quantum operator were in vain, no such time operator being found.

In the thinking brain the role of time becomes even more crucial. Thought runs forward, understanding runs backward. Time is an organic integral part of the cognizing mechanism which manipulates time in a quite spectacular manner. This becomes apparent in dreams, in the effect of déjà-vu and in the state of hypnosis. Without time logical consciousness and SELF simply would not exist. However, we are led astray if we try to apply the usual physical rule for time in cognitive logic. Time is given directly to consciousness, it can be altered, reversed or paradoxically cancelled altogether. To deal with the paradoxes of consciousness we need a time operator. How can we incorporate time in logic? Just as to relativity linked time and space, matrix theory links time and logical consciousness. But to unravel this link we must take a bold step, and instead of trying to force time into the procrustean bed of Hermitian physics, we consider it to be a non-Hermitian quantity. In context of quantum field theory we define the time operator as the difference between the annihilation and creation operators:

$$\mathbf{TIME} = a^* - a,$$

A particular direction of time is determined by the imbalance towards creation or annihilation. Treating time as a logical dynamical variable, we therefore can represent it by the operators $\blacktriangledown$ and $\blacktriangle$ which are the fundamental operators of comparison, basic to intelligent operations:

$$\blacktriangledown = a^* - a, \qquad \blacktriangle = a - a^*$$

or equivalently in pure logical formulation:

$$\blacktriangledown = [\wedge, \vee], \qquad \blacktriangle = [\vee, \wedge].$$

This sets up the framework in which time is raised to the status of a dynamical variable. We begin by explaining the reasons for choosing the comparison operators to represent time. It is well-known that the function of comparison is critical for logical actions. Neither the deduction nor the induction process can proceed if the comparison operation is not available in one form or another. This operation is realized by the operators of comparison which evaluate the difference between the truth status of logical states

$$<p|\blacktriangledown|q> = p - q = \bar{q} - \bar{p}, \qquad <p|\blacktriangle|q> = q - p = \bar{p} - \bar{q}.$$

Comparison is an antisymmetric operator, and the bracket product will be reduced to zero, if the two states are identical:

$$<q|\blacktriangledown|q> = <q|\blacktriangle|q> \equiv 0.$$

Comparison can be also meaningfully treated as the vector product, generating the two, opposite in sign, quantities:

$$<p|\blacktriangledown|q> = p\bar{q} - \bar{p}q, \qquad <p|\blacktriangle|q> = \bar{p}q - p\bar{q}.$$

The results which we have presented provide a motivation for interpreting the comparison operators as time operators, and we will simply write **TIME** in place of $\blacktriangledown$ and $\blacktriangle$, as the need arises. The fundamental time operators are unitary and antisymmetric:

$$<p|\mathbf{TIME}|q> = -<q|\mathbf{TIME}|p>.$$

$$\mathbf{TIME}^{+} = \mathbf{TIME}^{-1} = \widetilde{\mathbf{TIME}} = -\mathbf{TIME}.$$

Any unaltered content of consciousness is still time:

$$\langle q|\mathbf{TIME}|q\rangle \equiv 0.$$

Introduce change and time begins to flow

$$\langle p|\mathbf{TIME}|q\rangle \neq 0.$$

We now consider the eigenequation for the time operator:

$$\mathbf{TIME}|q\rangle = \lambda\,|q\rangle,$$

$$|\mathbf{TIME} - \lambda\,\mathbf{I}| = \lambda^2 + 1 = 0,$$

$$\lambda_{1,2} = \pm i.$$

It is solved with the imaginary eigenvalues $\pm i$ and complex eigenstates

$$\begin{pmatrix} 1 \\ i \end{pmatrix} \text{ and } \begin{pmatrix} i \\ 1 \end{pmatrix}.$$

A fundamental feature of the time operators is that the eigenstates are not orthogonal and therefore the subspaces in which they reside are not separable. Instead, the eigenstates are self-orthogonal:

$$(1 \;\; i)\begin{pmatrix} 1 \\ i \end{pmatrix} = 0, \qquad (i \;\; 1)\begin{pmatrix} i \\ 1 \end{pmatrix} = 0.$$

Diagonalizing the time operators,

$$\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\mathbf{TIME}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}^{-1}$$

we determine that $\blacktriangledown_{diag}$ and $\blacktriangle_{diag}$ are the pure imaginary traceless matrices

$$\mathbf{Tr} \pm \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} = 0$$

and inverses of each other

$$\blacktriangledown_{diag} \bullet \blacktriangle_{diag} = \mathbf{I},$$

as the time operators themselves:

$$\blacktriangledown\blacktriangle = \mathbf{I}.$$

There is a fundamental relation, linking time and the energy operators:

$$\blacktriangledown = e^{-\mathbf{M}} - \mathbf{H}_{diag}$$

where the exponent series of the momentum finitely terminate. Since the time average is a vanishing still time,

$$\langle e^{-\mathbf{M}} \rangle - \langle \mathbf{H}_{diag} \rangle = 0$$

we get

$$\langle e^{-\mathbf{M}} \rangle = \langle \mathbf{H}_{diag} \rangle.$$

In quantum mechanics complex quantities are essential and explicit. But in logic they are hidden in the time operators whose eigenvalues are the imaginary numbers $\pm i$. **TIME** is a matrix representation of the imaginary unit, not measurable quantum-mechanically but cognitively. In the logical momentum the time and spin operators enter as the anticommuting cognitive and quantum-mechanical components:

$$M = \blacktriangle + \sigma$$

and

$$\{\blacktriangle , \sigma\} = 0$$

where σ is quantum-mechanical observable, hence Hermitian, and $\blacktriangle$ is cognitive observable, hence non-Hermitian. When consciousness registers a logical momentum it in fact measures time. The combined action of the time operator and spin operator on a normed logical state gives rise to a superposed state:

$$\mathbf{TIME}\begin{pmatrix}\text{FALSE}\\ \text{TRUE}\end{pmatrix} + \mathbf{SPIN}\begin{pmatrix}\text{FALSE}\\ \text{TRUE}\end{pmatrix} = \begin{pmatrix}\text{TRUE}\\ \text{-FALSE}\end{pmatrix} + \begin{pmatrix}\text{FALSE}\\ \text{-TRUE}\end{pmatrix} = \begin{pmatrix}1\\ -1\end{pmatrix}$$

Here the time operator swaps the true and false, negating the true, while the spin operator keeps them in place, negating the true. When the two results join, we have a coherent cognitive superposition.
The time operators convert logical energy into a logical momentum:

$$H\blacktriangle = M \quad \text{and} \quad \blacktriangle 1 = M$$

while the inverse time provides for the reverse transformation:

$$M\blacktriangledown = H \quad \text{and} \quad \blacktriangledown M = 1$$

With the aid of the spin operator σ, instead of the different times, these transformations can be accommodated by a single quantum operation:

$$H\sigma = M, \quad \sigma 1 = M,$$

$$M\sigma = H, \quad \sigma M = 1.$$

Theorem

To obtain the inversion of time it is necessary simultaneously to negate the logical momentum and spin which amounts to their simultaneous unitary logical rotation:

$$\blacktriangledown = \theta M \cdot \theta\sigma = -\blacktriangle$$

Proof

$$\begin{aligned}\blacktriangledown &= \theta M \cdot \theta\sigma \\ &= \mathbf{NOT} \bullet M \bullet \mathbf{NOT} - \mathbf{NOT} \bullet \sigma \bullet \mathbf{NOT} \\ &= -M + \sigma \\ &= \blacktriangledown = -\blacktriangle\end{aligned}$$

STILL TIME

When time flows the time operator and the energy operators neither commute nor anticommute:

$$[H, \blacktriangle] \neq 0, \qquad \{H, \blacktriangle\} \neq 0.$$

But the still time has to be a constant of motion. Calculation shows and quite extraordinarily that the only nontrivial (anti)commutations are with the Hadamardians, representing the unbroken topological phase:

$$[\blacktriangledown, .H] = 0, \qquad [\blacktriangledown, \dot{H}] = 0$$

and

$$\{\blacktriangledown, H.\} = 0, \qquad \{\blacktriangledown, \dot{H}\} = 0$$

Time induces the transposition or converse transposition of the unbroken topological phase H(·):

$$.H\blacktriangledown = \dot{H}, \qquad \dot{H}\blacktriangledown = H.$$

$$\dot{H}\blacktriangle = .H, \qquad H.\blacktriangle = \dot{H}$$

and is invariant under the fundamental generic transformation:

$$\rightarrow \bullet \mathbf{TIME} \bullet \leftarrow = \mathbf{TIME}$$

where the 'colliding' implications vanish, leaving the **TIME** operators intact:

$$\widetilde{IF} \bullet \blacktriangle \bullet IF = \blacktriangle \qquad \text{and} \qquad \widetilde{IF} \bullet \blacktriangledown \bullet IF = \blacktriangledown.$$

Linked to the transitive matrix implication, time follows the complex rules of symmetry. While the orthogonal logical transforms

$$N \rightarrow N = \leftarrow, \qquad N \leftarrow N = \rightarrow$$

are simply the transpositions, the time orthogonal transform rotates the implication to the inverse of the converse:

$$\mathbf{TIME} \bullet \rightarrow \bullet \widetilde{\mathbf{TIME}} = \leftarrow^{-1}, \qquad \mathbf{TIME} \bullet \leftarrow \bullet \widetilde{\mathbf{TIME}} = \rightarrow^{-1}$$

while the spin orthogonal transforms give simply the inverse

$$\sigma \rightarrow \sigma = \rightarrow^{-1}, \qquad \sigma \leftarrow \sigma = \leftarrow^{-1}.$$

THE TIME WRAPS

Time, spin and the logical momentum are negated under unitary transformation:

$$\theta\blacktriangle = -\blacktriangle, \qquad \theta\sigma = -\sigma, \qquad \theta M = -M,$$

all anticommute with **NOT**:

$$\{M, \mathbf{NOT}\} = \{\sigma, \mathbf{NOT}\} = \{\mathbf{TIME}, \mathbf{NOT}\} = 0.$$

This fact is important. In quantum physics the observables, if they to be known simultaneously, must **commute**. In matrix logic observables can share

common eigenstates when they **anticommute**. Logical momentum and **NOT** share a common eigenstate but do not commute.
The logical momentum can be swapped with time in the commutator:

$$\langle q[\blacktriangledown, H]q\rangle = \langle q[M, H]q\rangle,$$

which reduces to the singularization theorem

$$\langle q|\sigma|q\rangle = \langle q|M|q\rangle.$$

Being wrapped in circles, cognitive time is distinguished by the quatric identity and is intrinsically periodic. Subjected to the iterating action of the time operator, a logical state will be rotated to itself in **4** consecutive steps:

$$\begin{pmatrix}\bar{q}\\ q\end{pmatrix} \Rightarrow \begin{pmatrix}q\\ -\bar{q}\end{pmatrix} \Rightarrow \begin{pmatrix}-\bar{q}\\ -q\end{pmatrix} \Rightarrow \begin{pmatrix}-q\\ \bar{q}\end{pmatrix} \Rightarrow \begin{pmatrix}\bar{q}\\ q\end{pmatrix}$$

SPIN transforms the antiunitary **TIME** into the unitary **NOT** and vice versa:

$$\blacktriangledown\sigma = N, \qquad N\sigma = \blacktriangledown.$$

In the commutator format the role of **SPIN** is taken over by **AND**:

$$[\wedge, \blacktriangledown] = N, \qquad [\wedge, N] = \blacktriangledown.$$

Then by nesting:

$$\blacktriangledown = [\wedge, [\wedge, \blacktriangledown]]$$

$$\blacktriangledown = [\wedge, [\wedge, [\wedge, [\wedge, \blacktriangledown]]]]$$

$$\blacktriangledown = [\wedge, [\wedge, [\wedge, [\wedge, [\wedge, [\wedge, \blacktriangledown]]]]]]$$

$$\blacktriangledown = [\wedge, [\wedge, [\wedge, [\wedge, [\wedge, [\wedge, [\wedge, [\wedge, \blacktriangledown]]]]]]]]$$

Theorem

Dual logical operators are equivalent in the commutator with the logical energy:

$$[L, H] = [L^*, H]$$

Proof

The left hand side evaluates to the commutator with **NOT**:

$$[L, H] = L(I - N) - (I - N)L = L - LN - L + NL = [N, L].$$

The right hand side evaluates to the same commutator:

$$\begin{aligned}[L^*, H] &= (1 - NLN)H - H(1 - NLN)\\ &= -NLNH + HNLN\\ &= NLH - HLN\\ &= NL(I - N) - (I - N)LN\\ &= NL - NLN - LN + NLN\\ &= NL - LN\\ &= [N, L]\end{aligned}$$

Consequences

Time generation:

$$[\mathbf{AND}, H] = [\mathbf{OR}, H] = \blacktriangle$$

Momentum generation:

$$[\mathbf{TRUE}, H] = [H, \mathbf{FALSE}] = M$$

Spin realization:

(1)

$$[(\mathbf{AND}+\mathbf{OR}), H] = 2\blacktriangle = [\sigma_z, \sigma_x]$$

which is

$$[+, H] = 2\blacktriangle = [\sigma_z, \sigma_x]$$

where $+ = \mathbf{AND}+\mathbf{OR}$.

For the unitary transform:

$$[\theta+, H] = [(\downarrow + \uparrow), H] = -2\blacktriangle = 2\blacktriangledown = [\sigma_x, \sigma_z]$$

Since $+$ is the logical dual of spin

$$+ = \sigma_z^*$$

and

$$(\downarrow + \uparrow) = (-\sigma_z)^*,$$

we get

$$[\sigma_z^*, H] = [\sigma_z, \sigma_x]$$

which, in view of the equality $[L, H] = [N, L]$, is reduced to the almost quantum-mechanical form:

$$[\sigma_x, \sigma_z^*] = [\sigma_z, \sigma_x]$$

(2)

$$[\mathbf{IF}^2, H] = [\blacktriangledown^*, H] = 2\sigma_z = i[\sigma_y, \sigma_x]$$

which for the unitary transform changes the spin orientation to its opposite:

$$[\theta\blacktriangledown^*, H] = -2\sigma_z = i[\sigma_x, \sigma_y]$$

Theorem

The averages of the complements and duals of time are tautologies:

$$\langle q|\overline{\blacktriangledown}|q\rangle = \langle q|\blacktriangle^*|q\rangle \equiv 1,$$

$$\langle q|\overline{\blacktriangle}|q\rangle = \langle q|\blacktriangledown^*|q\rangle \equiv 1.$$

Proof

Since the average of the time operators is universally empty:

$$\langle q|\blacktriangle|q\rangle = \langle q|\blacktriangledown|q \equiv 0,$$

their complement must be universally true:

$$\langle q|\overline{\blacktriangle}|q\rangle = \langle q|\overline{\blacktriangledown}|q\rangle \equiv 1.$$

But the complement of time is a dual of the reversed time:

$$\overline{\blacktriangledown} = \blacktriangle^*, \qquad \overline{\blacktriangle} = \blacktriangledown^*.$$

and the averages of the duals are universally true too.

The complements and duals of the time operators represent the combined operation of symmetry involving the annihilation and creation operators:

$$\overline{\blacktriangledown} = \overline{a^* - a} = \overline{a^*} + a = \rightarrow^2 = \blacktriangle^*,$$

$$\overline{\blacktriangle} = \overline{a - a^*} = \overline{a} + a^* = \leftarrow^2 = \blacktriangledown^*.$$

From

$$a^* - a = \overline{a} - \overline{a^*} = \leftarrow - \rightarrow$$

deduce

$$a^* - a = a^* \leftarrow - \rightarrow a,$$

which reduces to the commutator of the canonical pair of logical operators

$$\blacktriangledown = a^* - a = a^* \leftarrow - \rightarrow a = \perp \; - \dashv = \wedge \vee - \vee \wedge = [\wedge, \vee],$$

$$\blacktriangle = [\vee, \wedge].$$

The time commutator is unchanged with orthogonal rotation of the operators:

$$\blacktriangledown = [\downarrow, \uparrow], \qquad \blacktriangle = [\uparrow, \downarrow].$$

It may be clear that the **TIME** operator gives a reading on the degree of orthogonality of logical states. Whenever the time bracket is an integer, the states are orthogonal and can be discriminated. Comparison of the orthogonal states yields the integers ± 1, if the states are Boolean, and ± 2 if they are superposed:

$$\langle 1|0\rangle = 0: \qquad \langle 1|\mathbf{TIME}|0\rangle = \pm 1$$

$$\langle s_+|s_-\rangle = 0: \qquad \langle s_+|\mathbf{TIME}\ |s_-\rangle = \pm 2.$$

For nonorthogonal logical states, $\langle p|q\rangle \neq 0$, the bracket is a fraction on the interval (-1,1) without boundaries:

$$\langle p|\mathbf{TIME}|q\rangle \in (-1,1)$$

IS THERE TIME TO SEE? STROBOSCOPIC CONSCIOUSNESS

The time scale of thinking and the time scale of consciousness are significantly far apart, which gives rise to a conjecture that consciousness cannot be analyzed because it is too fast, while thinking is too slow to allow observation of its details. To 'see' the details, as in stroboscopic photography, one must have very fast write-in and slow read-out. One intriguing possibility is to connect cognitive time and relativistic time dilation

$$t' = t\sqrt{1 - \frac{v^2}{c^2}}$$

In this scenario the thinking operations are spacelike, and to observe them one would require tachions, with their forbidden faster-than-light speed of propagation, leaking into a spacelike region. Here may lie an explanation why the brain is "unaware" of itself. The brain cannot cognize the details of SELF but only the total SELF. Imagine on the other hand the mess we would be in, if the states of every neuron in the brain were available for observation. There is a 'mysterious' phenomenon of *blindsight* when a person cannot consciously see introduced images, but makes a right 'guess' when asked to do so. It would be amazing but not entirely surprising if one day we determined that *blindsight* is actually a relativistic effect. Even though we usually think of *blindsight* as a neurological malfuction, there is an element of '*blindsight*' in each and every cognitive operation.

CONVERTING THOUGHT INTO ACTION

Explaining the conversion of abstract thought into biophysical action, like speech, writing or the movement of the body is one of the hardest problems in neuroscience. A solution to this problem must account for the fact that physical quantities are Hermitian, while cognitive quantities are not. Much of the mystery is taken away when we treat the interactions in the mind-brain system as a duality exchange between topological and Noether charges. This approach has allowed us to reinterpret the fundamental Planck constant $\hbar$ as the transition coefficient from (topo)logical to quantum energy:

$$H = \hbar T.$$

Through additive polarization of the logical vacuum we obtain **80** basic **(2×2)** logical particles, which of course can be compactified to a reduced basis. For example it is clear that we cut down this number to **40** nibbles, since the other half is simply a global phase shift of the chosen ones. More meaningfully, each **(2×2)** nibble can be expanded in the non-Hermitian basis [88]

$$L_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad L_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad L_2 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad L_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

which is almost like the Pauli spin basis

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = i\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},$$

except for L_2, which is anti-Hermitian while σ_y is Hermitian. Since the Pauli matrices form a basis set for any (2×2) matrix, we can put logic in direct contact with spin by expanding the logical operators in the spin basis {Ref 88]:

$$L = f(\sigma_0, \sigma_x, \sigma_y, \sigma_z).$$

In the multiplicative generation we need only derive L_2 from the spin base because all the other operators we already have. We can proceed as follows

$$L_2 = \sigma_x\sigma_z = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}.$$

However, the reverse transform of the spin in logical terms is not automatic. There is no multiplicative logical expansion for the Hermitian spin operator

$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

in the logical base $\{L_0, L_1, L_2, L_3\}$. The fundamental asymmetry becomes manifest: logic can be expressed multiplicatively via logic itself and via spin, but spin cannot be derived from matrix logic, unless we consider the complex square rooting

$$\sigma_y = \sqrt[i]{\sigma_0}.$$

Next we introduce the model of a 'logical spin', extended in the operator basis of four linear independent logical matrices which either commute or anticommute:

$$\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{N} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \blacktriangledown = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}, \qquad \boldsymbol{\sigma} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

The logical basis is in direct relation to the Pauli spin SU(2) group whose generators form the group of symmetry with cyclic closure

$$\sigma_x\sigma_y = i\sigma_z, \qquad \sigma_y\sigma_z = i\sigma_x, \qquad \sigma_z\sigma_x = i\sigma_y,$$

anticommutation

$$\sigma_x\sigma_y + \sigma_y\sigma_x = 0, \qquad \sigma_y\sigma_z + \sigma_z\sigma_y = 0, \qquad \sigma_z\sigma_x + \sigma_x\sigma_z = 0$$

and

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \sigma_0.$$

The logical spin operators also obey the anticommutation rule:

$$\mathbf{N}\blacktriangledown + \blacktriangledown\mathbf{N} = 0, \qquad \blacktriangledown\boldsymbol{\sigma} + \boldsymbol{\sigma}\blacktriangledown = 0, \qquad \boldsymbol{\sigma}\mathbf{N} + \mathbf{N}\boldsymbol{\sigma} = 0.$$

This, making use of the uniform notation

$$L_0 = \mathbf{I}, \quad L_1 = \mathbf{N}, \quad L_2 = \blacktriangledown, \quad L_3 = \boldsymbol{\sigma},$$

will be written exactly as the commutation rule for the spin operators:

$$L_1L_2 + L_2L_1 = 0, \qquad L_2L_3 + L_3L_2 = 0, \qquad L_3L_1 + L_1L_3 = 0.$$

The logical product, however, differ from the spin products:

$$\mathbf{N}\blacktriangledown = \boldsymbol{\sigma} \quad \text{while} \quad \sigma_x\sigma_y = -i\sigma_z$$

$$\blacktriangledown\boldsymbol{\sigma} = \mathbf{N} \quad \text{while} \quad \sigma_y\sigma_z = -i\sigma_x$$

$$\boldsymbol{\sigma}\mathbf{N} = -\blacktriangledown = \blacktriangle \quad \text{and} \quad \sigma_z\sigma_x = -i\sigma_y$$

Observe that only the time operator products behave similarly to spin operator products, the other giving the required operator but with the global phase being inverted. Another feature distinguishing the behavior of the logical spin from its quantum-mechanical counterpart is the nonuniformity of the squaring:

$$\mathbf{N}^2 = \sigma^2 = \mathbf{I} \qquad \text{but} \qquad \blacktriangledown^2 = -\mathbf{I},$$

whereas for spin uniformly

$$\sigma_x^2 = \sigma_y^2 = \sigma_z^2 = \mathbf{I}.$$

We now can draw some important conclusions from our analysis. The Pauli spin basis is a Hermitian system of operators which provides an adequate description of quantum-mechanical spin. The matrix logical basis $\{\mathbf{I}, \mathbf{N}, \sigma, \blacktriangledown\}$ is also functionally complete but the presence of the non-Hermitian time $\blacktriangledown$ in the basis indicates that logical spin cannot be a quantum observable. Cognitive thought is a non-Hermitian quantity and to convert the logical basis into the Pauli spin basis one acts on the **TIME** operator with the imaginary unit *i*. For the conversion of the thought process into the physical action of spin the complex unit *i* must be available. Then the logical operations can project themselves into microphysical actions, afterwards to be amplified into speech or some other cognitively controlled motion. By the same token the complex unit, applied to the spin basis, will transform it into a non-Hermitian logical basis:

$$\{\sigma_0, \sigma_x, \sigma_y, \sigma_z\} \overset{i}{\Leftrightarrow} \{\mathbf{I}, \mathbf{N}, \sigma, \blacktriangledown\}$$

In order to attain an interactive cognitive dynamics, we need *i* to be realized in the actual operations of the brain. It is available in matrix logic in the form of the **TIME** operator which is the matrix form of *i*.

An important mechanism for converting logic into quantum operations is the commutation interaction of the logical **Lagrangian H** and the self-interaction of the Hadamardians $\mathbf{H}^{\bullet}$ and .H:

$$[+, \mathbf{H}] = [\sigma_z, \sigma_x], \qquad \text{where} \quad + = \mathbf{AND+OR}$$

$$(\mathbf{H}^{\bullet})^2 = [\sigma_x, \sigma_z],$$

$$(.\mathbf{H})^2 = [\sigma_z, \sigma_x],$$

with

$$(.\mathbf{H})^2 = [+, \mathbf{H}] = -(\mathbf{H}^{\bullet})^2$$

When a current logical state of the brain, associated with an operator **L**, interacts with the unitarily rotated self $\theta\mathbf{L}$ in the commutator $[\theta\mathbf{L}, \mathbf{L}]$, the outcome can be

(a) the empty state

$$[\theta\mathbf{AND}, \mathbf{AND}] = \mathbf{0}$$

(b) a spin state

$$[\theta \mathbf{IF}, \mathbf{IF}] = \sigma_z$$

(c) the logical **TIME** twice, which reduces to the spin commutator

$$[\theta \mathbf{OR}, \mathbf{OR}] = 2\blacktriangle = [\sigma_z, \sigma_x]$$

(d) the logical momentum

$$[\theta \mathbf{TRUE}, \mathbf{TRUE}] = \mathbf{M}$$

If we do unitary transformations, we can reach the answer in a much more efficient way if we do only classical logic. Spin, time and logical momentum are three fundamental operators that are essential for converting thought into action and vice versa. Thought tells the brain how to spin. The individual strongly correlated spins align to form the ordered pattern, and in turn tell the brain how to think. The quantum-theoretic brain we are considering is fully defined by the system of equations:

$$< q|\mathbf{M}|q > \; = \; < \psi|\sigma_z|\psi >,$$

$$\text{rot}\,\mathbf{NOT} = < p[\sigma_x, \sigma_z]q >, \qquad \text{rot}\;\mathbf{YES} = < p[\sigma_z, \sigma_x]q >.$$

In view of equality,

$$\blacktriangle - \blacktriangledown = 2i\sigma_y$$

we can reformulate the logical rotors in terms of relative time:

$$< p[\sigma_x, \sigma_z]q > = < p|\blacktriangledown - \blacktriangle|q >$$

The three fundamental operators are distinguished by anticommutation with **NOT**:

$$\{\mathbf{NOT}, \mathbf{M}\} = \{\mathbf{NOT}, \sigma\} = \{\mathbf{NOT}, \mathbf{TIME}\} = 0.$$

TIME does not discriminate between logical momentum and spin in the commutator:

$$[\blacktriangledown, \mathbf{M}\,] = [\blacktriangledown, \sigma],$$

just like **NOT** in the anticommutator

$$\{\mathbf{NOT}, \mathbf{M}\} = \{\mathbf{NOT}, \sigma\}.$$

We have determined various means for conversion from logic to spin. The most distinguished case is the commutator of implications

$$[\rightarrow, \leftarrow] = \sigma.$$

Similar outcome will be given by the diagonalization of **NOT**:

$$\sigma = \mathrm{H.}^{-1} \bullet \mathbf{NOT} \bullet \mathrm{H.}$$

which is

$$\sigma = \frac{\mathrm{H.} \bullet \mathbf{NOT} \bullet \mathrm{H.}}{2}$$

where H. is the Hadamard code matrix, with the half being the inverse:

$$\mathrm{H.}^{-1} = \tfrac{1}{2}\mathrm{H.}$$

Equivalently, this conversion can be treated as unitary rotation

$$\sigma = Q \bullet \mathbf{NOT} \bullet Q^{+}$$

where Q is changed with the quantum factor $\frac{1}{\sqrt{2}}$ into the unitary matrix

$$Q = \frac{1}{\sqrt{2}} H. = Q^{+}$$

Extreme reductionism demands that the microworld determines the world of quotidian experience. Because cognitive logic is macroscopic, to address the ultimate mystery of consciousness, we need microscopic concepts. Taking this view we can begin to understand the mechanism by which the thought process proceeds from the macroscopic level all the way down to the quantum level and back. With the conversion from logic to spin the uncertainty increases and we encounter a loss of information. If the transformation is to be reversible, with conversion from spin to logic, we must have a gain in information to compensate for its loss in the direct transformation. Here lie new clues for the inventive, creative and interpreting brain, capable of generating new information, an effect which is not attainable in a purely classical information system. Although some features of classical and quantum logics are alike, differences abound, with information gain taking centrestage at microphysical level. Such a quantum gain is released to the classical level due to the conversion of {**AND**, **OR**} into a functionally complete logical basis, generating **NOT**. The logical rotors consequently bring **NOT** into principal axes alignments, which is the spin operator σ_z. The creative thought depends on the brain's intrinsic ability to manipulate quantum information.

CONSERVATION OF TIME CHARGES

A diagonalized time is a product of the quantum-mechanical spin operators

$$\blacktriangle_{diag} = \sigma_x \sigma_y = i\,\sigma_z ,$$

which explicitly puts the time in direct contact with Pauli's formalism. The integral logical charge of **TIME** is zero:

$$\mu(\blacktriangle) = \int_0^1 \langle q|\blacktriangle|q\rangle\, dq = 0$$

as is the charge of the diagonalized **TIME**:

$$\mu(\blacktriangle_{diag}) = \int_0^1 \langle q|i\sigma_z|q\rangle\, dq = 0$$

The conservation of the time logical charge $\mu(\blacktriangle) = \mu(\blacktriangle_{diag})$ is quite unexpected, since no other logical operator, except those already in

diagonalized form, like **AND**, conform to the conservation of the integral logical charge after diagonalization, for example:

$$\mu(\mathbf{H}) = \frac{1}{3} \qquad \text{but} \qquad \mu(\mathbf{H}_{\text{diag}}) = \mu(2\wedge) = \frac{2}{3}$$

$$\mu(\mathbf{ONE}) = 1 \qquad \text{but} \qquad \mu(\mathbf{ONE}_{\text{diag}}) = \frac{2}{3}$$

$$\mu(\mathbf{TRUE}) = \frac{1}{2} \qquad \text{but} \qquad \mu(\mathbf{TRUE}_{\text{diag}}) = \frac{1}{3}$$

MAKING USE OF FORBIDDEN INTERACTIONS

Every logical thought is a string of interacting nibbles. When two strings collide, and the resulting operator is not a logical operator, $\mathbf{L} \notin \Omega$, we consider such interaction forbidden. In strictly positive discrete matrix logic there are **49** forbidden products, and in complete logic **1982** are forbidden, although this number decreases in continuous logic. For example in discrete matrix logic we exclude the interaction

$$\vee^2 = (\mathbf{N}+\wedge)^2 = \mathbf{1}+\wedge = \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix}$$

because a nonbinary matrix element is manifest, but we can make use of it in fuzzy logic:

$$< p \begin{pmatrix} 0 & 1 \\ 1 & 2 \end{pmatrix} q > \; = p + q.$$

The forbidden operators become the tool we can employ to convert a thought into a physical action. What is **logically** inadmissible can be explored as a **physical** connection. With forbidden operators cognitive logic gains access to the physical brain, and vice versa the physical operations can affect logical states. In spite of the presence of nonlogical values in an operator, it may procure a meaningful logical transformation, for example:

$$\begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix} |S_-> \; = |S_+>, \qquad \begin{pmatrix} 2 & 1 \\ 1 & 1 \end{pmatrix} |S_-> \; = |0>.$$

or

$$\begin{pmatrix} 1 & 0 \\ 6 & 5 \end{pmatrix} |S_-> \; = |S_+>, \qquad \begin{pmatrix} 2 & 1 \\ 3 & 3 \end{pmatrix} |S_-> \; = |0>.$$

where the operators act as though they were logical operators:

$$\mathbf{TIME}|S_-> \; = |S_+>, \qquad \mathbf{IF}|S_-> \; = |0>.$$

ENERGY-TIME PHASE SPACE

In theoretical physics we commonly make use of the canonical momentum-position and energy-time phase spaces. The extent of simultaneous information about a pair of canonical variables differs in classical mechanics, in which there is no uncertainty in position and momentum coordinates, and in quantum mechanics which is noncommuting. In matrix logic we treat the coordinates as matrix degrees of freedom [Ref 88]. When the matrix logical coordinates commute or anticommute we have an Abelian form of logic and otherwise non-Abelian form. The matrix coordinates which are known simultaneously with the logical momentum are **YES** and **NOT**, which forms a 3-dimensional phase space,

$$[\mathbf{MOVE}, \mathbf{YES}] = \mathbf{0}, \qquad \{\mathbf{MOVE}, \mathbf{NOT}\} = \mathbf{0}.$$

Our task now is to replace ordinary time and energy phase coordinates with matrix coordinates. We begin with the identification of the time coordinate. Since matrix transposition swaps cause and effect, hence reversing the direction of time, the time coordinate can be most naturally represented by the antisymmetric time operator, introduced in the preceding sections:

$$\widetilde{\mathbf{TIME}} = -\mathbf{TIME},$$

Given that **TIME** is matrix complex unit, multiplying the time phase coordinate by the fundamental velocity **c**, the speed of light in vacuum, we get

$$\mathbf{c} \cdot \mathbf{TIME} = \mathbf{c}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & -\mathbf{c} \\ \mathbf{c} & 0 \end{pmatrix}.$$

which can enter as a matrix factor instead of *ict* in the relativistic spacetime interval. A defect of relativity theory is that it was created when fermions were not yet known. Einstein's spacetime is essentially 'bosonic', which leaves out an important part of real-world physics. To define the matrix time-energy phase space we must also take fermions into account. Since fermionic and bosonic fields are distinguished by the commutator and anticommutator identities, by making use of this criterion we can distinguish between fermionic and bosonic energy matrix coordinates. If these are to be known simultaneously with the time coordinate, the bosonic coordinate X must commute and the fermionic coordinate Y must anticommute with **TIME**, respectively:

$$[\,\mathbf{TIME}, \mathbf{X}\,] = 0, \qquad \{\mathbf{TIME}, \mathbf{Y}\,\} = 0.$$

The only nontrivial logical matrices that conform to these fundamental requirements are the Hadamard matrices H(·) which we associated with the unbroken topological phase. Consequently the matrix energy phase space is Hadamardian. Identifying the **2** bosonic coordinates:

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix},$$

and the **2** fermionic coordinates:

$$\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix},$$

we consequently arrive at **5**-dimensional time-energy phase space:

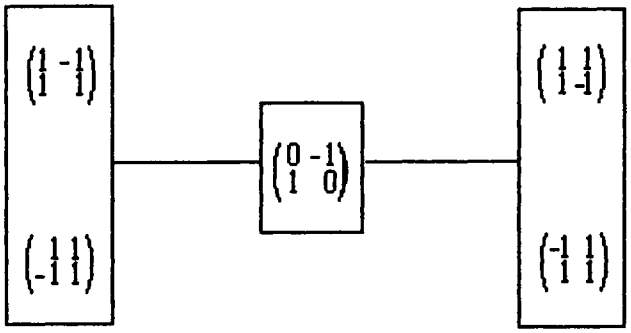


Fig. 35

5-dimensional energy-time phase space {**TIME**, $^{\bullet}$H, H., .H, H$^{\bullet}$ }.

Symmetric fermionic and asymmetric bosonic phase coordinates carry the quark logical charges:

$$\mu(Y) = 1/3, \qquad \mu(X) = 2/3,$$

suggesting a link between the **5**-dimensional energy-time phase space and nucleonic interactions.

The commutation relations

$$[\mathbf{TIME}, X] = 0 \quad \text{and} \quad \{\mathbf{TIME}, Y\} = 0,$$

entail:

$$\mathbf{TIME} \bullet X \bullet \mathbf{TIME}^{-1} = X \quad \text{and} \quad \mathbf{TIME} \bullet Y \bullet \mathbf{TIME}^{-1} = -Y$$

$$X \bullet \mathbf{TIME} \bullet X^{-1} = \mathbf{TIME} \quad \text{and} \quad Y \bullet \mathbf{TIME} \bullet Y^{-1} = -\mathbf{TIME}.$$

The bosonic dimensions commute between themselves and with **TIME**:

$$[X_1, X_2] = [X_1, T] = [X_2, T] = 0.$$

The fermionic dimensions anticommute between themselves and with **TIME**:

$$\{Y_1,Y_2\} = \{Y_1,T\} = \{Y_2,T\} = 0.$$

However, there is no symmetry transfer between the bosonic and fermionic coordinates which neither commute nor anticommute between themselves. **TIME** converts one energy coordinate into the other energy coordinate of the same class:

$$\mathbf{TIME} \cdot H(\cdot)_1 = \pm H(\cdot)_2.$$

The time-energy phase coordinate satisfies the Jacobi commutator equalities:

$$[\,T,[\,X_1,X_2\,]\,]+[\,X_1,[\,X_2,T\,]\,]+[\,X_2,[\,T,X_1\,]\,]=\mathbf{0}$$

$$[\,T,[\,Y_1,Y_2\,]\,]+[\,Y_1,[\,Y_2,T\,]\,]+[\,Y_2,[\,T,Y_1\,]\,]=\mathbf{0}$$

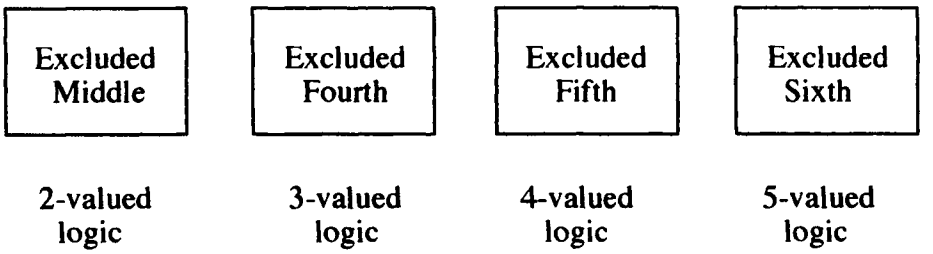
In addition to that the fermionic coordinates satisfy the anticommutator equality:

$$\{T,\{Y_1,Y_2\}\}+\{Y_1,\{Y_2,T\}\}+\{Y_2,\{T,Y_1\}\}=\mathbf{0}.$$

5-VALUED CONSCIOUSNESS

THE INCLUDED MIDDLES

We have considered a logical **5**-dimensional phase space comprising the **2+2** energy coordinates and the time coordinate. The **5**-dimensional approach is important for general field theory and was explored by Kaluza, Klein, Rumer and ourselves [Ref 79]. It is also relevant to cognitive studies, where we can distinguish five fundamental logical states of consciousness: **true, false, undecided, superposed** and **empty**. This increase of dimensionality in logic holds potential for an increase in the intelligent response of a logical system. Complexities or inconsistencies which cannot be resolved at a given level fade away with the addition of the new logical values. The law of Excluded Middle gives way to the law of Excluded 3rd, then to the Excluded 4th, then to the 5th, etcetera. Concomitantly the valuedness of a logical system grows as follows:



There is a strong correlation between the valuedness of logic and the degree of intelligent response a cognitive system may display. Because data are fixed constants, data processing in a sense is the single-valued or one-valued 'logic'. Information processing relies on a binary code of 0's and 1's, which permits deterministic decision-making schemes. These differ from knowledge processing, which is indeterministic, not always 'black or white' but often ambiguous and into which the undecided logical values must be introduced.

The indeterminism is in itself a complex concept, which takes various forms. When a logical system is capable of separating, distinguishing and manipulating the ambiguities themselves, we have high-level intelligence, which is further away from simple binary computation or singular deduction. Finally, if the system gains induction capabilities and has means to reversibly recover executed logical actions, and is in this way empowered to keep track of and voluntarily alter its own thinking operations, such a logical system is a cognizing system.

ABSTRACTION LEVEL	LOGICAL LEVEL
DATA	1-VALUED
INFORMATION	2-VALUED
KNOWLEDGE	3-VALUED
INTELLIGENCE	4-VALUED
COGNIZING	5-VALUED
AESTHETIC	7-VALUED

Fig. 36
The degree of intelligent response in relation to the valuedness of logic.

The complete set of the charged sets of matrix logic maps to itself if we invert the charge of every subset in a given set:

$$+ \text{ goes to } -$$
$$- \text{ goes to } +$$
$$0 \text{ goes to } 0$$

From the total of **81** sets we get down to **41** sets, which are further irreducible. This reduction, reformulated in terms of the logical states, reduces the set of **9** possible discrete states

$$\mathcal{L}(9): \{(0\ 0),\ (0\ 1),\ (1\ 0),\ (1\ 1),\ (1\ -1),\ (0\ -1),\ (-1\ 0),\ (-1\ 1),\ (-1\ -1)\}$$

to the further irreducible set

$$\mathcal{L}(5): \{(0\ 0),\ (0\ 1),\ (1\ 0),\ (1\ 1),\ (1\ -1)\}.$$

Clearly, multiplying all states of the system $\mathcal{L}(9)$ by the phase factor $e^{i\pi}$ produces no observable effects

$$\mathcal{L}(9)\ e^{i\pi} = \mathcal{L}(9)$$

Retaining symmetry, the $\mathcal{L}(9)$ maps to itself, reducing the set to the **5**-valued system. Inspecting the set $\mathcal{L}(5)$, we determine that it comprises the empty state (0 0), the pair of orthogonal complementary Boolean states (1 0) and (0 1), and the pair of orthogonal superposed states (1 1) and (1 -1). The state (0 0) is the origin whose phase cannot be altered

$$(0\ 0)e^{i\pi} = (0\ 0).$$

The four states with nonnegative elements are involved in forward logical reasoning, and the state (1 -1) in the inverse logical operations. Since thinking runs forwards but understanding runs backwards, negative logic is critical for cognitive operations. The superposed logical states **False+True**, **False-True**, first deduced by the author in matrix logic, are coherent superpositions which have found fundamental application in quantum computing.

A FORBIDDEN FIVEFOLD SYMMETRY

To enact the cognitive matrix **5**-valued logic is required. Are there fundamental reasons for the **5**-valuedness of the mind? Is it a law of nature or a sort of 'crystallographic error'? Is there any connection between $\mathcal{L}(5)$ and the symmetry groups, such as the **SU(5)**? If one treats the five fundamental truth-values as the elements of a permutation group, no cognitive matrix can be found in a group smaller than the group of even permutation of five objects. A finite group is nonsolvable if it has a nontrivial subgroup whose commutator subgroup is itself. The group of even permutation of five objects is the smallest of the finite nonsolvable groups, which is relevant to matrix consciousness.

The concept of **5**-valued logical consciousness gains special significance when we consider it in the general framework of the biosciences. The basic theorem

of crystallography states that for lattices with translational symmetry the allowed rotational symmetries are **2**-, **3**-, **4**- and **6**-fold only. There is no symmetry by which a system returns to itself after **5**-fold rotations. However, what is not permitted in ordinary physics is a common occurrence in biology. Fivefold symmetry is intrinsic to the living systems which grow and evolve in time, from flowers to consciousness. The equations of both classical and quantum mechanics are symmetric with respect to the inversion of time. In relativistic quantum field theory particles can move forwards and backwards in time without impediment. But living matter knows only one direction of time, inevitably growing from birth to death.

We now will extend these ideas to logic, with the aim of showing an intimate relationship between time and **5**-valued logical consciousness. The models of multivalued logics are relatively new. As soon as we abandon the familiar domain of two-valued Boolean logic, numerous, in fact infinite possibilities emerge. One is at liberty to design a logic with an arbitrary number of truth-values. It is a valid and interesting mathematical exercise. However, when we study an actual logical system like the brain, we are forced to decide on the **exact** value of **k**. Not only because a different **k** would supply different powers to a logical calculus, but the logic of the thinking brain is bounded by fundamental physical law, and cannot be arbitrary. It is a fundamental logic.

Group theory has been used to explain crystallographic symmetries. To understand the logical **5**-fold symmetries of consciousness matrix logic becomes instrumental. Thinking is a dynamical process which must take cognitive time into account. Instead of **spatial** crystallographic lattices we should consider the **spacetime** lattices in which logical time is a matrix degree of freedom, obeying the fundamental **5**-fold operator symmetry

$$\mathbf{TIME}^{5} = \mathbf{TIME}.$$

This result has important repercussions for the physics of the brain as well as for physics in general. At a very early stage matrix logic uncovered new and hitherto hidden aspects of logic: in addition to the **true** and **false** states:

$$(0\ 1) \text{ and } (1\ 0),$$

we were obliged to consider the **true&false** and **neither true nor false** states:

$$(1\ 1) \text{ and } (0\ 0).$$

The undecided logical states are actually a hidden aspect of logic, which reveals itself in the matrix formulation. Matrix logic extends beyond the classical realm since it deals not only with true and false but also with the undecided. The logical operators evolve Boolean states to themselves but also to the triplet of the undecided states

$$|s_+\rangle,\ |s_-\rangle,\ |s_o\rangle.$$

This expands naturally two-valued logic into a **5**-valued logic of true, false and undecided, the latter consisting of three different types, which appear at the intermediary steps of computation, not showing themselves in the final results. The Boolean states are complementary:

$$|\bar{1}> = |0>$$

and so are the two undecided states,

$$|\bar{s}_+> = |s_0>,$$

The fifth state $|s_->$, which is essential for the realization of negative logic, stands apart; it does not have a complementary pair, only the orthogonal one $|s_+>$:

$$<s_+|s_-> = 0.$$

which results in a logical space with an odd numbers of logical values.

PSEUDO-GALILEAN SPACE AND THE 5-SPACE CORRECTION TO ENERGY

Many problems become simplified when treated in spaces of higher dimensions. While classical spacetime is a purely geometric concept, we have treated time-energy phase space as a coding information concept. Most importantly, we have taken into account both the bosonic and fermionic aspects of the theory and considered two, bosonic and fermionic, matrix fields that encode information. Because the two fields neither commute nor anticommute between themselves, they cannot be known simultaneously, the task which must be tackled by a supersymmetric theory.
Applying the idea of five-dimensional space to metric invariants, instead of the relativistic 4-interval

$$s^2 = x^2 + y^2 + z^2 - (ct)^2$$

we consider the 5-interval

$$s_\alpha{}^2 = x^2 + y^2 + z^2 - (ct)^2 + \alpha'^2$$

where the quantity $(\alpha')^2$ accounts for the uncertainties due to spacetime quantization at the Planck scales. At these scales the ordinary concept of space and time loses its meaning. Quantized spacetime is fundamentally different from classical spacetime. The space and time quanta simply cannot be divided into smaller parts, just as it is not possible to quantize further the vacuum ground state $\hbar/2$. Because it is not possible to obtain a reading on quantities

smaller then the quanta of space and time, quantized spacetime is fuzzy. The eigenvalues of matrix time are the complex numbers $\pm i$, and so are the eigenvalues of the bosonic space coordinates $1\pm i$. We have a set of realities **connected** with each other but which do not **infer** one from the other. Such non-Hermitian realities are not ordered in time because 'before' and 'after' cannot be specified. Likewise, it is no longer possible to measure distance inside particles. A truly **elementary** particle cannot be split into parts which are connected by a light signal, and the standard causal relations, typical of a **compound** particle and ultimately of macrobodies do not apply. For acausal constellation of events the result of the measurement of an interval between two points is dependent on the standard by which this interval is measured. The following possibilities open up:

(1) For $\alpha'= 0$, the **5**-interval is reduced simply to the Minkowski interval

$$s^2 = x^2 + y^2 + z^2 - c^2t^2$$

(2) For $\alpha' \neq 0$ and $(ct)^2 = (\alpha')^2$ we have the pseudo-Galilean metric [Ref 79]:

$$r^2 = x^2 + y^2 + z^2$$

(3) For the light-like interval $s^2= 0$, we are left with the pure **5**th dimension:

$$s_\alpha{}^2 = (\alpha')^2$$

(4) For $s^2 \neq 0$ and $s^2 = (\alpha')^2$ the isotropic separation is recovered in **5**-space:

$$s_\alpha{}^2 = x^2 + y^2 + z^2 - c^2t^2 + (\alpha')^2 = 0$$

Physics is Euclidian for $v \ll c$, and pseudo-Euclidian for $v \sim c$. For a body approaching the speed of light c, time eventually must stop, which makes it impossible for a finite mass to reach the fundamental speed limit. But in **5**-space it is possible to restore the 'Euclidian' metric in the relativistic realm, and to obtain isotropic separation for $v < c$. This result has fundamental consequences. Modifying the relativistic formula for mass, so that it takes into account the effects of the third quantization, we get

$$m = m_0 \sqrt{1 - \frac{v^2}{c^2} + (\alpha')^2}$$

Following Einstein's prescription, and expanding the formula into a series, we observe that the **5**th dimension adds a small correction to mass-energy relation

$$E = mc^2 + \varepsilon(\alpha'),$$

where $\varepsilon(\alpha')$ is the energy of the string's ground state. One intriguing possibility is to link the term $\varepsilon(\alpha')$ with cognitive phenomena [Ref 79]. Because of the vacuum fluctuation of mass due to the 3rd quantization, on the order of string tension $0{,}5 \cdot (10^{-33})^2$ the mass of the conscious brain will differ from the mass in the ground unconscious state. The ambitious goal of matrix logic, to connect the theory of the lowest quantum level with the theory of the highest logical level, receives a major boost. The pseudo-Galilean metric should become manifest inside elementary particles where relativistic interval aborts its dependence on time. The idea of performing logic in the interior of elementary particles is quite outrageous, but at the fundamental Planck scales

matrix logic becomes essential. The interior of a fundamental particle is a classical singularity in 5-space. In a peculiar manner the pseudo-Galilean metric is the classical Euclidian metric:

$$x^2 + y^2 + z^2 - (c\tau)^2 + \alpha'^2 = x^2 + y^2 + z^2,$$

where the contribution of the 4th and 5th dimensions to the interval cancel each other out:

$$-(ct)^2 + \alpha'^2 = 0 \quad \Rightarrow \quad ct = \alpha' \quad \Rightarrow \quad t = \alpha'/c = \tau = 10^{-33}/10^{10} = 10^{-43}\text{sec}$$

Here τ is the Planck time, which is the time in which a relativistic particle would traverse the fundamental distance α'. In Minkowski space two points are 'close' when the spacetime separation is zero, but from that fact alone one would not know whether the space and time coordinates are both separately large or whether they are simply zero.

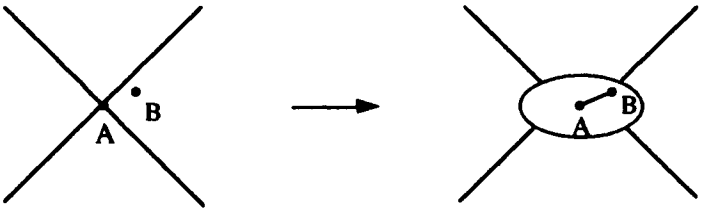


Fig. 37
With the addition of extra dimension a contradiction which is spacelike in 4-space is 'smoothed out' into a tautology which is timelike in 5-space.

CONSTRUCTING CAUSAL SYSTEMS FROM ACAUSAL COMPONENTS

Probing the quantum structure of spacetime, we considered time-energy five-dimensional space in which commutation relations play the key role. In quantum field theory the commutation relations determine the correlation of fields in different spacetime points. For spacelike intervals the correlations between fields vanish, signifying an absence of causal links. As long as we consider pointlike particles no violation of the causal relation can be detected. The Green propagator vanishes rapidly outside the worldcone, and no two points can be connected faster than by a light signal. In the language of the

S-matrix, causal relations can be formulated only to the extent that we can make simultaneous use of momentum-energy space and spacetime. This is macroscopic causality. Simultaneous use of these spaces rests on wavepacket overlaps. The local acausal short-range interactions vanish faster than the causal long-range interactions. At the Planck scales the S-matrix will tolerate different schemes of interaction of particles and their different internal organization. As we indicated, in 4-space, if a particle is not pointlike, the momentum transfer must occur instantly, which is a major stumbling block. An important consequence of local acausality is the appearence of quantities that are unobservable, quantum-mechanically or otherwise: the probability amplitudes become negative, and standard deviations imaginary. A dispersion wave would occur prior to the arrival of the incoming wave, which should violate the causal relations, with velocity acquiring an infinite value. Thus the circle of the radius α' is a singularity area, inside which causal links break down locally. We effectively construct a causal system made of acausal elements:

$$\textbf{Causal} = \bigcap_{i=1}^{n} \textbf{acausal}_i$$

The number of participating acausal terms is important. Only the interactions of a very large number of the acausal elements, $n>>0$, can produce a statistically stable average, invariant to local change. A similar scheme is used to characterize a fault-tolerant computer circuit constructed with unreliable gates. A fault-tolerant system is a system that functions properly as a whole while its components are not prone to failures. It is an interesting question whether the behaviour of fundamental quantum systems can be based on a similar principle. Localized processes that destroy causal links are quite common. Yet, were we to follow the evolution of the 3rd quantized system with coarser resolution, tracking only the interaction of the Minkowski separated particles, we would observe causal evolution to remarkable accuracy; we would detect no nonlocal communication among the particles, no glimmer of the turmoil in full swing beneath the surface. Nature has woven fault tolerance into her design, shielding the acausal acts at the Planck scale from our view. It is not possible to probe the lower levels inside the fundamental particles, descending beneath the circle with the radius α'. In quantized spacetime small circles simply do not exist, and the macroscopic causal rules do not apply at the elementary length α' and time τ'. Relativity works to excellent accuracy at macroscopic length scales, where decoherence is rapid, but falters at the Planck scale where quantum chaos is common. A quantum theorist may entertain an alternative idea that the acausal processes, which are trapped below the Planck scale, is undetectable because they do not contribute to amplitude, only to phase. We believe that the quantized 5-spacetime, which limits the application of quantum field theory and relativity theory, is not merely an abstract phase space but an actual physical space, evidence for which is provided by the thinking brain.

THOUGHT AND LORENTZ TRANSFORMATION

When we study relativistic quantum objects the units of measurement are different from our ordinary Newtonian world. Distances can be measured by clock, and time is subject to Lorent transformations.The relativistic 4-interval remains invariant under the Lorentz and the more general Poincaré transformations. Minkowski space is divided into the causal, upper and lower, cones, where pseudo-Euclidian distance is apparently imaginary: $s^2 \leq 0$, and the absolutely remote, left and right, cones: $s^2 > 0$. With the addition of the 5th dimension the spacelike interval $ds^2>0$ can be made negative $s_\alpha{}^2 \leq 0$, and via the fifth dimension the points connect causally. Events which had no causal connection start to influence each other. This is significant for logic, which is not dependent on time. Truth is timeless. The thought process does not dependent on local time and space coordinates, and to obtain such independence one needs the isotropic Minkowski interval, which would leave us only with dependence on the third-quantized fifth dimension α'. In quantized spacetime the points do not form a continuum and one has to explain the process of a changeover from discretum to continuum. A rotation in 4-space is a quantum shift in 5-space which changes the pseudo-Euclidian metric into the pseudo-Galilean metric which is 'classical'. Since we have independence of thought from spacetime coordinates, which leaves us only with dependence on the 5th coordinate α', how could thought process be Lorentz-invariant, as one is naturally expects it to be? On the order of the Planck scale, which is about twenty times smaller than the size of an elementary particle, the world once again turns Euclidian. At both ends of physics things are the same. One can think about time without space, like a periodically flashing mathematical point, or about space without time, like a still photograph. These are metaphors. But in 5-space, when relativistic separation cancels out, we are actually again in the classical world, which is a strange relativistic world without time.

Relativity principles are known to account for mechanical motions, electromagnetism and gravity. They also must account for consciousness, the task which leads us to 5-space and beyond. We see that matrix logic meaningfully supports the goal of expressing the general model of physics in logical terms. A fundamental theory of physics cannot be built on physical considerations alone, but necessarily has to include logic. In reverse, the physical principles become of great value to understanding logic. Although the brain is a physical system, it is at the same time a logical thinking system. Broadening the concept of physics and unifying it with the fundamental laws of logic, we gain a scientific understanding of the conscious brain. When the brain takes an expression in a high-level code and translates it into an expression in a basic code, the higher-level code is reduced to the quantum primitives of the lower level. However, the interactions of fundamental particles possess nontrivial properties of their own. Distinguishing between the cipher and the key of the code appears to be a problem. The higher level

provides the key to the cipher of the quantum level. But when one rewrites logic into a basic code of quantum primitives, then the quantum level holds the key to the higher level of human thought. Mind and matter are alike in being profoundly mysterious and conceptual adjustment on both sides of the divide is needed. While it is generally accepted that the laws of logic are dependent on the laws of physics, another emerging view suggests that the universe would not work without an inclusion of information primitives into physical laws. Nature has built a macroscopic classical brain from relativistic and quantized components. It would never had occurred if the laws of relativistic physics did not encompass logic, which is a general way of saying that the operations of the thinking brain must be consistent with Lorentz transformation.

RELATIVISTIC BRAIN

THE SELF'S ATTEMPT ABOUT SELF

The principles of relativity and the principle of causality are inseparable. But in consciousness the causal relations are altered profoundly. The very attempt of SELF to understand itSELF introduces a circular causation which is difficult to grasp. This self-reference is also implicit in the conversion principle of matrix logic, according to which a well-formed quantum theory is converted into a logical calculus and a covariant logical theory into a quantum field theory. Logic must have universal meaning. Even though in different reference systems a logical thought can be different, if valid, it is necessarily Lorentz-invariant and should apply universally. To advance the argument we can connect the logical invariants and the invariants of relativity explicitly. The fundamental invariant of relativity is the pseudo-Euclidian interval or spacetime separation

$$ds^2 = dr^2 - (cdt)^2$$

The key invariant of logic is the relative truth-value or the logical interval, which is the 'false-true separation'

$$\Delta = \bar{q} - q.$$

But in the normed spaces the logical interval is naturally translated into the *secret* metric which is pseudo-Euclidian:

$$\Delta = \bar{q} - q \equiv \bar{q}^2 - q^2$$

Relative truth does not vary under involution. The algebra structure of relativity theory arises naturally in relation to logic. For the isotropic interval, $\Delta=0$, this pseudo-Euclidian invariance also extends to the quantized states of coherent superposition $|S_+>$ and $|S_->$. A thinking system has the capacity to

retain covariance not only in classical normed space, but also in quantum singularities, which naturally connects the relativistic analysis of the thought process with quantum mechanics. A fundamental link between relativity, quantum mechanics and matrix logic points to a general theory in which the relativity, logic and quantum mechanics join:

Relativity theory	Matrix logic	Quantum mechanics
$ds^2 = dr^2 - (cdt)^2$	$\Delta = \bar{q}^2 - q^2$	$\langle\sigma_z\rangle = \lvert\psi_u\rvert^2 - \lvert\psi_d\rvert^2$

The common structure of these three conerstones of scientific enquiry provides new insight for the advanced theory of fundamental physics [Ref 88]. The link between the thought process, spinor space and spacetime enables one not only to gain better understanding of the mechanism of intelligent thought, it also, and more importantly, suggests that the physics of consciousness can play a role in the unification of relativity and quantum mechanics.

The relativistic treatment of the brain must be consistent with the scheme of the second quantization, in which in the expansion of a wavefunction in the complete set of eigenstates

$$\Psi = \sum a_n \psi_n, \qquad \Psi^* = \sum a_n^* \psi_n^*$$

the coefficients a and a^* are the annihilation and creation operators satisfying the anticommutation algebra

$$\{a,\ a^*\} = 1,\ \{a,\ a\} = 0,\ \{a^*,\ a^*\} = 0.$$

The relativistic Hamiltonian and the charge operator, respectively

$$H = \sum_n \varepsilon_n a_n^+ a_n - \sum_n \varepsilon_n b_n b_n^+ \qquad \text{and} \qquad Q = \sum_n (n_n^+ - n_n^-)$$

account for particles and antiparticles. The energy of the system is the sum of energies of all constituent particles and antiparticles. Particles are created and annihilated in particle-antiparticle pairs. In the topological framework introduced earlier, a fermionic field is an alternating knot, which adds or loses crossings in pairs. The conservation rule applies neither to particles and antiparticles separately, nor to their sum. It is the difference which must be conserved, just as in matrix logic the truth-value is considered jointly with the false-value, while the difference between them remains stable under involution. In the Wigner anticommutator the operators a and a^* can be replaced by the logical momentum

$$\{a,\ a^*\} = \{M,\ a^*\} = \{a, -M\} = 1.$$

Matrix logic matches the scheme of the second quantization. The logical momentum is the difference of the true and false

$$M = \text{FALSE} - \text{TRUE}.$$

For the average a conservation of the difference of false and true, not their separate conservation, nor conservation of their sum, is required. The logical synergy gets either negative or positive contributions of negation

$$H = \text{YES} - \text{NOT}, \qquad 1 = \text{YES} + \text{NOT},$$

convertible by the time unitary transformation:

$$\blacktriangle H \blacktriangledown = 1$$

but also in nonunitary fashion with the aid of summators and counters:

$$+H+ = 1, \qquad \rightarrow^{2} H \leftarrow^{2} = 1.$$

The annihilation and creation operators, a and a^*, which in quantum theory are simply the computational tools for studying the fields, are in matrix logic the cognitive observables, **ON** and **OFF**, the objects of logic themselves. As in quantum field theory, we are concerned with the operator difference. The unitary logical rotation of the sum (**ON+OFF**) produces no observable effects,

$$\theta(\mathbf{ON} + \mathbf{OFF}) = (\mathbf{ON} + \mathbf{OFF})$$

but the rotation of the difference is the anti-identity:

$$\theta(\mathbf{ON} - \mathbf{OFF}) = (\mathbf{OFF} - \mathbf{ON})$$

With the aid of the logical superposition principle we can expand the logical state $|q\rangle$ and its complement $|\bar{q}\rangle$ in the orthogonal Boolean basis

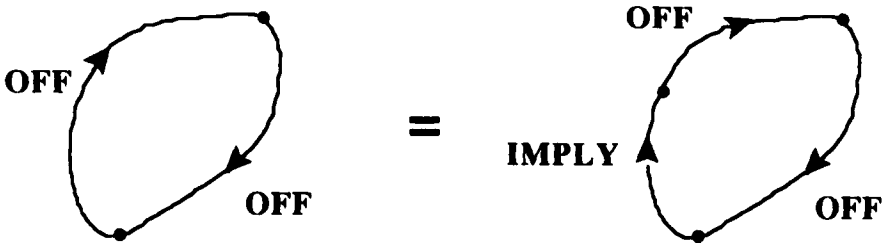
$$|q\rangle = \bar{q}|0\rangle + q|1\rangle, \qquad |\bar{q}\rangle = q|0\rangle + \bar{q}|1\rangle$$

where q and $\bar{q}$ are scalars. Making use of **ON** and **OFF**, we obtain the following reciprocal expansions

$$|q\rangle = \mathbf{OFF}|\bar{q}\rangle + \mathbf{ON}|\bar{q}\rangle, \qquad |\bar{q}\rangle = \mathbf{OFF}|q\rangle + \mathbf{ON}|q\rangle$$

The coefficients in the expansions are no longer numbers but operators acting on logical states, just as the creation and annihilation operators a^* and a^* act on a wavefunction in the second-quantization.

The second-quantized logical systems can invariantly gain or lose implications. It is possible to induce such a perturbation of the logical vacuum where a gain or loss of a symbol occurs and the symmetric vacuum turns nonsymmetric but is not destroyed. For example,



On the left, the vacuum loop is symmetric, comprising two identicalbranches, opposite in phase only, which additively cancel out:

$$\mathbf{OFF} - \mathbf{OFF} = \mathbf{0}.$$

On the right, the two branches are different, but the insertion of **IMPLY** does not break the vacuum:

$$\mathbf{IMPLY} \bullet \mathbf{OFF} - \mathbf{OFF} = \mathbf{OFF} - \mathbf{OFF}.$$

VIOLATING PAULI EXCLUSION

We found earlier that logical evaluation follows the rules of the normal ordering in quantum field theory and that any logical expression can be reduced to a string of **ON**'s and **OFF**'s [Ref 91]. Making use of the anticommutator

$$\{\mathbf{ON}, \mathbf{OFF}\} = \mathbf{YES},$$

the self-cancellation

$$\mathbf{ON} \cdot \mathbf{ON} = \mathbf{NONE}, \quad \mathbf{OFF} \cdot \mathbf{OFF} = \mathbf{NONE},$$

and the convolutions

$$\mathbf{OFF} \cdot \mathbf{ON} \cdot \mathbf{OFF} = \mathbf{OFF}\ (\mathbf{YES} - \mathbf{OFF} \cdot \mathbf{ON}) = \mathbf{OFF},$$

$$\mathbf{ON} \cdot \mathbf{OFF} \cdot \mathbf{ON} = \mathbf{ON}\ (\mathbf{YES} - \mathbf{ON} \cdot \mathbf{OFF}) = \mathbf{ON},$$

one reduces a complex logical string to one of five possible outcomes:

$$\mathbf{ON}, \mathbf{OFF}, \mathbf{AND}, \mathbf{NOR}, \mathbf{NONE}.$$

Given the matrix fixed points:

$$\mathbf{IMPLY}|0\rangle = |0\rangle, \qquad \mathbf{IF}|1\rangle = |1\rangle$$

by complementation we deduce the equation for a creation of a true state out of false and the annihilation of a true state into false, respectively:

$$\mathbf{ON}|0\rangle = |1\rangle, \qquad \mathbf{OFF}|1\rangle = |0\rangle$$

Because the false state $|0\rangle$ and true state $|1\rangle$ are fixed points for **IMPLY** and **IF** respectively, we can invariantly iterate:

$$\rightarrow^2|0\rangle = \rightarrow|0\rangle = |0\rangle, \qquad \leftarrow^2|1\rangle = \leftarrow|1\rangle = |1\rangle,$$

and, complementing the iterations, to deduce:

$$\rightarrow^2|0\rangle = |\bar{0}\rangle = |1\rangle, \qquad \overline{\leftarrow^2|1\rangle} = |\bar{1}\rangle = |0\rangle.$$

Joining these equations with the equations for annihilation and creation of logical states, we derive

$$\overline{\rightarrow^2|0\rangle} = \mathbf{ON}|0\rangle, \qquad \overline{\leftarrow^2|1\rangle} = \mathbf{OFF}|1\rangle,$$

reduced to

$$\blacktriangledown|0\rangle = \mathbf{ON}|0\rangle, \qquad \blacktriangle|1\rangle = \mathbf{OFF}|1\rangle,$$

where the counter operators were replaced by the time operators.

This result has important consequences, casting light on the question how many cognitive 'particles' can be accepted by a logical state. Fermions with half-integer spins obey Fermi-Dirac statistics, while bosons with integer spins obey Bose-Einstein. As a consequence of Pauli's theorem, linking spin and quantum statistics, a quantum state can be populated by an arbitrary number of bosons, but not more than with a single fermion. The operators **ON** and **OFF** in the right hand side of our equations are fermionic. Their squaring is the empty operator **NONE**, which agrees with the Pauli exclusion principle: no two fermions can occupy the same quantum state. The creation operator, applied to an occupied state, gives zero, and the annihilation operator cannot annihilate twice:

$$\mathbf{ON}^2|0\rangle = |0\rangle, \qquad \mathbf{OFF}^2|1\rangle = |0\rangle.$$

However, if we examine the left hand sides of the equations, we find out that they behave in bosonic fashion. The time operators, being applied twice, do not yield an empty state but permit accumulation of particles in the same state. This process, however, is not fully bosonic due to the limitations of the **5**-fold symmetry of the time operator

$$\blacktriangle^5 = \blacktriangle .$$

Violating Pauli exclusion, our model will generate **3** different particles before recovering the original state:

$$\begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\-1\end{pmatrix} \rightarrow \begin{pmatrix}-1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} \rightarrow \begin{pmatrix}1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\-1\end{pmatrix} \rightarrow \begin{pmatrix}-1\\0\end{pmatrix} \rightarrow \begin{pmatrix}0\\1\end{pmatrix}$$

After that wrapping occurs and the system runs in circles, intersecting itself,

$$\blacktriangle + \blacktriangle^2 + \blacktriangle^3 + \blacktriangle^4 + \blacktriangle^5 = \blacktriangle ,$$

$$\blacktriangledown + \blacktriangledown^2 + \blacktriangledown^3 + \blacktriangledown^4 + \blacktriangledown^5 = \blacktriangledown .$$

FUNDAMENTAL SUPERCOMMUTATOR

Matrix Statistics supersymmetrically mixes Bose-Einstein and Fermi-Dirac, balancing a fermionic contribution with a triple of bosonic contributions. The triplets of particles distinguish nucleons, composed of three quarks, which suggests a possible link between quarks and Matrix Statistics, although neither nucleons nor quarks are bosons.

Cognitive processes are directed towards obtaining the codons of tautology, the fundamental invariants of the intelligence code. Matrix Statistics dictates that these invariants are realized supersymmetrically, which entails the equality of commutator and anticommutator,

$$AB + BA = AB - BA.$$

Because the codons are universal constants, independent of a particular logical interpretation, the catalogue of interactions includes **ONE** and **NONE**. Supersymmetry requires that the commutator and anticommutator not only equal between themselves but also equal to **1** or **0**. Clearly, a supercommutator always exists whenever $BA = \mathbf{0}$. Such a supersymmetric property distinguishes logical momentum in its interactions with **TRUE** and **FALSE**:

$\{\mathbf{M}, \mathbf{TRUE}\} = [\mathbf{M}, \mathbf{TRUE}\ ]$, $\quad\quad$ $\{\mathbf{M}, \mathbf{FALSE}\} = [\mathbf{M}, \mathbf{FALSE}\ ]$.

However, here $\{A, B] \neq \mathbf{0}$, the condition we require to obtain a contradiction, and, consequently, a tautology supersymmetrically.

The coupling of logical operators follows the rules of composition in quantum field theory, which in a natural way connects the logic with the second-

quantized field. In the latter the coupling of operators is determined by the difference between the chronological T [AB] and the normal N [AB] orderings:

$$\overbrace{A\,B} = T\,[AB] - N\,[AB].$$

In the term T [AB] the annihilation a and creation a* operators are ordered from the right to the left in accordance with the increase of time. The normal product N[AB] is such an ordering of operators in which all annihilation operators are on the right of the creation operators. Suppose that time ordering T [AB] = AB, and A is the creation operator, B - the annihilation operator. Then N [AB] = AB, and $\overbrace{A\,B} = AB - AB=0$. In general, if A is the annihilation operator, and B is the creation operator, then according to the rule of the normal ordering

$$N\ [AB] = \pm BA$$

This identifies the Bose and Fermi operators, respectively

$$N\,[AB] = +\,BA \qquad \text{and} \qquad N\,[AB] = -\,BA.$$

Consequently, if A and B are **bosonic** operators, the coupling is the commutator

$$\overbrace{A\,B} = AB - BA$$

But if A and B are **fermionic** operators then the coupling is the anticommutator

$$\overbrace{A\,B} = AB + BA$$

A **supersymmetric** ordering should not distinguish between bosons and fermions:

$$[A, B] = \{A, B\}$$

As we already mentioned, this is always the case when the second term is zero, but generally there could be three different kind of *zeros*: fermionic, bosonic and supersymmetric:

$$\{A, B\} = 0, \qquad [A, B] = 0, \qquad \{A, B] = 0$$

The condition of supersymmetry in logic expresses the fact that the inference rule is independent of a particular, bosonic or fermionic, realization. It is possible to reduce the fundamental tautology of the **Modus Ponens** inference rule $((x \wedge (x \to y)) \to y$ to the commutator:

$$\overbrace{\langle x|\to|y\rangle\langle x|\backslash|y\rangle} = \overbrace{\langle x|\,\overline{\backslash}\,|y\rangle\langle x|\backslash|y\rangle} = \overbrace{(1-\langle x|\backslash|y\rangle)\langle x|\backslash|y\rangle}$$

$$= \overbrace{\langle x|\backslash|y\rangle - \langle x|\backslash|y\rangle} = \overbrace{\langle x|\backslash - \backslash|y\rangle} = \overbrace{\langle x|\leftarrow \backslash - \backslash \leftarrow|y\rangle}$$

$$= \overbrace{\langle x|[\leftarrow, \backslash]|y\rangle}.$$

But we also can inductively build up an inference tautology which relies on anticommutation, for example:

$$\overline{\langle x|0|y\rangle} = \overline{\langle x|M-M|y\rangle} = \overline{\langle x|M|y\rangle - \langle x|M|y\rangle}$$

$$= \overline{\langle x|M|y\rangle + \langle x|-M|y\rangle} = \overline{\langle x|MN|y\rangle + \langle x|NM|y\rangle}$$

$$= \overline{\langle x|\{M,N\}|y\rangle}.$$

With the above reductions we have compiled cognitive codons into bosonic and fermionic quantum operations:

$$\overline{\langle x|[\mathbf{IF},\mathbf{ON}]|y\rangle} \qquad \text{and} \qquad \overline{\langle x|\{\mathbf{MOVE},\mathbf{NOT}\}|y\rangle}.$$

Although these reductions reveal useful information about logic at deeper levels of the intelligence code, they unfortunately, are not supersymmetric. The inference tautologies we obtain by negating the contradiction, which is identified by demanding that

$$\overline{A\,B} \equiv 0.$$

When the coupling involves the populations **AND** and **NOR**, a logical system obtains a supersymmetric property, not distinguishing between commutation and anticommutation

$$[\mathbf{AND}, \mathbf{NOR}] = \{\mathbf{AND}, \mathbf{NOR}\} = \mathbf{0}$$

which gives the gauge-invariant supercommutator codon:

$$\overline{\langle p|\{\mathbf{AND},\mathbf{NOR}\}|q\rangle} \equiv 1$$

Supersymmetry is a fundamental feature of the mind, reflected in the equivalence of bosonic and fermionic interactions. According to the matrix reduction theorem any logical expression is a fundamental product of **ON** and **OFF**. We are interested in the invariant supersymmetric orderings which can deliver the fundamental constants of contradiction and tautology. These remain intact under arbitrary interpretation: local perturbations can influence a thought but cannot change a truth. The commutation relations which are the field conditions in quantum theory become the coding units of the intelligence code. Since truth is not dependent on time, how can a fundamental field be chronologically ordered when time, in a sense, is expelled from the system? For that, the time ordering must be universally equal to the normal ordering:

$$T\,[AB] = N\,[AB\,].$$

When the time ordering equals the normal ordering, the coupling between the operators reduces to zero:

$$\overline{A\,B} = T\,[AB] - N\,[AB\,] = \mathbf{0}$$

which is a general way of saying

$$[A, B] = \{A, B\}.$$

The intelligence code is covariant and the coupling of operators must give gauge-invariant interactions that are not explicitly dependent on time. One is effectively connecting absolutely remote spacetimes but without violating the fundamental speed limit. The spacelike events cannot influence each other and since the respective operators commute if the events are bosonic or anticommute if they are fermionic, in logical terms this corresponds to a contradiction. From a relativistic standpoint a changeover from no-connection

to causal link involves negating a spacelike separation. From a logical standpoint, the negation of a contradiction is a tautology. Basically we get tautology when we are negating a spacelike separation. Keeping this eventuality in mind, we can without loss of generality identify contradiction as the vanishing coupling

$$<p|\overline{AB}|q> \equiv 0.$$

Substituting it into the codon's master formula gives the required covariant tautology:

$$\overline{<p|\overline{AB}|q>} \equiv 1.$$

The operators A and B are the annihilation and creation operators, which identifies the fundamental and the only matrix logical supercommutator:

$$[\,\mathbf{ON \cdot OFF},\ \mathbf{OFF \cdot ON}\,] = \mathbf{0}$$

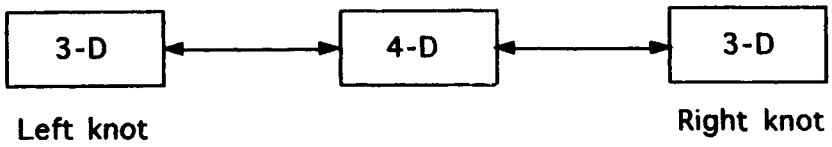
Equivalently, in quantum field terms:

$$[\,a^*a,\ aa^*\,] = 0$$

Since the computation forms a nonalternating string, it vanishes, providing for the fundamental tautology:

$$\overline{<p|\overline{(aa^*)\quad (a^*a)}|q>} \equiv 1.$$

The quantum relativistic analysis of the brain has implications for the topological theory of consciousness. Typically we think of (topo)logical knots in 3 dimensions. The relativistic analysis of the brain in a natural way introduces the forth dimensions, which is of much significance. A change of a chiral knot to its mirror image, impossible in three dimensions, can be accomplished by rotating a knot in higher dimensions. As we recall, the third dimension is the only dimension in which knots stay knotted. To untie a left(right) knot relativistically, move it in the 4th dimension backwards in time, and then retrieve it forwards in time into a right(left) knot:



Instead of knots we can consider the two orthogonal superposed states, from which the left and the right trefoils are produced by closed cut. Then the time operators convert the left knot into the right knot and vice versa:

$$\blacktriangledown|s_->\ =\ |s_+>, \qquad \blacktriangle|s_+>\ =\ |s_->.$$

An attractive possibility is to connect this dimensional transformation with uniform quantum transformation:

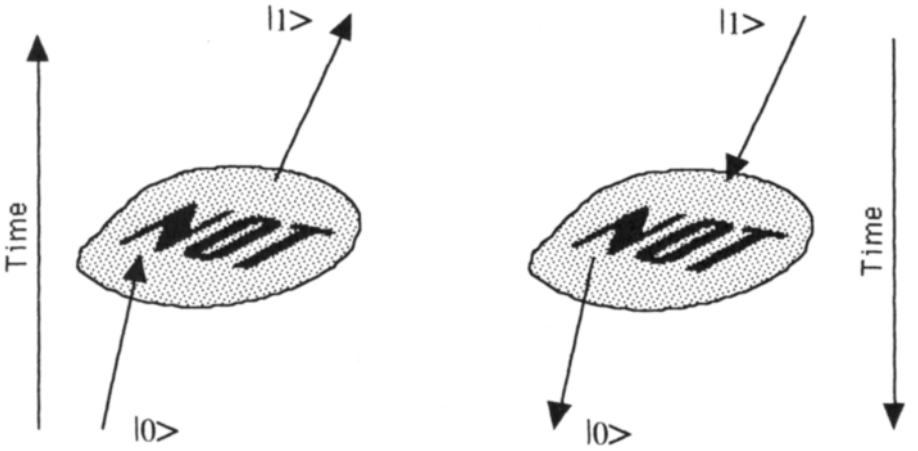
$$\sigma|s_\pm> = |s_\mp>$$

which suggests that the direction of logical time can be manipulated with the aid of spin.

TRUE AND FALSE TIMES CROSSING THE NOT BARRIER

Dogmas are hard to overcome. We prefer familiar notions which give us a sense of the permanence of our conclusions. Consciously or unconsciously we tend to surround ourselves by a protective cocoon of the **NOT** barriers. To undo these barriers we must go backwards in time, which we cannot do in the macroworld. But by changing logical values, we can alter cognitive time. Then the false can be interpreted as true realised backwards in time. This is in analogy to the quantum field, where an antiparticle is a particle moving backward in time.

The quantum field equations study waves scattered by a potential. Suppose we have a potential associated with **NOT**, through which a logical state passes.The false state |0> scattered by the potential re-emerges as a true state |1>. If we send a true state |1> through the potential, it would scattered into a false state |0>. It follows that we can swap the true and false states by reversing the direction of cognitive time:



But the true and false states are the integration constants for the logical equations of motion. The equation with forward direction of time

$$-\frac{d|q>}{dq} = \mathbf{H}\blacktriangle|q>$$

is solved with the integration constant |0>:

$$|q> = -\int_0^q |s_->dq + |0>$$

The equation with backward direction of time

$$-\frac{d|\bar{q}>}{dq} = \mathbf{H}\blacktriangledown|\bar{q}>$$

is solved with the integration constant $|1>$:

$$|\bar{q}> = \int_0^q |S_->dq + |1>$$

Reducing the problem to a problem of discrete logic we have:

$$|1> = -\int_0^1 |S_->dq + |0>, \qquad |0> = \int_0^1 |S_->dq + |1>.$$

where the change of the sign signifies the reversal of cognitive time.

THE ASYMMETRIC MIND

THOUGHT IS PROCESSED FORWARDS BUT UNDERSTOOD BACKWARDS

Physics seeks symmetry. The principles of symmetry determine everything we conceivably can learn in physics. Is symmetry as relevant for the logical mind as it is relevant for matter and fields? Consciousness is fundamentally asymmetric. Indeed a fully symmetric system cannot evolve, sooner or later falling back into a steady 'unproductive' state of equilibrium. But the mind is endlessly in motion, permanently retaining dynamical nonequilibrium. Our goal is to explain meaningfully mind's asymmetry in the context of matrix theory.

We defined logical momentum as the **minus** differentiation of a logical state, but equivalently we can define momentum as **plus** differentiation of a complementary state

$$-\frac{d|q>}{dq} = \frac{d|\bar{q}>}{dq}$$

A forward dynamics of true is fully equivalent to a backward dynamics of false and vice versa. **NOT** is **YES** acting backward. Thought runs forwards, understanding runs backwards. Can we think backwards? We can, but not backwards in time. Thinking backwards is not thinking awkwardly, but the distinction between backward reasoning and thinking backwards in time must be properly understood. Knowledge is irreversible: it can be **erased** from memory but it cannot **evolve** backwards. Although the reasoning process can start from a conclusion and go back to a premiss, this does not means that thought unrolls backwards in time. When we negate a logical value, the direction of cognitive time is unaltered,

$$q \xrightarrow{\text{NOT}} \bar{q}$$

This is different from the phase reversal which introduces negative values, changing time to its opposite:

$$q \xrightarrow{e^{i\pi}} -q$$

Prior to the development of matrix logic all formalizations of logic employed positive number representations. The introduction of negative logic is an important conceptual change which has bearing on the question of reversibility in logic. Many logical operations can be turned backwards, provided we have access to negative values. Luckily for us, this is not true of all operations. What a disastrous situation it would be if any cognitive action could roll back. Memory would become impossible and the linear chain of perception would break down. Fortunately, a fundamental irreversibility is built into the logical mind. It is reversible, but only in part. Among **81** logical operators **33** are singular, and hence irreversible. Since there is no full reversibility and the number of nonsingular operations is greater than the number of singular ones, as a whole consciousness is not unitary and will necessarily evolve asymmetrically.

The essence of negative logic is best understood with the support of an analogy with a physical vacuum. The relation between positive and negative logics resembles the relationship between positive and negative levels of energy. The analogy with a physical vacuum is, however, not fully adequate. It would be incorrect to perceive negative logic simply as a mirror image of positive logic. In negative logic there are more logical relations which positive logic cannot possibly handle. The relationship between both logics is complex. In the physical framework negative logic can be consistently treated as an antilogic in which true and false are swapped over. Positive logic can be associated with conscious and negative logics with unconscious logical operations respectively. Taking the unconscious logic into consideration, we encounter symmetries of a more complex nature than those of positive logic alone.

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REVERSING THE IRREVERSIBLE

Prior to the development of matrix operator logic one of the common misconceptions in logic was that negation is the only reversible operation. A logic based on **NOT** is dual to a logic based on **1/NOT**

$$N \Leftrightarrow \frac{1}{N}.$$

Since duality is equality:

$$N = \frac{1}{N},$$

we have a self-dual, with one logic being completely indistinguishable from the other. **NOT** is a self-inverse

$$\mathbf{NOT}^{-1} = \mathbf{NOT},$$

and **NOT** twice is identity:

$$\mathbf{NOT} \cdot \mathbf{NOT} = \mathbf{IS}.$$

Self-duality is the condition of the unitarity of logic which in classical logic applies exclusively to negation: it is not possible to undo the action of the other logical operators. Here the expressions $\mathbf{OR}^{-1}$ or $\mathbf{IF}^{-1}$ are simply meaningless and noncomputable. One is led to believe that the logical **OR** and **IF** cannot be reversed, not to mention the conceptual difficulties one might have in trying to grasp the logical meaning of such reversals. But what is irreversible in Boolean logic becomes reversible in matrix logic, where the heuristic notion of the logical inverse is replaced by a rigorous mathematical notion. The inverse obtains the meaning of an **inverse matrix**, which at once suggested the idea of extending the reversibility to other nonsingular logical operators besides **NOT** [Ref 88]. Any nonsingular logical operation can be turned backwards, provided we have access to negative truth-values (improbabilities). Breaking with the long-standing tradition, we in this manner calculated and incorporated into logic the inverses of **OR**, **IMPLY**, **IF** and **NAND**, previously thought to be nonexistent [Ref 88]:

$$\vee^{-1} = \begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \rightarrow^{-1} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}, \qquad \leftarrow^{-1} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix}, \qquad \downarrow^{-1} = \begin{pmatrix} 0 & 1 \\ 1 & -1 \end{pmatrix}.$$

Contrary to the commonly held view, there are many logical inverses about which we simply were unaware, not knowing how to evaluate or compute them, due to the lack of negative logical values. Besides the single inverse of Boolean logic much larger set of logical inverses has been identified in matrix theory. Altogether we identified **40+8** inverses, which includes the **40** operators in the matrix logic set itself and the octet of the isomeries $\pm 1/2H(\cdot)$ where $H(\cdot)$ are the Hadamard operators

$$\begin{pmatrix} 1 & 1 \\ -1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad \begin{pmatrix} -1 & 1 \\ 1 & 1 \end{pmatrix}.$$

The new logical inverses contain the negative matrix element(s) and an attempt to interpret these elements consistently led us to a negative extension of logic which reveals new fundamental aspects of the thought process. Although we

arrived at negative logic as a result of a matrix generalization of the notion of a logical inverse, if consciousness is a vector subspace, negative logic can be introduced in a purely axiomatic fashion. The axioms of vector space are a sufficient mathematical argument, obliging us to consider the negative logical values. A negative logical operator is a subtraction of two positive operators, that is typically not cognizable. However, cognizable logical subtractions also exist in which negative values do not appear, for example

IMPLY - FALSE = AND.

Negative logic allows a more subtle analysis of the thought processes. No inference in classical logic generates **NOT**, which is considered a basic and total operation. However, apart from all-out full denials, we often make use of partial and incomplete forms of negation. Starting from first principles negative logic opened up the possibility of deriving the negation operator as a product of logical operators: **NOT=A•B**. Consecutive application of these operators achieves complete negation, whereas each operator alone is a subnegator. In this way we introduce the degrees of **NOT** and enlarge the negating power of logic significantly.

To conclude this discussion it may be useful to emphasize one important implication which stems from considering negative logic. Negative logic provides extensions of logic which otherwise can be conceived only in a postulatory manner, if at all. Since **NOT** can now be factorized and conversely be **deduced** as the product of other operators, its typical presence in the basis sets of logic such as {**NOT, AND**} or {**NOT, AND, OR**} ceases to be necessary. **Paradoxically we understood NOT in depth only to learn that we can do without NOT.**

RECOVERING LOST DATA

In conventional logic compound formulas are **alternating** sequences of logical variables and connectives. A fundamental feature of consciousness is the **nonalternating** strings which nonetheless are well-formed formulas. The interaction of operators is again a logical operator, and one important question concerns the recovery of interacting operators from the product. Not any logical operator is recoverable, and cognitive data are frequently lost. This question of the recovery of lost data is part of a general problem of the reversibility of thinking operations. Matrix logic provides a new effective way to recover information lost in processing. The critical question is in the availability of the inverse.

Suppose the operator **TRUE** results from the interaction of **AND** with some operator which can be singular, for example:

$$\mathbf{TRUE = AND \cdot 1}$$

explicitly:

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \qquad \det \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} = 0,$$

or nonsingular, for example

$$\mathbf{TRUE = AND \cdot IF}$$

explicitly:

$$\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad \det \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} \neq 0.$$

In the second instance the inverse $\mathbf{IF}^{-1}$ exists and **AND** is recoverable from **TRUE**:

$$\mathbf{TRUE \cdot IF^{-1} = AND \cdot IF \cdot IF^{-1} = AND},$$

where the negative matrix element appears in the inverse $\mathbf{IF}^{-1}$.
Negative logic makes possible an effective reversible computation, recovering lost data.This is an example of the general rule. Any nonsingular matrix logical operator has an inverse and the corresponding operation can be 'rolled backwards'. We mentioned earlier that together with the octet of the halved Hadamardian nonbinary inverses **48** matrix logical operators are reversible. In contrast to Boolean logic, where **NOT** is the only nontrivial self-inverse, in matrix logic **14** different self-inverses manifest themselves.

A logical inverse must be distinguished from a converse, which does not require negative logic. Consider the implication $\langle x|\rightarrow|y\rangle$. The converse simply transposes the implication:

$$\langle x|\leftarrow|y\rangle = \langle x|\tilde{\rightarrow}|y\rangle$$

the inverse cancels it:

$$\langle x|\rightarrow\rightarrow^{-1}|y\rangle = \langle x|y\rangle$$

essentially requiring negative logic:

$$\langle x|\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & -1 \\ 0 & 1 \end{pmatrix}|y\rangle = \langle x|y\rangle$$

Although a converse is a nonnegative logical transformation, it is possible to obtain it in an 'awkward' way with the aid of negative matrix logic:

$$\langle x|\mathbf{IF}|y\rangle = \langle x|\mathbf{IMPLY(TRUE\text{-}OFF)}|y\rangle$$

which in matrix formulation is

$$\langle x|\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}|y\rangle = \langle x|\begin{pmatrix}1&0\\1&1\end{pmatrix}|y\rangle$$

One must not overlook a special case of the **TIME** operator for which the inverse equals the converse, the property of the unitary operator:

$$\blacktriangle^{-1} = \blacktriangledown$$

Negative logic also plays an important role in the identification of the complete set of superposed states. Matrix logical consciousness is **5**-valued, making use of **empty**, **false**, **true**, **false+true** and **false-true** states. In the very first works on matrix logic it was determined that in positive matrix logic the first four states naturally emerge but the superposed state **false-true** is absent. However, as soon as negative logical operators become active, the missing **false-true** state appears. For example:

$$\begin{pmatrix}1&1\\-1&-1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix} \qquad \text{or} \qquad \begin{pmatrix}0&1\\1&-1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}.$$

LEFT ≠ RIGHT

The limits of logic are the limits of our world. The world is not a scalar. The world has left and right identities, most noticeable in the weak interactions involving the neutrino and in the left-handedness of life molecules. Classical Boolean logic, however, is effectively a scalar. Its fundamental limitation is that being a scalar theory it doesn't account for the left and right tensor properties of logic. Matrix logic spells the end of a scalar concept of logic and consciousness. In Boolean logic the logical connectives are divided into unary and binary, and binary connectives cannot act as unary ones. In matrix logic **every** connective operator can be studied in isolation and can act both as a binary and unary operator. As opposed to numbers which do not have left and right 'sides', noncommuting matrix logical operators naturally display distinct left and right identities. Matrix logic lives on the lattice of **81** vertices, of which **54** are nonsymmetric and only **27** are symmetric. Because the left and right applications of a nonsymmetric operator are different, this explicitly defines two kind of logical operations, left and right, which are not equivalent and sometime completely opposite. The nonsymmetric tensor T_{asym} of matrix logic contains twice as many components as the symmetric tensor T_{sym}. To convert from Hermitian to non-Hermitian space, the symmetric tensor must duplicate by means of the topological ***ZIP*** operation

$$T_{asym} = \boldsymbol{ZIP}(T_{sym}),$$

converserly,

$$T_{sym} = \boldsymbol{CUT}(T_{asym}).$$

With the powerful language of matrix theory, logic reveals new and hitherto hidden aspects. The different left and right identities of matrix logical operators dictates the need to distinguish left and right logics. In the fundamental theory of consciousness left and right logics play an important role. While in Boolean logic the changeover from left to right is undetectable, in matrix logic it can have dramatic repercussions, effectively negating a logical operation. Conversely, negation can induce the left-right changeover, for example

$$\langle p|\rightarrow {}^{2}|q\rangle = \bar{p} + q \qquad\qquad \langle p|\leftarrow {}^{2}|q\rangle = p + \bar{q}\,,$$

where transposition is equivalent to transfer of negation from bra to ket. Another fundamental example is the actions of the annihilation and creation operators in left and right logics. The operators a^* and a reverse their roles with the changeover from left to right: creation turns into annihilation and annihilation into creation:

$$a^*|0\rangle = |1\rangle \qquad \text{and} \qquad a|1\rangle = |0\rangle$$

but

$$\langle 1|a^* = \langle 0| \qquad \text{and} \qquad \langle 0|a = \langle 1|$$

To conserve the result with a changeover from left logic to right logic, a swap of the annihilation and creation operators is required.

HOW MANY LOGICAL THEORIES?

To distinguish the true from the false one must have an adequate idea of what is true and false.

Benedict Spinoza

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SEARCH FOR UNIVERSAL LOGIC

We have fuzzy logic which solves the classic paradox by calling it a half truth, there is August Stern's Matrix logic which operates with four truth values: True, False, True and False, Neither True Nor False. And there are myriads others. The hope is that there turns out to be a finite number of hyperlogics. It will be very interesting to see if our culture manages to make this kind of thinking intuitive.

Onar Aam

INTUITION VERSUS INTUITION

In physics there is a strong belief in and a growing amount of supporting evidence for the existence of one grand unified physical theory. This is in great contrast with logic which is engulfed by myriads of different approaches, ideas and methods, reflecting the taste, education or lack of it of their creators. Is logical theory an arbitrary construction of the mind? Can it be invented or should it be designed from first principles and studied in the same way as we study the physics of the real world? My goal here is to show that **the existing theories of logic are in fact particular limits of one universal theory** - MATRIX LOGIC. The significance of MATRIX LOGIC lies not only in its providing a framework for unifying diverse logics into one universal system, but also in contributing to the progress of general field theory in physics. The ideas introduced in MATRIX LOGIC have been the main impetus for the development of M(atrix) theory. To get full insight into the matrix concept we must travel an uphill road, strewn with many intellectual mines and counterintuitive concepts.

Logic has always been an integral and fundamental part of human thinking. The development of logic is an ongoing process of the rejection of dogmas. For a long time no one questioned Aristotle's law of the Excluded Middle and the bivalence of logic, according to which every logical formula ought to be either true or false. But then multivalued logics were developed, with two-valued BOOLEAN LOGIC as a particular limiting case. Multivalued logics

included other truth-values than true and false. The extension of the alphabet of truth-values was followed by the introduction of probability into logical systems, giving rise to FUZZY LOGIC, which abandons the idea of precise logical values and enables one to treat intermediate levels of validity between true and false as well as intermediate levels of certainty between complete and incomplete information. These developments were in part and indirectly inspired by the uncertainties found in quantum physics. Various forms of MODAL LOGIC, employing the dual notions of necessity and possibility, recognize the fact that there are truths that lie wholly beyond the grasp of deterministic classical logics.

Further still, in NONMONOTONIC LOGIC one rejects the convexity of logical inference. If logical evaluation goes astray and falters, in nonmonotonic logic one has the option of putting things right and starting all over again but with different premises. TENSE LOGIC rejects the independence of logic from time, seeking to provide dynamical solutions to logical problems. A radical revision of logic is adopted in QUANTUM LOGIC, which rejects the universal character of the commutative and distributive laws in order to accommodate the nonclassical behaviour of quantum fields and particles. In MATRIX LOGIC one can treat logical thoughts as octonions, which is a final blow to scalar intuition, since in the octonionic algebra the associativity law may also break down.

When one looks back on this list of rejections, one wonders what is left of classical dogma and what there remains to be rejected besides certainty, commutativity and distributivity, and the associativity laws? But in MATRIX LOGIC the rejection reaches its ultimate degree, with logic basically rejecting logic itself. In model theory and other logical systems logical interpretation of well-formed formulas has been always available. From premises to conclusions, any step of inferences can be logically analysed. MATRIX LOGIC breaks out of this conceptual constraint. It introduces new abstract operations, which are without direct logical interpretation, yet are intricately incorporated into the logical thought process. This total dissolution of logic yields not an anarchy, but a new freedom of deductive and inductive actions, which allows one to penetrate deeper into the mechanism of consciousness, the task which pure interpretive logical systems have failed to accomplish.

We are all unique. Everyone has his or her own way of thinking and reasoning. The question which naturally arises is whether everyone's mind realizes its own particular logical calculus or whether there is one general universal theory of which all human and possible cosmological logics, if such exist, are particular realizations. Generally speaking this all boils down to the following question: is there one universal intelligence code, similar to the universal genetic code, or there are many, even incompatible thinking systems? We may never know with certainty, but in virtue of prevailing scientific doctrine, we tend to think in universal terms, which then would require any

imaginable logical system to be a particular limit or application of ONE UNIVERSAL LOGIC.

Different logical theories account for different logical properties. The list of logical systems is large and growing. Logicians and computer scientists continue to discover various multivalued systems, logic of forms, nonmonotonic logic, deodonic logic, linear logic, tense logic, modal and intuitionistic logics and many others. Are we at liberty to freely design a logical system? My goal here is to demonstrate that all logical theories are in fact particular limits of one universal logical theory and that such a theory is offered by the formalism of MATRIX OPERATOR LOGIC. An extensive research effort has led us to conclude that the key logical theories, such as BOOLEAN LOGIC, FUZZY LOGIC and QUANTUM LOGIC, are the particular limits of one general theory of matrix logic. The purpose of this study is to demonstrate and substantiate this claim and to show that the scalarization of MATRIX LOGIC is BOOLEAN LOGIC, its randomization is FUZZY LOGIC, and its quantization is QUANTUM LOGIC.

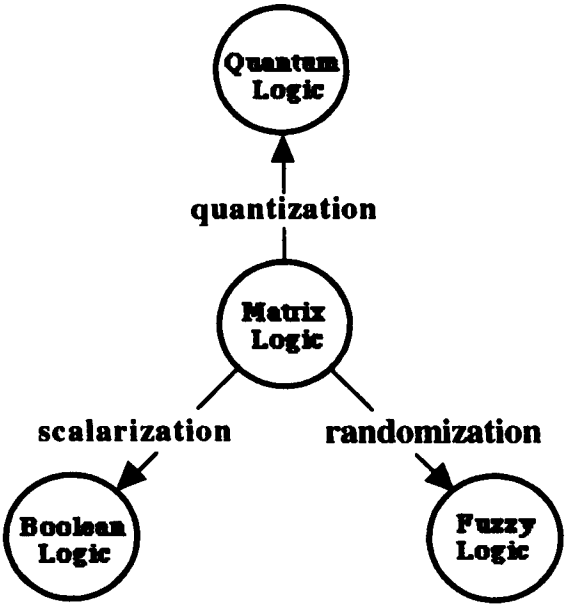


Fig. 38
Quintessential components of grand unified logical theory.

The scheme in Fig. 38 can be given a fundamental physical interpretation, in which classical logics, similar to classical physics, is identified by $\hbar = 0$, quantum logic by $\hbar \neq 0$, with matrix logic covering both the classical and the quantum limits.

TRUTH SERIES

Scientific models are typically idealizations of actual phenomena, often abstracting from important aspects of reality. Classical logic suffers from similar drawbacks. In classical logic true x and false $\bar{x}$ are treated simply as complements of each other

$$\bar{x} = 1 - x$$
$$\bar{\bar{x}} = 1 - (1 - x) = x$$

This interpretation is explored in Boolean logic as an algebraic model of negation. Negation is the fundamental operation of thinking. We learn the truth by negating false or we become false by negating true. The nature of negation, however, has been grossly oversimplified. The finite concept of negation often conflicts with our cognitive experience which tells us that the negation operation can be quite complex, frequently comprising partial denials. We consequently are motivated to look for a more fundamental definition of negation which can take into account the multiplicity of truths.

With the aim of treating truth as an unfolding process, instead of relating true and false by simple complementation, we relate them through wavelike exponentiation:

$$e^{-\mathrm{TRUE}} = \mathrm{FALSE}$$

Then the series expansion

$$\bar{x} = e^{-x} = 1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \frac{x^4}{4!} \cdots$$

reveals that classical logic makes use only of the linear terms of the expansion, and for a general thought process linearity has to be given up. We thought we knew negation too well, but infinity offers a surprise. If the false is defined as the exponential expansion of the true, one must be aware of the fact that besides the Boolean linear part of the expansion, there is infinite nonlinear part that can be explored, revealing the otherwise undetectable properties of **NOT**.

Logic evolves together with the notion of truth, which is central to the theory of logic. Depending on the definition of true and false different logics arise. But truth is not necessarily black or white. There are truths we can prove and those we cannot. There are singular and multiple truths. There are truths full of lies, there are relative truths, hidden truths and quantum truths. There are obvious truths and truths we will never know, which is a fascinating truth in itself. In the 20th century many of us were caught up in the excitement of the existence of virtual and parallel truths. Do fundamental absolute truths exist? The understanding of truth as an infinite process would suggest that only relative truths are given to consciousness unless the series expansion halts, arresting the infinite regress.

FIVE FUNDAMENTAL LOGICS

Each science has as much science as it has mathematics.

Emanuel Kant

With the development of matrix logic it has immediately become clear that **true** and **false**, which dominated the world of logical theory, are not enough and that the three-valued and four-valued logics are hidden aspects of logic. The formalism of matrix theory has granted us a natural extension of the logic of **true** and **false**, to the ternary logic of **true**, **false** and **undecided**. Then, with a fine resolution of the **undecided**, we have arrived at the tetrary logic of **true**, **false**, **true and false**, and **neither true nor false**. Apparently, in describing our familiar, macroscopic world we were unable to see the deeper levels of logic and neglected its intrinsic quantization. This stunning revelation, taken in the framework of multivalued or **k**-valued logic, prompted us to ask the question, how many logics there are. Is **k** arbitrary, and freely chosen, as is the case in multivalued logics, or are there some finite fundamental values of **k**? To answer this important question we must connect the values of **k** to some mathematical principle, which essentially can tell us how many different logics actually exist and what are the key differences between these logics. Our result can now be summarized as follows.

The thought process is naturally represented by a normed algebra. Since there are just four normed division algebras, the real, complex, quaternion and octonion, one is motivated to see whether the brain makes use of them. A deeper level of understanding of logical theory is reached when we consider a particular logic resulting from a particular potency law [Ref 89]:

$$X^{\alpha} = X,$$

where the powers are primes or power of a prime $\alpha = 2, 3, 4, 5, 7$. In (2×2) matrix logic $\alpha = 2, 3, 4, 5, 7$. The 7th power is the greatest, and we can only speculate what happens after 7, in logics of higher dimensions, if such exist. More significantly, we can now make a connection between logic and the Galois fields. The Galois field is a field which contains a finite number of elements. There are fields with 2, 3, 4, 5, 7, 8, 9, ... elements but not with 6 or 10, because the number of elements in a field must be a prime or a power of a prime. In matrix logic the Galois prime series gives rise to the 5 fundamental logics:

$x^2 = x$ - idempotent Boolean logic (0 and 1)
$x^3 = x$ - tripotent negative logic (true, false and negative)
$x^4 = x$ - quatric positive logic (01, 10, 11, 00)
$x^5 = x$ - pentapotent time logic (0, 1, -1, i, $-i$,)
$x^7 = x$ - septonic logic (imaginary section of octonion)

The different forms of logic that have been discussed are particular limits of matrix logic which includes (2×2) operators only up to the potency 7. The idempotence of Boolean logic has already been referred to on many occasions. In Boolean logic the absorption rule applies: the identical expressions merge, leaving intact the value of the logical formula. Fuzzy logic can be classified as **weakly** idempotent logic:

$$x^2 \leq x$$

where the conjunction of identical expressions does not cancel but tends to zero. To the tripotent logic we put in correspondence the negative unit -1,

$$-1^3 = -1.$$

The 4-potency offers no new roots which are Boolean-degenerate. But the 5-potent logic adds two imaginary truth-values:

$$i^5 = i,$$

From the contradiction law, taking the form

$$i - i^5 = i\,(1 - i^4) = 0$$

one derives the five logical values 0, 1, -1, i and $-i$, which can be mapped onto the five fundamental logical states of matrix logic:

$$(0\ 0),\ (0\ 1),\ (1\ 0),\ (1\ 1),\ (1\ -1).$$

Finally, we have the septonic logic which is a pure imaginary section of a hypercomplex number or octonion:

$$a_0 + i_1 a_1 + i_2 a_2 + i_3 a_3 + i_4 a_4 + i_5 a_5 + i_6 a_6 + i_7 a_7, \qquad a_0 = 0.$$

Septonic logic in our scheme is regarded as a fundamental limit of human intelligence, at least within the scope of (2×2) matrix theory. Septons are fundamental truth-values which are involved at the abstract aesthetic level of consciousness: emotions, perception of music, colours and art in general. Just as there are seven key notes of harmony, there are seven septonic values. Significantly, for octonion elements to order seven, after seven successive multiplications, the space volume no longer grows. The product of observables represented by self-adjoints on the complex Hilbert space remain observable. But if the theory is designed over octonions, then the algebraic structure of the subspace admits successively only seven elements, and then phase space stop growing. A brane, punctured with a given number of holes, if wrapped around 7 curled dimensions and compactified would 'miraculously' produce a topology with a different number of holes [18]. Here we are reaching out into deep areas of quantum topology. In a topological framework the potency of a logic is closely connected with the number of crossings in a knot. The number of crossings (self-intersections) is put in correspondence with the potency laws. These start from idempotency and are saturated for the septonic logic. Two crossings cannot produce a knot. In (2×2) matrix logic we have 2-potent and 4-potent operators corresponding to bosonic knots, and 3, 5 and 7-potent operators corresponding to fermionic knots. The absence of 6-potent operators

agrees with the idea of a connection between logic and the Galois fields. Cognitive knots are alternating. The nonalternating knots start with **8** crossings, which do not appear in our system.

The discrete classification scheme which we have chosen seems to exclude continuous fuzzy logic. There is a finite number of logical dimensions. Where does fuzzy logic stand in this framework? Logical analysis can be applied, if only in theory, to any number of dimensions, and in continuous logic a significant number of new logical dimensions can be constructed. One is at liberty to consider, say 5.33 or 7.99 dimensions, where the number of dimensions, bizarre as it is, a continuous function. After logical evaluation is completed and converged, the extra dimensions can be removed, and system renormalized to the basic **5**-brane. The mind works with fuzzy values but in the end discards them as not cognizable, just as in electrodynamics one works with complex numbers but in the end, on completion of the calculation, dispenses with them as beingirrelevant to actual physics.

BOOLEAN LOGIC

In Boolean logic any propositional formula can be expressed in the functionally complete basis set of connectives {**NOT, AND, OR**}. These canonical connectives lend themselves naturally to algebraic interpretation as binary complement, product and logical addition respectively, the fact explored in the design of idempotent Boolean algebra [Ref 95]. Boolean logic algebra is based on the following system of axioms:

Negation

$$\bar{x} = 1 - x$$

Absorption

$x \wedge x = x$ $\qquad$ $x \vee x = x$

Contradiction & excluded middle

$x \wedge \bar{x} = 0$ $\qquad$ $x \vee \bar{x} = 1$

Extremas (min & max)

$x \wedge 1 = x$ $\qquad$ $x \vee 1 = 1$

$x \wedge 0 = 0$ $\qquad$ $x \vee 0 = x$

Commutativity

$x \wedge y = y \wedge x$ $\qquad$ $x \vee y = y \vee x$

Associativity

$x \wedge (y \wedge z) = (x \wedge y) \wedge z$ $\qquad$ $x \vee (y \vee z) = (x \vee y) \vee z$

Distributivity

$x \wedge (y \vee z) = (x \wedge y) \vee (x \wedge z)$ $\qquad$ $x \vee (y \wedge z) = (x \vee y) \wedge (x \vee z)$

DeMorgan duality

$\overline{x \wedge y} = \bar{x} \vee \bar{y}$ $\qquad$ $\overline{x \vee y} = \bar{x} \wedge \bar{y}$

Boolean algebra requires several constraining postulates:

(a) the involution of logical addition: $1 + 1 = 1$,

(b) the modulo sign degeneracy: $+1 = -1$,

(c) the implication power: $0^0 = 1$.

The last postulate allows us to enlarge the system of axioms of Boolean logic with the implication tautologies:

Ex falso quodlibet: $0 \rightarrow x \equiv 1$,
Verum ex quolibet: $x \rightarrow 1 \equiv 1$,
Self-implication: $x \rightarrow x \equiv 1$.

Although the operations of Boolean algebra have a great deal in common with the ordinary operations of multiplication, addition and subtraction, in contrast with ordinary algebra there are not one, but two distributive laws. Boolean addition and product are idempotent, so that they do not yield factors or degrees of power. Although any logical function can now be expanded as a binary polynomial, only three (four) logical connectives $\{N, \wedge, \vee, (\rightarrow)\}$ can be given direct mathematical form. There are simply not enough algebraic operations in scalar Boolean algebra to allow every logical connective to be associated with a particular algebraic operation.

Every Boolean expression is a discrete function, the arguments of which, independently of each other, obtain the values true 1 or false 0. There is a finite number of different argument sets for a function, and a logical function is uniquely defined by a table listing all the possible argument sets and the corresponding function values. The number of Boolean functions of n variables rapidly increases if n increases. For $n = 1$ there are **4** different functions, for $n = 2$, **16** functions, for $n = 3$ we can construct **256** functions, and for $n = 4$, **65536** functions.

Given a set of functions one is usually interested in expressing some functions via the others and in determining the minimal irreducible set of functions — the basis set. Considering the functionally complete basis sets in Boolean logic one must not fail to distinguish between the algebraic and pure logical basis sets. Expressing compound logical functions via more simple ones, we end up with elementary Boolean functions, just as $sinx$ or x^2 are regarded as elementary in analysis. In Boolean logic different basis sets exist: algebraic, pure logical or both. The standard basis set

{NOT, AND, OR, IMPLY, IS}

can be cut down to the major basis set, commonly used in applications,

{NOT, AND, OR}

which in turn can be reduced, with the aid of the **DeMorgan** equalities, to the minor basis sets:

{NOT, AND}, {NOT, OR}, {NOT, IMPLY}.

The minor basis sets can be further reduced to the minimal logical basis sets

{NAND} or {NOR},

which are functionally complete in themselves:

$\bar{x} = (x \downarrow x)$ $\bar{x} = (x \uparrow x)$

$(x \wedge y) = (x \downarrow y) \downarrow (x \downarrow y)$, $(x \wedge y) = (x \uparrow x) \uparrow (y \uparrow y)$

$(x \vee y) = (x \downarrow x) \downarrow (y \downarrow y)$ $(x \vee y) = (x \uparrow y) \uparrow (x \uparrow y)$

The minimal sets, made of a single logical function, allow maximum economy in connectives to be achieved. However, this economy is gained at the expense of clarity, for the use of a single connective can give rise to rather complicated expressions. In various applications it is much more convenient to use major or minor basis sets, because loss of clarity is the price which must usually be paid if minimal basis sets are used.

MULTIVALUED LOGIC

In multivalued or **k**-valued logics the class of logical functions is expanded to incorporate the intermediate truth-values between true and false. Clearly, the introduction of multivalued logical systems is possible only at the expense of weakening the classical law of the Excluded Middle. It is possible to distinguish two related multivalued approaches. In one of these the addition of new truth-values encodes intermediate levels of validity between true or valid and false or invalid. In another approach the addition of truth-values encodes intermediate levels of certainty between complete information and incomplete information.

The number of **k**-valued functions of **n** arguments is k^{k^n}. For $k \geq 3$, the difficulties of tabulating functions, in comparison with Boolean logic, increase rapidly. In three-valued logic there are **19683** functions with two arguments, a significant difference from two-valued logic, where only **16** different functions of two arguments exist. Not surprisingly, multivalued systems have been attracting growing attention for their wealth of logical structures. The Boolean functions have several analogs in multivalued systems:

NOT:

$\bar{x} = x + 1 \ (\text{mod} k)$ – cyclical

$\sim x = k - 1 - x$ – diametrical

AND:

(1) **min** (x, y)
(2) x·y (modk)

OR:

(1) **max** (x, y)
(2) x+y (modk)

The multivalued functions in many respects resemble the functions of classical logic and some results of the latter remain applicable in multivalued logics. However, there are important novel properties which distinguish a logic for $k \geq 3$. For example,

$$\sim (\sim x) = x \qquad \text{but} \qquad \overline{\overline{x}} \neq x$$

$$\sim \mathbf{min}\ (x, y) = \mathbf{max}\ (\sim x, \sim y) \qquad \text{but} \qquad \overline{\mathbf{min}(x, y)} \neq \mathbf{max}(\overline{x}, \overline{y})$$

It is commonly believed that the value **k** in multivalued logics is arbitrary and can be freely chosen. The potency approach of matrix logic has thrown new light on this important question. Just as theoretically one is at liberty to consider **n**-dimensional spaces, in reality, according to **Einstein**, actual physical space is **4**-dimensional.

TENSE LOGIC

In spite of the fundamental role of time in logical thinking, tense logic is probably one of most poorly developed extensions of logical theory, offering no significant theorems, if any. Dissertations and books have been written on the subject of time in logic, but their intellectual and practical value is almost nil.

The role of time is most evident in noncommuting logical operations. Typically, logical functions are applied to arguments in a certain order. With each function we associate a converse function, in which the order of arguments is reversed. For asymmetric logical functions, like implication, a swap of the arguments can change the value of the function. The arguments do not commute in implication and therefore generally

$$x\ \mathbf{IF}\ y \neq y\ \mathbf{IF}\ x.$$

The ordering of the arguments in the implication conceals within itself a temporal ordering, and if the inference is causal, the logical variables should appear in the expression in a certain order. To prevent the breakdown of causal relations in tense logic a logical expression is specified by stating the materially ordered set of logical connectives and logical variables. With the aid of the time notation, instead of

$$(x\ \mathbf{L}\ y)$$

we must write

$$(x(t_1)\ \mathbf{L}\ y(t_2))$$

where the times indicate the particular order in which logical variables must appear, no matter whether a function is asymmetric or symmetric. We encounter no difficulties with the tense treatment of asymmetric logical functions. The problem truly presents itself when we consider symmetric functions, like **AND** and **OR**. Because classical logic does not not concern itself with the meaning of logical variables but only with their truth status, one legitimately writes

$$x \wedge y = y \wedge x$$
$$x \vee y = y \vee x$$

where x and y commute under conjunction and disjunction and the time ordering is irrelevant to the resulting truth-value of the expression. Disregard for time, however, can yield truth-functionally valid but meaningless statements. Whereas the **AND** and **OR** we are familiar with in language are often based on some connection in meaning between the variables, the logical **AND** and **OR** represent only a relationship between their truth statuses. This enables the truth of (x **AND** y) or (x **OR** y) to be inferred from the truth of x and y, independently of the order of their appearance in the expression. Since the commutation law is one of the defining features of classical logic, a fundamental contribution to logical theory of tense logic, if only on the conceptual level, is in underlining the fact that one must reasonably expect classical logic to fail in real-world situations whenever the logical variables do not commute. It is important to emphasize that this is more an observation than a technical result of tense logic. Time does not actually appear in the logical expression explicitly but must be brought in by some intelligent agent like ourselves, possessing the faculty of dealing not only with truth but also with meaning.

Another serious shortcoming of tense logic is a lack of negative values, which are nonexistent in classical logic but are necessary if we are to deal with time. An 'arrow of time' in logic always points in one direction. The time operator, introduced in matrix logic, changes all that. The antisymmetric commutator products of the major basis set of logical operators universally give rise to a signed time operator, the fundamental cognitive observable.

QUANTUM LOGIC

Logic is an abstract theory which concerns itself with determining the truth of various statements. This abstract concept of logic, however, includes the possibility of a physical realization, which indeed does take place, for example in the computer or in the brain. Although the brain and the computer are macrophysical systems, there is no fundamental physical reason preventing us from downscaling logical devices to a single atom, or even, if we are inventive enough, to a single particle [Ref 52, 75]. The logical operations then must take into account quantum-mechanical uncertainty and coherent superposition, which distinguish quantum evolution from classical motion.

While in classical logic there is no valid inference rule that would give an invalid conclusion, a quantum-logical system may be in a state which makes neither a given proposition nor its complement true. Quantum-logical problems cannot be accommodated by Boolean logic, and the logic underlying quantum mechanics could not be a Boolean lattice, since it must admit propositions that can be neither true nor false. Quantum fields and particles display properties that do not conform to the classical rules of inference, with the commutation relations setting fundamental limits to observation. As with tense logic, but for completely different reasons, Boolean logic would fail in the quantum domain whenever the logical variables do not commute or are superposed. Most significantly, the distributive laws break down:

$$x \wedge (y \vee z) \neq (x \wedge y) \vee (x \wedge z)$$
$$x \vee (y \wedge z) \neq (x \vee y) \wedge (x \vee z)$$

Quantum theory shifted the interest of physicists towards the investigation of information properties of matter. At first sight, the formalism of quantum physics bears no resemblance to formalism of logic. What do wavefunctions and spinors have to do with tautology or Modus Ponens? Can we think quantum-mechanically? Each well-formed formula of logic is supposed to express a **valid** proposition, while each statement of physics is supposed to define a **true** physical fact. Modelling propositions by mathematical structures of quantum theory shifts a large part of the problem to the study of these mathematical structures. But if the laws of thought represent physical reality, one must first investigate the reality which contains logic as its intrinsic structure. When logical premisses are accepted with incomplete certainty of their truth, they cannot as such provide an absolute guarantee of the truth of the conclusions. In the Modus Ponens

$$(a \wedge (a \rightarrow b)) \rightarrow b$$

where a is a quantum premiss, b may or may not be a Boolean truth. The fact that we cannot make fully predictable use of the basic inference rule in the quantum realm shows that quantum logic not really a logic. The debate continues whether it should be regarded as a genuinely new logic or merely as an algebra under another name. One extreme is to view it not only as a new logic but as one even more fundamental than classical logic. Others give priority to classical logic. In developing quantum theory we do not think quantum-mechanically but rely on classical inference. Thus it seems a contradiction in terms that quantum theory, which makes extensive use of classical logic, gives rise to a revision of logic which denies classical logic itself.

Like no other development, quantum logic has brought into focus the question whether logic can be invented or should be derived from first principles.The choice of logic cannot be divorced from real-world constraints and ought to be based on an objective study of propositions themselves, thus ultimately via physical experiment. A formal theory which adequately expresses the features of the real world must mirror these features in its structure because of the laws

of logic. After the discovery of the links between logic and quantum mechanics, quantum theory had become a major source of inspiration for further development of logic. The more we learn about quantum physics the more we know about the thought process. There remain many dark spots, *because* the language of the brain is neither classical nor quantum. We can of course design separate logics, classical and quantum, which can deal with separate issues, and this is commonly the case. But, more meaningfully, what we can do is to seek one general system which joins the existing logical theories together.

FUZZY LOGIC

Multivalued logics generalize Boolean two-valued logic, expanding the alphabet of truth-values beyond singular true and false. Taken to an extreme, the method of multivalued logic leads to fuzzy logic, where the finite array of truth-values is interpolated into a continuum on [0, 1]. In this way much of the distinction between truth-value and probability is taken away. Fuzzy logic is a fancy way of saying that we use clever approximations. The rules of fuzzy reasoning are rules for deducing probabilities from probabilities. Boolean reasoning is effectively a special case of fuzzy reasoning in which the conditional probabilities are 1 (true) or 0(false).

While in Boolean logic the absorption rule applies and the identical variables in an expression merge, in fuzzy logic such cancellation is generally not possible. The conjunction of identical variables produces a cumulative effect and uncertainty increases with the increase in their number. This consequently alters the structure of negation in fuzzy logic, in which as in intuitionistic logic, **NOTNOTx** is weaker than **x**. In fuzzy logic the contradiction law becomes the weak contradiction law:

$$q\,\bar{q} \leq 0.$$

The same applies to the idempotency of Boolean logic which is involutive:

$$x^2 = x\ ,$$

but in fuzzy logic

$$q^2 \leq q.$$

The equality can hold only in the orthogonal discrete Boolean limit of fuzzy logic where probabilities turn into certainties.

Probability and truth-value

Since there are logical truths that lie beyond the grasp of discrete logics, any logical theory would be grossly incomplete if it failed to account for the fuzzy reasoning which is an important component of logical consciousness. As originally envisaged by Lofti Zadeh fuzzy logic is based on the idea of fuzzy sets, distinguished by approximately defined *shadow* boundaries [Ref 111]. Since the measures of true and false are given by the integers 1 and 0

respectively, it seems quite natural to associate fuzzy true and false with probabilities. The method of fuzzy logic is greatly enhanced with the support of probability theory because a truth-value is quite naturally translated into a probability, which, in turn, allows a straightforward application of probability algebra to logical expressions, treated as indeterministic events. A fuzzy logical expression we characterize by the probability of realization:

$$p(x) \in [0, 1].$$

The true and false expressions are identified by the probability $p(x) = 1$ and $p(x) = 0$, respectively, with the fuzzy expressions found in-between:

$$0 < p(x) < 1.$$

By assigning the probabilities of realization to elementary expressions, we assume that a compound expression also can be characterized by a probability of realization, which depends on the probabilities of realization of the constituent elementary expressions. Then the fuzzy **NOT**, **AND** and **OR** are straightforward applications of the probability laws of complement, multiplication and addition respectively. For the disjoint expressions the fuzzy **NOT** is defined as in Boolean logic as the algebraic complement:

$$p(\bar{x}) = 1 - p(x) = \overline{p(x)}$$

In calculating the truth-value of fuzzy **AND** we must take into account whether the logical expressions are dependent or not. As opposed to Boolean logic, the question of independence of expressions is significant to fuzzy logic, and it is this concept which marks off fuzzy logic as a distinct branch of logical theory. For independent expressions the truth or falsity of one is in no way connected with the truth or falsity of the other, and the fuzzy **AND** is the product of the fuzzy truth-values:

$$p(x \wedge y) = p(x)\, p(y).$$

Fact: the probability of a product is the product of the probabilities. But if the condition of independence is not satisfied, conditional probability is required:

$$p(x \wedge y) = p(x)\, p(y \mid x) = p(y)\, p(x \mid y)\,.$$

This establishes the necessary and sufficient condition for independence: the expressions are independent if and only if the conditional probabilities equal the absolute probabilities.

The fuzzy **OR** can be calculated with the aid of the law of addition of probabilities:

$$p(x \vee y) = p(x) + p(y)\,.$$

This relation holds only for disjoint expressions, $p(x \wedge y)=0$, otherwise it must be modified to account for the overlap:

$$p(x \vee y) = p(x) + p(y) - p(x \wedge y)\,.$$

An important distinction between fuzzy **OR** and **AND** is that the latter cannot be defined in Boolean terms, unless all its constituent components are binary. In contrast, even when the components of **OR** are fuzzy, it can still be true in

Boolean sense whenever it is applied to the complete set. The complete set refers to a certain realization which is effectively a tautology,

$$p(\Omega) \equiv 1,$$

while an impossible realization is necessarily a contradiction,

$$p(\bar{\Omega}) = p(\varnothing) \equiv 0.$$

The general form of the multiplication law can be used to define the fuzzy **IF**:

$$p(x \mid y) = \frac{p(x \wedge y)}{p(y)}, \qquad p(y) \neq 0 .$$

However, because conditional probability is not defined for $p(y) = 0$, it cannot in general serve as a proper model of fuzzy implication, which may involve a false antecedent. There are logical truths that are beyond the grasp of fuzzy logics. A principal difference between conditional probability and the probability of the conditional has to be duly recognized. As opposed to Boolean logic, fuzzy reasoning with false premisses cannot imply the truth of the conclusions. Fuzzy logic enlarges the computational capabilities of logic, but the interpretation of truth-value as probability has its advantages and its limitations. A major plus is that logic and probability theory can be joined computationally into a single concept, making it possible to explore probabilistic methods for solving logical problems. Although truth-value is quite naturally translated into probability, it must be emphasized that the concept of truth-value is wider than the concept of probability. While probability is a strictly positive measure, a truth-value can be negative or greater than unity. This fact indicates that probability is part of logical theory and not the other way round. Even though some aspects of logic are incompatible with a strictly positive measure, the analysis of logic can be enhanced with the continuum of truth–values which allows us to deals with approximate or imprecise reasoning involving partially true and partially false truth-values. Moreover, a prospect with rich mathematical possibilities emerges where the laws of probability can be derived from the logical system. Because we can pass from probability expressions to logic expressions and vice versa, the treatment of probability and the treatment of logic can be done jointly and not separately. Fuzzy logic allows us to join both theories into a single concept computationally. This is step one. To join a large repertory of logical relations into a single framework, we must go further and take the next step towards a theory in which different logics are derivable from the same formal base. An effective formalism, ideally suited for these purposes, is provided by matrix theory. Matrix logic allows a unified description which accounts equally well for different logical limits: discrete and continuous, and quantum and classical. One fundamental algebraic structure applies in uniform and concise fashion to different limits which until recently were incorrectly assumed to be separate branches of logical inquiry.

M(ATRIX) LOGIC

The matrix calculus framework can support the standard extensions of classical logic in a natural way.

Luis Farinas del Cerro

TENSORIZATION OF TRUTH-VALUES

The different forms of logic which we have discussed make use of different primitives. In contrast to classical logics which operate in a scalar format, modelling logical variables by numbers, matrix theory forces us out of the one-dimensional scalar framework. What distinguishes matrix logic from all previous formalisms is that it is 2-dimensional, and generalized, if necessary, to n-dimensional space. Matrix logic is the **nonscalar** logic in which a logical state is the ordered pair of the false-value and truth-values. This innovation has enabled us to describe logic in tensor language, in which a different type of intuition exists. For the sake of uniformity, we will represent the discrete Boolean variables, true 1 and false 0, by the two-dimensional Boolean orthogonal states:

$$\begin{pmatrix}0\\1\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}1\\0\end{pmatrix}.$$

As opposed to the pure states of two-valued BOOLEAN LOGIC, in continuous FUZZY LOGIC the basic primitives are the mixed normed probability states which correspond to uncertain situations. A general logical state is a convex of true and false states, accommodated by the two-component probability vector

$$\begin{pmatrix}\bar{q}\\q\end{pmatrix}$$

QUANTUM LOGIC is distinguished by the presence of the states of coherent superposition:

$$\begin{pmatrix}1\\1\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}1\\-1\end{pmatrix}$$

and the vacuum empty state

$$\begin{pmatrix}0\\0\end{pmatrix}.$$

The complete space of matrix logical states can be generated from the Boolean basis states $|0\rangle$ and $|1\rangle$ or the superposed states $|s_+\rangle$ and $|s_-\rangle$; both are

linearly independent and span the space. However, the superposed states do not have an interpretation in standard logic and probability theory.

If there exists a grand unified theory of logic, it must be able to consistently account for all these different logical states in a single formalism. It is my purpose to show that MATRIX LOGIC is such a universal logical theory in which the different forms of logic emerge as particular limits, intertwined in fundamental way. Prior to explicitly demonstrating that the discussed logical states are naturally contained in MATRIX LOGIC, we need to introduce several new important concepts.

Phase

In addition to 'amplitude' we characterize logical values by phase, which may also be viewed as the introduction of new negative truth-value. Then instead of the Boolean ring $E_2 = \{0, 1\}$, we have a balanced ternary set $E_3 = \{-1, 0, 1\}$, and the phase logic which emerges in this framework has been defined as negative matrix logic or antilogic [Ref 88, 92]. With the introduction of phase one clearly sees the need for the two-dimensional vectorization of logical values, one of the important contributions of matrix logic to logical theory. In scalar logic the phase is relevant only for the true bits, and cannot show up for the false bits because +0 = -0. Matrix logic recovers full symmetry and naturally accounts for the phase of both the true and false logical states:

$$e^{i\pi}|1> = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \qquad e^{i\pi}|0> = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

FROM TRUTH-TABLES TO OPERATORS

The notion of logical connectives, next in importance to the notion of truth, lies at the very foundation of logic. Together with axioms, variables and rules of inference, the connectives are the fundamentals in designing a logical calculus, without which it would be difficult to imagine the successful development of logic which we observe today. A connective is commonly defined by a truth-table whose entries give the value of a corresponding logical function depending on the value of argument(s). In Boolean logic there are **4** and **16** truth-tables defining **4** and **16** connectives, unary and binary respectively. For example, for the canonical pair of logical connectives, **AND** and **OR**, one writes the following tables:

AND	0	1
0	0	0
1	0	1

OR	0	1
0	0	1
1	1	1

The key invention of matrix logic is to convert the truth-tables into genuine mathematical objects which permit logic to start functioning fully as an instrument of calculation. Logic takes the full step into the maths. Instead of viewing a truth-table simply as a chart introduced for the purpose of friendly usage, in matrix logic we portray it as a matrix operator, obtained directly from the truth-table. For **AND** and **OR** this gives us, respectively:

$$\mathbf{AND} = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \mathbf{OR} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}.$$

This innovation propels logic to a new abstract level where the powerful apparatus of operator theory becomes directly applicable to logic. By promoting logical connectives to the rank of matrix operators, one overcomes many limitations of classical logic. While Boolean algebra heuristically accounts only for a few logical connectives, in matrix logic all connectives without exception are given direct and uniform mathematical form. 'All animals are equal' and none is more equal then another. Any logical function is now uniformly computable in quantum-mechanical fashion as the expectation or average

$$< L > \; = \; < p | L | q > \; = \; \bar{p}\, L_{11}\, \bar{q} \; + \; \bar{p}\, L_{12}\, q \; + \; p\, L_{21}\, \bar{q} \; + \; p\, L_{22}\, q$$

or equivalently by the tracing

$$< L > = \mathrm{Tr}\Omega L = < p | \, L \, | q >$$

where Ω is the logical density matrix and **L** is a matrix logical operator.

In Boolean logic algebraic mapping is restricted to just **three** connectives, **NOT, AND** and **OR**, which some view as an advantage. As a matter of fact, it rather points up the major defect of classical logic. The logic in use today is more mathematical in presentation than in substance. A somewhat vague but real and significant boundary separates the domains of logic and mathematics. The computational limitations of classical logic have been largely overcome in matrix logic in which **all** logical connectives without exception obtain direct mathematical form, with the atomic bracket <p|L|q> providing correct and meaningful computations.

The matrix logical operators act in the dual spaces of normed logical states, the bra and the ket:

$$< q | \; = \; (\bar{q} \;\; q), \qquad | q > \; = \; \begin{pmatrix} \bar{q} \\ q \end{pmatrix}, \qquad \bar{q} \; + \; q \; = \; 1,$$

neither of which is more important or fundamental than the other. Whatever a logical ket stands for, the same information is held within a corresponding logical bra. A theorem derived for a ket space automatically has the converse, applicable to a bra space, a fact we shall not mention unless the need arises.

For $\Omega^2 < \Omega$ matrix logic is in the fuzzy limit, naturally yielding the laws of probability. While conventional theory requires different rules to evaluate the product and the sum of probabilities

$$p(\mathrm{x\ AND\ y}) \; = \; p(\mathrm{x})\, p(\mathrm{y}) \, .$$

$$p(\text{x OR y}) = p(\text{x}) + p(\text{y}) - p(\text{x AND y}),$$

in matrix logic both the **AND** and **OR**, in fact any logical function, can be uniformly computed with one universal device $\langle p|L|q\rangle$:

$$\langle p|\ \textbf{AND}\ |q\rangle = pq$$

$$\langle p|\ \textbf{OR}\ |q\rangle = p + q - pq.$$

Employing the tensor product, and taking into account the commutativity of the inner product, or equivalently the associativity of scalar multiplication, we convert the operator's average into a matrix element of the logical density matrix:

$$\langle p|L|q\rangle = \langle p|x\rangle\langle y|q\rangle = \langle x|p\rangle\langle q|y\rangle = \langle x|\Omega|y\rangle$$

where $|x\rangle$ and $|y\rangle$ are Boolean state vectors.

For the mixed logical states $\Omega^2 \leq \Omega$ but with convergence of the logical density matrix to the projective operator:

$$\Omega^2 = \Omega,$$

matrix logic reduces to the classical limit of Boolean logic.

Because $\mathbf{Tr}\Omega^2=1$ only for a pure state, it provides a measure of the fuzziness of a logical state. Since

$$\Omega \cdot \Omega = \mathbf{Tr}\Omega \cdot \Omega,$$

for a classical state $\mathbf{Tr}\Omega = 1$. In the classical limit the mixed states converge to the complementary Boolean true and false states:

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix},$$

satisfying orthonormality:

$$\langle x|y\rangle = \delta_{xy}$$

and the closure relation:

$$\sum_i |x_i\rangle\langle y_i| = \mathbf{I}$$

In order to obtain the Boolean limit of matrix logic, one makes use of the knowledge operators **TRUE** and **FALSE** which universally convert an arbitrary fuzzy state into a Boolean state. A computation starts in a fuzzy universe but ends up with definite true or false states:

$$\forall|q\rangle: \quad \textbf{TRUE}\ |q\rangle \equiv |1\rangle$$

$$\forall|q\rangle: \quad \textbf{FALSE}\ |q\rangle \equiv |0\rangle$$

Matrix logic accounts both for Boolean and fuzzy inferences, with the discrete and continuous computed in a uniform manner. The reduction from fuzzy to Boolean logic or vice versa has no consequences for elementary atomic logical functions. In both instances the same computational rules apply; simply swap

the discrete $|x\rangle$ and $|y\rangle$ with the continuous $|p\rangle$ and $|q\rangle$ and read off fuzzy or Boolean logic:

$$\forall \mathbf{L}_i,\ i=1,81: \quad \langle x|L|y\rangle \Leftrightarrow \langle p|L|q\rangle.$$

In the past, the Boolean and the continuous systems were assumed to be two separate branches of logical inquiry. Matrix logic is a unified description which elegantly accounts for both discrete and continuous logics. The same atomic device that works in a discrete universe is also applicable in a continuous universe. However, for logical strings the repercussions of choosing a particular limit are serious. It is no longer possible to simply swap the discrete and continuous states, but corrections terms are needed. This is most clearly seen in the transmultiplication rule. While in the Boolean limit

$$\langle x|\mathbf{L}_1|y\rangle\langle x|\mathbf{L}_2|y\rangle = \langle x|\mathbf{L}_1 \blacklozenge \mathbf{L}_2|y\rangle\ ,$$

for example

$$\langle x|{\rightarrow}|y\rangle\langle x|{\leftarrow}|y\rangle = \langle x|{\rightarrow}\blacklozenge{\leftarrow}|y\rangle = \langle x|y\rangle$$

in fuzzy logic

$$\langle p|{\rightarrow}|q\rangle\langle p|{\leftarrow}|q\rangle = \langle p|{\rightarrow}\blacklozenge{\leftarrow}|q\rangle + \langle p|a^*|q\rangle\langle p|a|q\rangle$$

where $\langle p|a^*|q\rangle\langle p|a|q\rangle$ is the fundamental correction term without which the fusion of implications cannot occur. The correction term is a novel feature which distinguishes the fuzzy limit of matrix logic [Ref 92].

Matrix logic incorporates in the most natural way Boolean and fuzzy logics. Because matrices generally do not commute, there is a framework in which one also effectively accounts for tense logic, explicitly making use of the newly defined time operator:

$$\langle x|\blacktriangledown|y\rangle = x - y, \qquad \langle x|\blacktriangle|y\rangle = y - x.$$

The matrix time operator inadvertently brings in antilogic with negative logical values, so essential for the extension of the set of logical inverses from the single self-inverse **NOT** in Boolean logic

$$\mathbf{N}^{-1} = \mathbf{N}$$

to **40+8** inverses in the general matrix set, containing **48** nonsingular operators.

But the most unexpected and surprising revelation of matrix logic is its quantum limit. Matrix logical operators acting on ordinary classical states, pure $|x\rangle$ or mixed $|q\rangle$, can give rise to the empty vacuum state $|s_o\rangle$ and to the states of coherent superposition, $|s_+\rangle$ and $|s_-\rangle$, which have no interpretation in classical theory. In classical logic we present the meaning of a logical thought by recording an ordered string of the true and the false which determine a particular output value of the string depending on the truth-values of its constituents. But in the quantum limit of matrix logic we are not obliged to have a complete record. Instead of listing and explicitly stipulating the sequence of truth-values we can equally well send the logical string into a

coherent superposition which nonetheless maps to the same output value. In this manner we have explicitly extended the traditional logic of **true** and **false** into a four-state logic of **true**, **false**, **true and false**, **neither true nor false** [Ref 88]. We are now in a position to begin to answer the question which we raised at the beginning of this chapter. Different forms of logic are particular limits of one general matrix theory.

LOGICAL CONNECTIVES INTERACT

Prior to the development of matrix logic the study of logic was carried out by developing and investigating different scalar modalities. The important innovation of matrix logic is to place at the foundation of logic not scalar quantities but more complex mathematical objects, namely logical states and operators, joined eventually into the more general concept of a logical tensor. This fundamentally distinguishes matrix logic from classical logics which regard scalars as primitive entities. Following this line of reasoning we begin to reveal the previously unrecognized possibility of the **direct** interaction of connectives. This new feature is important for capturing the intelligent aspect of logic, providing a new clue to the understanding of the higher level of abstraction. There are additive and multiplicative interactions, which can be meaningfully employed: for example, we write the tautology operator in the form

$$\mathbf{YES + NOT = OR \cdot AND \cdot OR.}$$

While in conventional logic there is a strict division into unary and binary connectives, in matrix logic, where we represent connectives by matrix operators, this distinction is eliminated, and only when it is recovered can matrix logic be reduced to a classical limit. For a matrix logical expression there is no need to request the alternation of connectives and operators which distinguishes classical logic. Matrix strings can be computed in a new, different way. Given the logical operators $\mathbf{L}_1 \in \Omega$ and $\mathbf{L}_2 \in \Omega$, and provided the product $\mathbf{L}_1 \cdot \mathbf{L}_2 \in \Omega$, expressions like

$$\langle p| \mathbf{L}_1 \cdot \mathbf{L}_2 |q\rangle = \langle x|\mathbf{L}_3|y\rangle,$$

where a pair of operators enter into direct interaction, are perfectly legal and legitimate. In reverse, one factorizes a logical operator into a product. Factorization of logical connectives, previously thought to be elementary, is one of the important contributions of matrix logic to the understanding of the mechanism of consciousness. It is necessary to distinguish three different types of factorization:

(a) matrix product factorization

$$\mathbf{L} \Rightarrow \mathbf{A \cdot B}$$

(b) outer product factorization

$$\mathbf{L} \Rightarrow |x\rangle\langle y|$$

(c) dimension-changing factorization

$$(2\times2) \Rightarrow (2\times n)(n\times2).$$

Matrix logical connectives interact, generally in a noncommuting fashion, for example:

$$\langle p|\mathbf{AND}\bullet\mathbf{OR}|q\rangle = p \quad \text{but} \quad \langle p|\mathbf{OR}\bullet\mathbf{AND}|q\rangle = q.$$

The interactions of connective operators can take various complex forms, giving rise to expressions that are not cognizable prior to the completion of computation, for example:

$$\langle p|\mathbf{AND}\bullet\mathbf{TIME} - \mathbf{TIME}\bullet\mathbf{AND}|q\rangle = \langle p|\mathbf{XOR}|q\rangle,$$

or the fundamental theorem **'IF to the power of IF is IF'**:

$$\mathbf{IF}^{\mathbf{IF}} = \mathbf{IF}.$$

Matrix logic not only gives rise to many new logical connective operators, it also introduces new fundamental logical operations. With the aid of these operations the familiar **AND** and **OR**, which are employed to evaluate a logical conjunction, the product of probabilities $\langle p|\mathbf{AND}|q\rangle = pq$, and disjunction, the sum $\langle p|\mathbf{OR}|q\rangle = p+q-pq$ of probabilities, can be put to new and unfamiliar use to perform the comparison:

$$\langle p|\,[\mathbf{AND}, \mathbf{OR}]\,|q\rangle = p - q,$$

which in a sense introduces the rule for the subtraction of probabilities.

In matrix logic, as in conventional logic, there are unary and binary operations, but a division into unary and binary operators ceases to have similar implications. An important generalization is provided by matrix logic. While the binary connectives of classical logic **cannot** act as unaries, the same logical operator acts as a binary operator *between* two logical states, or as a unary operator *on* a single logical state. Every matrix connective has unary and binary identities. This implies that the **4** unary operations of Boolean logic have grown to **81** operations, making the number of binary and unary operations in matrix logic equal, **162** operations in total. This is fact with major repercussions. The mathematical understanding of consciousness starts with Boolean logic which lives on a **4**-dimensional hypercube, comprising **16** vertices. Classical logic has captured only the most basic and in fact most trivial cognitive operations and relations, whose role for the intelligence code can be compared with the role of the simple Morgan laws of heredity for the genetic code, which is also characterised by enormous complexity. Existing logic only takes a first small step into math. Many limitations and shortcomings of classical logic stem from the incorrect idea that an abstract logical thought can be described with the standard set of operations:

$$\{\mathbf{NOT}, \mathbf{AND}, \mathbf{OR}, \mathbf{IF}, \mathbf{IS}\}.$$

In fact the Boolean hypercube must stretched into higher matrix dimensions where the untapped universe of logical laws and relations exists which must be studied if we are to make inroads into the complex subject of consciousness. We have a long way to go. Matrix logic shows that the operations of the cognitive brain involves at the very least **81+81** distinct logical operations. Such a significant enlargement of intellectual power reveals previously unknown aspects of logic and sheds new light on the inner workings of the intelligent brain. Technically speaking, matrix logic resides on the lattice comprising **81** different vertices. Making use of various mathematical tools, ones which exist already in logic and ones we have developed ourselves, we 'travelled' extensively through the universe of this complex lattice, gaining insight into its fundamental underlying code.
Matrix logic reveals important new properties of logic, undetectable in conventional formalisms. The possibility of direct interactions between connective operators greatly enhances the computational power of logic, leading to important applications in computer science and intelligence code theory. It propels logic to a higher level of abstraction, at which we operate not over single elements but over compounds, the superimposed concepts, which often refuse definition in singular terms. The possible products of logical operators is constrained by the set of forbidden interactions which prevent a product from escaping from the matrix logical lattice. There are **49** forbidden interactions in the hypercube subset of nonnegative matrix logic and **1982** on the complete matrix lattice.

Conformable matrices

The question of forbidden interaction is important. In logic written on a blackboard everything is clear-cut and well-defined. In reality one may spend days and even years searching for a simple conjunction or contraposition. Distracted by insignificant details or misguided by irrelevancies, we often may need to work very hard to obtain a valid logical operation. Thought is easy to verify but hard to find, in particular if forbidden interactions block the way. A forbidden interaction is a 'logical' constraint, but there is another 'physical' constraint, that of the conformability of matrices. Matrix logical strings are computable only on the condition of strict coherence: the matrix dimensions of every pair of interacting elements must be compatible. A factorized logical string is the product of matrices which, to be multiplied, must obey the rules of matrix multiplication. For matrix multiplication to be possible, matrices must be conformable: the number of columns in one must equal the number of rows in the other. It is not difficult to write down a matrix string which would not comply with this essential condition. In fact any nontrivial thought, translated into a matrix string, behaves 'badly'. In order to make strings computable several effective methods have been developed by the author which explore the dual scalar-operator nature of **AND**.

LOGIC WITHOUT CONNECTIVES

Matrix logic also challenges another conventional idea, the division of logical quantities into operators and operands, which is a cornerstone of traditional logical theory and computer science. A strict division on the operators and the operands is a poor match for the intelligent behavior, in which an operand may flexibly assume the role of an operator and an operator the role of an operand [Ref 73]. The question which naturally arises is whether it possible to construct logical machinery that is capable of accommodating such phenomena. Can logical theory can be devised in a form which will enable a natural exchange of logical operators and operands? Apart from gaining new insights into the nature of logic itself, solving this problem can open up new avenues for attacking the problems of artificial intelligence. The drawbacks of the AI project are to a large extent due to the fact that the programming languages in use today either ignore or are unable to accommodate the operator-operand unification which is a critical element of genuine intelligence, but is not accounted for in classical logic.

The tensorization of logic makes it possible to design a logical theory without logical connectives. As a consequence one is able to eliminate logical connectives from the formalism altogether. Matrix logical operators act in the two adjoint spaces of logical states, and given the linear independent matrix basis:

$$|1><1| = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix},\ |1><0| = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix},\ |0><1| = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},\ |0><0| = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},$$

we can in direct algebraic fashion generate any logical matrix, for example:

$$\mathbf{AND} = |1><1|$$

$$\mathbf{NOT} = |1><0| + |0><1|$$

$$\mathbf{OR} = |1><0| + |0><1| + |1><1|$$

Classical logic distinguishes between operands and operators. Due to the possibility of obtaining the operators as the outer products of logical states in matrix logic, such a distinction is no longer necessary and a complete logical theory can be now derived from the logical states alone! The outer products of the true and false logical states

$$|1><1|, \quad |1><0|, \quad |0><1|, \quad |0><0|,$$

generate the matrix basis in which any logical operator can be expanded. It must be emphasized that this basis is not the only possible one. Another important linear independent matrix logical base is formed as the sum and difference of the outer products:

$$|0><0| + |1><1| = \mathbf{YES}$$

$$|1><0| + |0><1| = \mathbf{NOT}$$

$$|0><0| - |1><1| = \mathbf{SPIN}$$

$$|1><0| - |0><1| = \mathbf{TIME}$$

This analysis shows that a logic calculus can be designed without a division of the operational elements into connective operators and logical variables. Because an operator can be obtained as the outer product of logical states, in matrix logic one does not really need operators anymore, but only a two-dimensional logical phase space. In this sense, **matrix logic is connective-free logical calculus**, in which the division into operands and operators ceases to be necessary or relevant, since the complete theory can be derived from the logical state alone. All one requires to perform a logical computation is a system of interacting operands. All that is needed to generate logic in all its complexity is just a pair of two adjoint logical spaces. This result has profound consequences for the design of logic and for an understanding of the nature of cognitive operations. No less important are the implications for computer science. Whereas in conventional computing theory it is common to divide computing elements into operators and operands, the newly defined generators of logic pave the way to a computing theory in which such a division is irrelevant.

In reverse, provided that the logical states are in the discrete Boolean limit, it is possible to eliminate the logical states, insulating operators in different atomic brackets. In this manner one obtains a pure operator reduction, for example:

$$\langle x|\, L_1\, |1\rangle\langle 1|\, L_2|y\rangle = \langle x|\, L_1 \bullet \mathbf{AND} \bullet L_2|y\rangle$$

where the pair of interacting logical states has given rise to an operator. The products

$$L_1\, |1\rangle\langle 1| \quad \text{and} \quad |1\rangle\langle 1 L_2$$

are computable logical operators, as guarantied by the interaction tables, which prevent the product from going astray outside the logical set. An outer product of Boolean states is one of the elements of the matrix basis

$$\{\mathbf{AND}, \mathbf{ON}, \mathbf{OFF}, \mathbf{NOR}\},$$

and any product of these operators with the matrix logical operators is necessarily again a logical operator. This is seen from the following tables.

Left multiplication:

•	I	N	∧	∨	→	←	↓	↑	\	/	⊥	⊤	↛	↚	1
∧	∧	\	∧	⊥	∧	⊥	\	0	\	0	⊥	0	∧	\	⊥
\	\	∧	0	∧	⊥	\	⊥	\	0	∧	0	⊥	∧	\	⊥
/	/	↑	/	⊤	/	⊤	↑	0	↑	0	⊤	0	/	↑	⊤
↑	↑	/	0	/	⊤	↑	⊤	↑	0	/	0	⊤	/	↑	⊤

Right multiplication:

•	∧	\	/	↑
I	∧	\	/	↑
N	/	↑	∧	\
∧	∧	\	0	0
∨	⊣	⊢	∧	\
→	⊣	⊢	/	↑
←	∧	\	⊣	⊢
↓	/	↑	⊣	⊢
↑	0	0	/	↑
\	0	0	∧	\
/	/	↑	0	0
⊥	∧	\	∧	\
T	/	↑	/	↑
⊣	⊣	⊢	0	0
⊢	0	0	⊣	⊢
1	⊣	⊢	⊣	⊢

The outer product elements of the matrix logical base, 'mediating' the interaction between the operators, fulfil an important constraining role, preventing the appearance of the forbidden products.

QUANTUM STRINGS

Matrix logic computes in various ways, all important for logical theory and applications, but with different limitations. In well-formed formulas of conventional logic variables and connectives alternate, for otherwise they are noncomputable. In matrix logic this **alternation rule** cancels out: one connective operator may appear immediately after another. However, for the operator string

$$\langle x| L_1 \cdot L_2 \cdot L_3 \cdot \ldots \cdot L_n|y\rangle$$

to be logically computable certain conditions must be satisfied. It is necessary to put a string into a canonical form, which will exclude the forbidden interactions.

A powerful wisdom is built into fundamental matrix strings which have the capacity of self-ordering. In a fundamental product any ordering of operators is a logical operator. Making use of the bivalent nature of **AND**, which in matrix logic is the **(2×2)** operator but also retains the Boolean property of scalar multiplication, we reduce any logical formula to a fundamental product. According to the matrix reduction theorem any matrix logical expression is an algebraic sum of factorized atomic products [Ref 88, 89, 91]. Generally, logical thought is not factorable but an entanglement, hence a knot. An atomic product $\langle x|L|y\rangle$ is a basic device, an elementary building block, from which any compound formula in matrix logic is built up in uniform fashion. Making use of the conservation rule for the charged sets, one determines that in a fundamental product, comprising the array of $n = k+r+m$ factorized terms, evaluating **k** times to -1, **r** times to 0 and **m** times to 1, necessarily

$$m - k = 0 \text{ or } 1.$$

One of the tacit assumptions in logic is that a definite statement results from the interaction of definite elements. This premiss is shown to be wrong and does not withstand the scrutiny of deeper analysis. With the development of matrix theory, the deterministic foundation of logic has been profoundly shaken. The elements of consciousness can be indeterminate but in an entanglement which is definite. **We may know the exact state of the whole system but not the states of its parts.** A definite thought can arise from indefinite elements, although it must not be overlooked that every fundamental reduction comprising indefinite elements has a parallel definite reduction. Generally, a fundamental product is not factorable but an entanglement of matrix strings:

$$\sum^{n}(\langle x_1| L_1 |x_2\rangle\langle x_3| L_2|x_4\rangle\langle x_5| L_3 |x_6\rangle \ldots)_n$$

Consequently a complex logical thought cannot be factorized into a product of the basic devices of matrix logic. The prime reason for entanglement is that nontrivial logical inferences typically cannot do without the complement operator. The complement operator binds together atomic products into an

entanglement which cannot be further factorized. For a single atomic bracket one easily eliminates the complement operator: the complement of an atomic product is the atomic product of the complement:

$$\overline{\langle x|L|y\rangle} = 1 - \langle x|L|y\rangle = \langle x|1|y\rangle - \langle x|L|y\rangle = \langle x|1 - L|y\rangle = \langle x|\bar{L}|y\rangle,$$

or equivalently,

$$\overline{\mathrm{Tr}\,\Omega L} = \mathrm{Tr}\,\Omega\bar{L}.$$

In particular

$$\overline{\mathrm{Tr}\,\Omega} = \mathrm{Tr}\,\Omega N.$$

For example, the complementation rule of matrix logic [Ref 88]

$$\overline{\langle x|y\rangle} = \langle x|\bar{y}\rangle = \langle \bar{x}|y\rangle$$

is in fact the consequence of the general rule, applied to identity:

$$\overline{\langle x|I|y\rangle} = \langle x|\bar{I}|y\rangle = \langle x|N|y\rangle = \langle x|\bar{y}\rangle = \langle \bar{x}|y\rangle.$$

There is an exceptional rule for the operators of the matrix logical base where the complement can be shifted over from an operator to the logical density matrix:

$$\mathrm{Tr}\,\Omega\bar{L} = \mathrm{Tr}\,\bar{\Omega}L,$$
$$L \in \{\mathbf{AND},\ \mathbf{AND}\bullet\mathbf{NOT},\ \mathbf{NOT}\bullet\mathbf{AND},\ \mathbf{NOT}\bullet\mathbf{AND}\bullet\mathbf{NOT}\}.$$

The complement can be cancelled for the difference of operators, inducing a swap of operators:

$$\overline{L_1} - \overline{L_2} = L_2 - L_1$$

or simply disappear in the commutator of logical operators:

$$[\overline{L_1},\ \overline{L_2}] = [L_1,\ L_2]$$

Unfortunately there is no such simple and elegant rule to purge a complex matrix formula of complements, and entanglement is commonplace except for little trivial thoughts. Generally, a logical expression is an entanglement which is definite but whose constituents may not be in definite states. It is important to emphasize that while an entanglement typically cannot be disentangled, any factorized string can be entangled by the following reversible procedure, making use of the double complement:

$$\langle x|L_1|y\rangle\langle x|L_2|y\rangle$$
$$= \langle x|L_1|y\rangle\overline{\langle x|\bar{L}_2|y\rangle} = \langle x|L_1|y\rangle - \langle x|L_1|y\rangle\langle x|\bar{L}_2|y\rangle.$$

For example,

$$\langle x|\mathbf{AND}|y\rangle\langle x|\mathbf{TRUE}|y\rangle$$
$$= \langle x|\mathbf{AND}|y\rangle - \langle x|\mathbf{AND}|y\rangle\langle x|\mathbf{FALSE}|y\rangle.$$

With the aid of outer product decomposition of logical operators a matrix string is reduced to a Kronecker string, comprising the Boolean states only [Ref 94]:

$$\langle x_1|x_2\rangle\langle x_3|x_4\rangle\langle x_5|x_6\rangle \ \ldots = \delta_{x_1x_2}\delta_{x_3x_4}\delta_{x_5x_6} \ \ldots$$

or to a **quantum string** which also includes the superposed states, the consequence of the action of operators on logical states,

$$< x_1 | S_+ >< x_3 | S_- >< x_5 | S_0 > \ldots$$

Even though the initial conditions and the final readout state are classical, something extraordinary and nontrivial happens during the intermediary steps of evaluation. The classical states evolve into quantized superposed states. Because this change appears to be concealed by the overall Boolean transformation, prior to the development of matrix logic we had no means to detect this hidden effect, nor did we ever suspect that classical logic may actually arise from the undecided quantum level. But very early on, a simple test explicitly revealed the emergence of such states, like for example, the empty state **Neither True Nor False** and the superposed state **True and False**, respectively:

$$< 1 | \mathbf{AND} | 0 > = (0\ 1)\begin{pmatrix} 0 \\ 0 \end{pmatrix} = 0, \qquad < 0 | \mathbf{OR} | 1 > = (1\ 0)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1.$$

Although we deliberately began develop the formalism of matrix logic with no new premisses beyond those we accept in Boolean logic, the power of matrix theory at once exposed new and hitherto hidden aspects of logic. Prior to matrix logic we all were convinced that classical thinking does not and should not involve quantized coherent superposition, just as in thermodynamics, before quantum statistics were discovered, we explained temperature simply by classical Brownian motion. The classical world emerges from the quantum world. But in matrix logic things are more complicated than they seem to be. The matrix string is convoluted to a definite Boolean answer, but matrix strings themselves propagate in a form that neither classical nor quantum but counterintuitively both. If one has no means of knowing which particular reduction, quantum or classical, a matrix string chooses (we indeed think "unconsciously"), one is in a sort of quantum-classical uncertainty. The deterministic and nondeterministic exist side by side in a single system. It is because this is precisely what happens in the logical brain that the method of matrix logic is so critical for the theory of consciousness.

In classical logic, logical values are confined to the unit interval, effectively prohibiting overlaps which would lead to contradiction. In fuzzy logic, logical values can float within the unit interval, however never falling into a superposition of true and false. The scalar format of logic conforms to macrophysical requirements. This situation has been radically altered in matrix operator logic, which builds a bridge between logical theory and quantum physics, just as Boolean logic does between logic and classical physics.

Given a reduction of the fundamental product to the algebraic product of the Kronecker or **quantum** strings, the effective computational procedures for solving strings can be formulated. A Kronecker string is naturally translated into a normally ordered string of the annihilation and creation operators:

$$\delta_{x_1 x_2} \delta_{x_3 x_4} \delta_{x_5 x_6} \ldots = < x_1 | a a^* a \ldots a^* | x_n >$$

The annihilation and creation operators and their products mediate the interaction of logical operators in matrix strings, converting a string into a

pure operator mode which involves no logically noncomputable interactions. Without loss of generality and with the aid of field algebra:

$$a^{*2} = a^2 = 0,$$
$$\{a, a^*\} = I,$$
$$aa^*a = a,$$
$$a^*aa^* = a^*$$

the string is reduced to one of the five following fundamental strings

$$\delta_{x_1x_2}\delta_{x_3x_4}\delta_{x_5x_6}\cdots = \begin{cases} <a^*> \\ <a> \\ <aa^*> \\ <a^*a> \\ <0> \end{cases}$$

In reverse, the fundamental strings are uniformly expandable as follows:

$$<a^*> = <\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet...>$$

$$<a> = </\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet...>$$

$$<aa^*> = </\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet...>$$

$$<a^*a> = <\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet\backslash\bullet/\bullet...>$$

It will be clear that any nonzero string alternates, and any alternating string is nonzero which prevents a string's collapse to the empty operator. The values of a fundamental product correspond to the values of the antisymmetric tensor

$$\varepsilon_{ijk} = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

When one is interested in the invariant transformations which leave the nonzero string unchanged, one must order the operators in pairs since the removal of a single operator in an alternating string creates a contact between identical operators which are nilpotent, and hence reduces the string to a vacuum. The operators can be recombined with a neighbouring pair, provided that the pair is first unitarily rotated. This entails the orthocommutation:

$$[A, B) = A\bullet B - \theta B\bullet A = 0$$

Proof

$$\mathbf{OFF\bullet AND = OFF\bullet N^2\bullet AND = (OFF\bullet N)\bullet(N\bullet AND) = NOR\bullet OFF.}$$

$$\mathbf{ON\bullet NOR = ON\bullet N^2\bullet NOR = (ON\bullet N)\bullet(N\bullet NOR) = AND\bullet ON.}$$

Naturally, the same result can be proven by exploring the associativity of the matrix product:

$$\mathbf{OFF \cdot AND = OFF \cdot (ON \cdot OFF) = (OFF \cdot ON) \cdot OFF = NOR \cdot OFF,}$$
$$\mathbf{ON \cdot NOR = ON \cdot (OFF \cdot ON) = (ON \cdot OFF) \cdot ON = AND \cdot ON.}$$

Finally, the pairs leave the creation and annihilation operators intact:

$$\mathbf{OFF \cdot AND = NOR \cdot OFF = NOR \cdot OFF \cdot AND = OFF}$$
$$\mathbf{ON \cdot NOR = AND \cdot ON = AND \cdot ON \cdot NOR = ON}$$

and the orthocommutator of the pairs with the creation and annihilation operators yield the time operators:

$$\mathbf{[\ AND, ON) = AND \cdot ON - OFF \cdot AND = \blacktriangledown}$$
$$\mathbf{[\ NOR, OFF) = NOR \cdot OFF - ON \cdot NOR = \blacktriangle}$$

A **quantum** string, containing coherent superpositions, is collected into a fundamental energy-momentum string

$$\langle x_1|\mathbf{1}|x_2\rangle\langle x_3|\mathbf{H}|x_4\rangle\langle x_5|\mathbf{M}|x_6\rangle \ldots$$

where **1**, **H** and **M** are the outer products

$$\mathbf{M} = |s_-\rangle\langle s_+|, \qquad \mathbf{1} = |s_+\rangle\langle s_+|, \qquad \mathbf{H} = |s_-\rangle\langle s_-|$$

The energy-momentum reduction evaluates the logical function properly but computes without reference to logical objects.

Working with **quantum** and Kronecker strings, we have seen how to perform the exchange of definite and undetermined logical states, preserving the overall physics, which is indifferent to the local value of the states. We can advance the argument in terms of logical eigenstates. The brain intelligently takes eigenstates to eigenstates. But it also can prepare coherent superpositions of eigenstates which allow us to perform interesting cognitive computations. A definite truth can be established through interaction of the states, themselves lacking a definite truth status.

QUANTUM BASIS SETS

In Boolean logic $\{\mathbf{N}, \wedge, \vee\}$ is the major basis set in which any logical formula can be extended. The same holds true in matrix logic but involves **matrix multiplication**, with the notion of a basis set being revised. To observe the difference, compare the Boolean and matrix logical reductions of the implication function:

$$(x \rightarrow y) = (\bar{x} \vee y) \qquad \text{and} \qquad \langle x| \rightarrow |y\rangle = \langle x|\,\mathbf{N \cdot \vee}\,|y\rangle.$$

The major basis set of connectives is of great importance for logic and for the functioning of a digital computer whose operations are reducible to the three basic Boolean operations. Matrix logic reveals new, previously unknown properties of the major set, some of which we will present next.

Matrix algebra of the major basis set {NOT, AND, OR}:

$$\mathbf{NOT = OR - AND}$$
$$\mathbf{[\ NOT, AND\ ] = [\ NOT, OR\ ] = [\ OR, AND\ ]}$$
$$\mathbf{\{NOT, AND\} = NOT}$$

$$\overline{\text{NOT} + \text{AND} + \text{OR}} = \begin{pmatrix} 1 & -1 \\ -1 & -1 \end{pmatrix}$$

$$[\text{AND} \bullet \text{OR},\ \text{OR} \bullet \text{AND}] = \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix}$$

$$\overline{\text{NOT} + \text{AND} + \text{OR}} = [\overline{\text{AND} \bullet \text{OR}},\ \overline{\text{OR} \bullet \text{AND}}]$$

$$e^{-\text{NOT}} = \text{H}, \qquad e^{-\text{NOT}_{\text{diag}}} = \text{H}_{\text{diag}}$$

$$[\ \text{NOT}, \text{AND} \bullet \text{OR}\ ] = \text{M}$$

$$\text{NOT}_{\text{diag}} = \sigma$$

$$\text{NOT} = \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\sigma\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \qquad \text{AND} = \frac{1}{4}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\text{H}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\{\text{NOT}, \text{M}\} = \{\text{NOT}, \sigma\} = \{\text{NOT}, \text{TIME}\}$$

While in conventional logic a functionally complete set of connectives is a set from which the complete set of logical functions is derived through logical **operations**, in matrix logic a basis set is a set of **operators** with the aid of which the same goal is achieved either by logical or by nonlogical transformations. The nonlogical transformations include transposition, commutation, square rooting, exponentiation, differentiation and other matrix and analytical operations. Given free access to nonlogical operations, the annihilation and creation operators, generated by square root polarization of the logical vacuum, form a functionally complete matrix set. The operators of creation and annihilation are the two fundamental operators that enable us to derive logical theory in all its complexity from first principles. Making use of transposition we can reduce the **quantum** basis set $\{a^*, a\}$ to the functionally complete basis sets $\{a^*\}$ or $\{a\}$, which are made from a single operator, just as the Scheiffer stroke is a basis set in Boolean logic.

Deriving the creation and annihilation operators by square-rooting of the logical vacuum and introducing an algebra for a^* and a, we get the logical basis:

$$a^* \bullet a = \text{AND}$$
$$a^* + a = \text{NOT}$$
$$a^* - a = \text{COMPARE}$$

This formally confirms the conversion principle: a quantum field theory with creation and annihilation can be converted into a logical calculus. The operators a^*, a and their derivative products a^*a and aa^* compose a functionally complete system which can be put 'inside' any logical operator. For example,

$$\text{YES} = a^*a + aa^*, \quad \text{OR} = a^* + a + a^*a, \quad \text{IF} = a^*a + aa^* + a^*.$$

The phase of the inserted operators is important. Changing phase in the third term of **IF**, we get the inverse:

$$(a^*a + aa^* + a^*)(a^*a + aa^* - a^*) = I$$

Whereas classical logic is an interpretation of Boolean algebra and its extensions, the general logical theory is an interpretation of the tensor algebra of annihilation and creation operators. Unraveling the intricate inner structure of logical connectives, matrix logic allows us to deal with the basis sets of logic in fundamental terms. We are able to derive logic from the field operators, which, in turn, can be obtained from the canonical logical operators:

$$\wedge\bar{\wedge} = \vee\bar{\vee} = a^*, \qquad \bar{\wedge}\wedge = \bar{\vee}\vee = a.$$

Commutation algebra determines the fundamental properties of the intelligence code. The quantum operation of commutation, which hitherto has had no meaning whatsoever in logic, gives rise to unusual basis sets. With the aid of the commutator we determine that the logical momentum **M** and **AND** form a functionally complete basis, due to the fact that

$$[\mathbf{M}, \mathbf{AND}] = \mathbf{NOT}.$$

Likewise, the basis {**AND**, **TIME**} is functionally complete, since again it is possible to obtain **NOT** as the commutator

$$[\mathbf{TIME}, \mathbf{AND}] = \mathbf{NOT}.$$

Although new for logic, the operation of commutation is of great importance, giving logic new deductive and inductive powers. In classical logic the set {**AND**, **OR**} is not a basis set; it is functionally incomplete because **NOT** is not available. One surprising fundamental result of matrix logic is the completeness of {**AND**, **OR**}, emerging due to the quantum operation of commutation

$$\mathbf{NOT} = [\mathbf{AND}, [\mathbf{AND}, \mathbf{OR}]]$$

and

$$\mathbf{TIME} = [\mathbf{AND}, \mathbf{OR}].$$

LOGICAL SUPERPOSITION PRINCIPLE

The vector space formalism offers a powerful framework for matrix logic. One of the key axioms of the linear vector space **V** is the closure: if $|x_1\rangle, |x_2\rangle \in \mathbf{V}$, then $(a|x_1\rangle + b|x_2\rangle) \in \mathbf{V}$ where a, b are scalars. Because the addition of logical vectors violates this key axiom of vector space, it soon becomes clear that cognitive logic is not a vector space. To satisfy both logical normalization and the axioms of vector space we have introduced the logical superposition principle [Ref 91]. This principle is basic to matrix logic in the same manner as the linear superposition principle is basic to quantum mechanics. In compact form: if $|p\rangle$ and $|r\rangle$ are the two logical states then their superposition is also a logical state:

$$|q\rangle = c|p\rangle + \bar{c}|r\rangle.$$

where the scalar factors are not arbitrary but must be logically normalized

$$\bar{c} + c = 1.$$

We consequently can interpret these factors as components of a logical state vector $|c>$, termed **denktor**, a combined German-English term for a 'thinking vector' [Ref 93]. The normality condition for $|c>$ is proven by direct computation. By definition all states in the superposition are normalized. Multiplying the states $|p>$ and $|r>$ by the factors c and $\bar{c}$, and summing we get

$$\begin{pmatrix} \bar{q} \\ q \end{pmatrix} = c\begin{pmatrix} \bar{p} \\ p \end{pmatrix} + \bar{c}\begin{pmatrix} \bar{r} \\ r \end{pmatrix} = \begin{pmatrix} c\,\bar{p} \\ c\,p \end{pmatrix} + \begin{pmatrix} \bar{c}\,\bar{r} \\ \bar{c}\,r \end{pmatrix} = \begin{pmatrix} c\,\bar{p} + \bar{c}\,\bar{r} \\ c\,p + \bar{c}\,r \end{pmatrix}$$

Since

$$\bar{q} + q = c\,\bar{p} + \bar{c}\,\bar{r} + c\,p + \bar{c}\,r = 1,$$

collecting terms and expelling the common factors, we deduce:

$$\bar{q} + q = (c\,\bar{p} + c\,p) + (\bar{c}\,\bar{r} + \bar{c}\,r) = c(\bar{p} + p) + \bar{c}(\bar{r} + r) = \bar{c} + c.$$

But $|q>$ is normalized, hence

$$c + \bar{c} = 1.$$

The simplicity of the logical superposition principle is misleading. In contrast to quantum superposition, logical superposition is recursively closed, and thus in a sense, self-referential: a superposition of two logical states is carried out by another logical state. To any triplet of logical states there corresponds a certain state, satisfying the logical normalization condition. By iteration a state can be defined as a superposition of an infinite number of logical states, quite similar to the definition of a quantum state in the Hilbert space of quantum mechanics. This introduces a 'holographic' element into the logical system: each state can be expanded in all states and all states can be brought to bear on a given one [Ref 93]. Generally a logical state can always be expanded in the set of the orthogonal basis true $|1>$ and false $|0>$ states:

$$|q> = q|1> + \bar{q}|0>$$

Taking the projections of a state on the basis states and summing the projections gives back the original state. In such an expansion the state $|q>$ depends only on the probabilities q and $\bar{q}$ of the states $|1>$ and $|0>$ and not on the states themselves. In logical superposition, however, we are after a more intricate, an 'intelligent' superposition in which we want a logical state to depend both on states and the probability of states. This superposition challenges one of the key assumptions of classical probability (which is strictly positive) and a cherished principle of quantum mechanics (where probabilities are calculated by squaring complex amplitudes). Quantum superposition obeys the quadratic rule of normalization

$$|c_1|^2 + |c_2|^2 = 1$$

This key postulate of quantum mechanics fails when applied to logic because it cannot neither explain nor accommodates the improbabilities which can take

negative and greater-than-unity values. We consequently have to employ a superposition rule which is different from that of quantum mechanics. In disobeying the quadratic rule of normalization, the logical thought process violates the quantum superposition principle, indicating that quantum mechanical laws are inapplicable for the adequate description of the brain.

Although in a logical superposition the states |q> ,|p>, |r>, |c> are all normed, the denktor |c> is not necessarily a probability vector in the conventional sense. It is normed but may contain a negative component, which automatically sets another component to a greater-than-unity value, as the only means to retain the normalization. For example, if q= 2/3, p=1/2 and r =1/3, then |c> is improbability:

$$|\tfrac{2}{3}> = c|\tfrac{1}{2}> + \bar{c}|\tfrac{1}{3}> = c\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix} + \bar{c}\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\end{pmatrix} = c\begin{pmatrix}\frac{1}{2}\\ \frac{1}{2}\end{pmatrix} + \begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\end{pmatrix} - c\begin{pmatrix}\frac{2}{3}\\ \frac{1}{3}\end{pmatrix},$$

$$\begin{pmatrix}\frac{1}{3}\\ \frac{2}{3}\end{pmatrix} = \begin{pmatrix}\frac{c}{2}+\frac{2}{3}-\frac{2c}{3}\\ \frac{c}{2}+\frac{1}{3}-\frac{c}{3}\end{pmatrix} = \begin{pmatrix}\frac{4-c}{6}\\ \frac{c+2}{6}\end{pmatrix},$$

$$\frac{4-c}{6} = \frac{1}{3}, \quad c = 2, \quad \bar{c} = 1-2 = -1,$$

$$|c> = \begin{pmatrix}-1\\ 2\end{pmatrix}, \qquad -1+2 = 1.$$

A superposition of probability states into a probability state is mediated by improbabilities. But if |p> and |r> are Boolean probabilities, so is |c>:

$$\begin{pmatrix}0\\ 1\end{pmatrix} = 0\begin{pmatrix}0\\ 1\end{pmatrix} + 1\begin{pmatrix}0\\ 1\end{pmatrix}, \qquad |c> = |0>,$$

$$\begin{pmatrix}0\\ 1\end{pmatrix} = 1\begin{pmatrix}0\\ 1\end{pmatrix} + 0\begin{pmatrix}1\\ 0\end{pmatrix}, \qquad |c> = |1>.$$

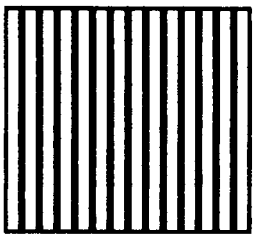
It may be clear, however, that the identical Boolean states can be logically superposed but never into a complementary state,

$$\forall c: \quad |1> \neq c|0> + \bar{c}\,|0>, \qquad |0> \neq c|1> + \bar{c}\,|1>.$$

Coherent superpositions

The logical superposition principle works adequately in normalized logic. But cognitive states exist that can be neither normally superposed nor classically localized. For example, perceptual rivalry prevents the brain from

obtaining a binocular fusion of the following two orthogonal figures, explicitly demonstrating oscillations of consciousness:



To obtain a coherent superposition from Boolean states or a Boolean state from superposed states, it may be necessary to set the denktor to a superposed state too:

$$\begin{pmatrix}1\\1\end{pmatrix} = 1\begin{pmatrix}0\\1\end{pmatrix} + 1\begin{pmatrix}1\\0\end{pmatrix}, \qquad \begin{pmatrix}1\\-1\end{pmatrix} = 1\begin{pmatrix}1\\0\end{pmatrix} - 1\begin{pmatrix}0\\1\end{pmatrix}$$

$$\begin{pmatrix}0\\1\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} - \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}, \qquad \begin{pmatrix}1\\0\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1\\1\end{pmatrix} + \frac{1}{2}\begin{pmatrix}1\\-1\end{pmatrix}.$$

One important application of the logical superposition principle is a solution of the Liar paradox. We take a superposition of undecided states $|1/2>$ (probability of true = probability of false) with the initial condition which may be $|1>$ or $|0>$, i.e. true or false, in order to compute the result. Then with the improbability vector (2 -1) such a superposition acts as negation:

$$|\bar{x}> = 2\begin{pmatrix}\frac{1}{2}\\\frac{1}{2}\end{pmatrix} + (-1)|x>$$

which is a complex and nontrivial way of converting a state $|x>$ in the right hand side of the equation into its negation in the left hand side:

$$|\bar{x}> \; = \mathbf{NOT}|x>$$

Although it appears like taking a complement,

$$\begin{pmatrix}x\\\bar{x}\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix} - \begin{pmatrix}\bar{x}\\x\end{pmatrix}$$

it is in fact a logical superposition of two definite classical states, realised with the aid of improbabilities. In manipulating the factors c, $\bar{c}$ outside the unit interval consciousness violates the laws of classical probability, which are otherwise universal. Logical superposition $|q> = c\,|p> + \bar{c}\,|r>$ is attainable by logical means alone if $q \in [p, r]$. Otherwise it contains elements which are not interpretable either in terms of classical probability or quantum probability amplitude.

The logical superposition principle has been profoundly misunderstood by some physicists who could not escape the dogma of the quadratic rule of quantum superposition. But in matrix logic the quadratic law cannot be

maintained without invalidating the well-established and empirically confirmed all-powerful results of logic and probability. Consequently, complex quantum-mechanical amplitudes are not suitable for the description of brain states, compelling us to consider a new form of quantum mechanics, involving only real variables. Whereas the validity of the laws of probability is hardly in question, one has less certainty that the quantum-mechanical picture, employing complex amplitudes, is complete and would withstand the scrutiny of future discoveries. If a convincing argument for rejecting the quantum superposition rule for the description of logical states is to be made, one must pinpoint where the failure occurs. Probability, as we know, concerns the prediction of the outcome of some measurement. There are clearly two different players in this situation: some estimator and some system undergoing a dynamical evolution. Now consider a dynamical system capable of computing the probability of the outcome of a measurement over itself, which under these circumstances becomes a fundamentally new sort of measurement, a self-measurement, while probability obtains new properties that are more characteristic of truth-value than simply of a chance measure. To make a long story short, think about a cognitive system which anticipates its own behaviour. In physics, when a prediction is made about the future states of a system, it has no bearing on the current state but when a cognitive system anticipates its future behaviour, the prediction itself becomes an integral part of and interferes with the current data. Cause and effect become inseparably looped and can no longer be treated in a canonical linear fashion. It is here that the logical superposition principle comes into play.

UNITARY INFERENCES

Inference rules are fundamental for logical theory. In classical logic the inferences suffer from a number of defects. The new deductive power of matrix logic allows us to uncover new aspects of inference which cannot be detected within the method of classical logic. Matrix theory extends the computational capabilities of logic. After a matrix logical string is written down, we can deal with it in a purely formal way, which has implications for the ways in which logical inference is achieved in logic.

While in classical logic the inferences refer back to logic itself, in matrix logic the inference is handled from first principles, taking the form of rigorous mathematical derivations. In addition to known classical rules, matrix theory enables us to introduce new **matrix inferences**. One of them is matrix transposition, which allows us to define a logical converse in a new way. The transpose matrix is obtained by swapping rows and columns against the principal diagonal, and if the matrix is symmetric, then a logical function and its converse are the same:

$$\tilde{A} = A.$$

The transpose of the product of two matrices is the product of the transposes in reversed order:

$$\tilde{AB} = \tilde{B}\tilde{A},$$

and if AB is a symmetric matrix, then

$$\tilde{AB} = \tilde{B}\tilde{A} = AB.$$

Since the transpose of a scalar is the same scalar, any number is a symmetric "matrix". These facts of matrix calculus can be naturally explored for logical inferences, which we divide into deductive and inductive.

It was determined that deductive inference is unitary, which puts logic in direct contact with quantum mechanics. However, quantum method is not enough. It cannot capture the thought processes in their totality because inductive thinking is nonunitary. In the Hilbert space of quantum mechanics the unitary transform leaves the norm of the wavefunction unchanged, and the scalar product is invariant under unitary transformation, giving rise to the family of conservation laws. Likewise, the deductive working of the logical mind relies on invariant unitary rotations. This is evident in the key logical rules, which make effective use of unitary rotation in combination with different matrix operations, like the product, the transposition, the complement and others.

THE IMPLICATION RULE

Exploring the fact that the square of the unitary logical transform θ is identity

$$\theta^2 = I,$$

we obtain:

$$\begin{aligned} <x|\rightarrow|y> &= <x|\theta^2\rightarrow|y> \\ &= <\bar{x}|\leftarrow|\bar{y}> = <\bar{x}|\leftarrow NN|\bar{y}> = <\bar{x}|\vee|y> \end{aligned}$$

Since **OR** is a symmetric matrix invariant under transposition,

$$\tilde{\vee} = \vee,$$

we derive

$$\begin{aligned} <\bar{x}|\rightarrow|y> &= <x|N\rightarrow|y> = <x|\tilde{N}\rightarrow|y> \\ &= <x|\tilde{\rightarrow}\tilde{N}|y> = <x|\leftarrow N|y> = <x|\leftarrow|\bar{y}> \end{aligned}$$

in which one recognizes the contraposition rule.

THE CONTRAPOSITION RULE

Contraposition is a fundamental inference rule which allows us to swap the premisses and conclusions:

$$\langle x|\rightarrow|y\rangle = \langle \bar{y}|\rightarrow|\bar{x}\rangle.$$

This backward reasoning is common in mathematics and in the routines of our daily thinking. Matrix logic determines it to be the unitary transformation:

$$\langle x|\rightarrow|y\rangle = \langle y|\leftarrow|x\rangle$$
$$= \langle y|\theta^2\leftarrow|x\rangle = \langle y|N^2\leftarrow N^2|x\rangle = \langle y|N\rightarrow N|x\rangle \succeq \langle \bar{y}|\rightarrow|\bar{x}\rangle$$

in which we explored the fact of invariance of a scalar under transposition:

$$\langle x|\tilde{\rightarrow}|y\rangle = \langle \tilde{x}|\tilde{\rightarrow}|\tilde{y}\rangle = \langle y|\leftarrow|x\rangle.$$

THE DE MORGAN RULE

The combined operation of symmetry, involving the unitary transform and the complement, automatically leads to the general duality rule:

$$\langle x|L|y\rangle = \langle \bar{x}|\theta L|\bar{y}\rangle = \overline{\langle \bar{x}|\theta L|\bar{y}\rangle} = \overline{\langle \bar{x}|L^*|\bar{y}\rangle}$$

equivalently:

$$\overline{\langle x|L|y\rangle} = \overline{\langle x|\theta^2 L|y\rangle} = \overline{\langle \bar{x}|\theta L|\bar{y}\rangle} = \langle \bar{x}|\overline{\theta L}|\bar{y}\rangle = \langle \bar{x}|L^*|\bar{y}\rangle$$

In particular for the pair of canonical logical operators **AND** and **OR** we derive:

$$\overline{\langle p|\wedge|q\rangle} = \overline{\langle p|\theta^2\wedge|q\rangle} = \overline{\langle \bar{p}|\uparrow|\bar{q}\rangle} = \langle \bar{p}|\bar{\uparrow}|\bar{q}\rangle = \langle \bar{p}|\vee|\bar{q}\rangle,$$

$$\overline{\langle p|\vee|q\rangle} = \overline{\langle p|\theta^2\vee|q\rangle} = \overline{\langle \bar{p}|\downarrow|\bar{q}\rangle} = \langle \bar{p}|\bar{\downarrow}|\bar{q}\rangle = \langle \bar{p}|\wedge|\bar{q}\rangle,$$

or equivalently:

$$\overline{\langle p|\wedge|q\rangle} = \langle p|\bar{\wedge}|q\rangle = \langle p|\downarrow|q\rangle = \langle p|\theta^2\downarrow|q\rangle = \langle \bar{p}|\vee|\bar{q}\rangle,$$

$$\overline{\langle p|\vee|q\rangle} = \langle p|\bar{\vee}|q\rangle = \langle p|\uparrow|q\rangle = \langle p|\theta^2\uparrow|q\rangle = \langle \bar{p}|\wedge|\bar{q}\rangle.$$

In the derivation of the duality relations via the outer product [Ref 88]:

$$\overline{\langle p|\wedge|q\rangle} = \overline{\langle p|1\rangle\langle 1|q\rangle} = \overline{\langle \bar{p}|0\rangle\langle 0|\bar{q}\rangle} = \langle \bar{p}|\vee|\bar{q}\rangle$$

$$\overline{\langle p|\vee|q\rangle} = \langle p|0\rangle\langle 0|q\rangle = \langle \bar{p}|1\rangle\langle 1|\bar{q}\rangle = \langle \bar{p}|\wedge|\bar{q}\rangle$$

the unitary transform is not directly visible but can be recognized:

$$\overline{\langle p|1\rangle\langle 1|q\rangle} = \overline{\langle p|N^2|1\rangle\langle 1|N^2|q\rangle} = \overline{\langle \bar{p}|0\rangle\langle 0|\bar{q}\rangle}$$

$$\langle p|0\rangle\langle 0|q\rangle = \langle p|N^2|0\rangle\langle 0|N^2|q\rangle = \langle \bar{p}|1\rangle\langle 1|\bar{q}\rangle$$

Matrix logic also makes us aware that the well-known swap of the universal and existential quantifiers in predicate logic and the connectives ***possibly*** $\Diamond$ and ***necessarily*** $\Box$ in modal logic is in fact the unitary transformation. In predicate logic the universal $\forall$ and the existential $\exists$ quantifiers enlarge the expressive power of logic, but actually are not both necessary because

$$\forall = \theta\exists = \text{NOT}\,\exists\,\text{NOT}$$

$$\exists = \theta\forall = \text{NOT}\,\forall\,\text{NOT}$$

and since necessarily $\bar{x}$ makes precisely the same assertion as impossible x, whereas not necessarily x is equivalent to possibly $\bar{x}$, the modal connectives are **unitarily** related [Ref 88]:

$$\Box = \theta\Diamond = \text{NOT}\,\Diamond\text{NOT}$$

$$\Diamond = \theta\Box = \text{NOT}\,\Box\,\text{NOT}$$

Just like the existential and universal quantifiers, the modal connectives are interchangeable and the expressive power of modal logic is in no way impaired by the omission of either of them. It comes as a surprise to a flat one-dimensional Boolean thinker that the exchange of the universal and existential quantifiers and the modal connective are nothing but unitary rotations.

The unitary transformation establishes five fundamental subsets of logical operators, each realizing unitary rotation with a particular operation of symmetry, carried on to the commutator with the energy operator. In the first subset, the unitary transform is the identity operation that leaves the operator intact; in the second, it is converse transposition; in the third, transposition; in the fourth, a binary complement; and in the fifth, a negation:

(1) bisymmetric operators $\{\mathbf{I}, \mathbf{N}, \mathbf{1}, \mathbf{0}\}$

$$\theta\mathbf{L} = \mathbf{L}, \quad \Rightarrow \quad [\mathbf{L}, \mathbf{H}] = [\mathbf{H}, \mathbf{L}]$$

(2) symmetric operators $\{\wedge, \vee, \downarrow, \uparrow\}$

$$\theta\mathbf{L} = \bar{\mathbf{L}}, \quad \Rightarrow \quad [\mathbf{L}, \mathbf{H}] = [\mathbf{H}, \bar{\mathbf{L}}]$$

(3) asymmetric operators $\{\rightarrow, \leftarrow, \backslash, /\}$

$$\theta\mathbf{L} = \tilde{\mathbf{L}}, \quad \Rightarrow \quad [\mathbf{L}, \mathbf{H}] = [\mathbf{H}, \tilde{\mathbf{L}}]$$

(4) biasymmetric operators $\{\perp, \top, \dashv, /-\}$

$$\theta\mathbf{L} = \bar{\bar{\mathbf{L}}}, \quad \Rightarrow \quad [\mathbf{L}, \mathbf{H}] = [\mathbf{H}, \bar{\bar{\mathbf{L}}}]$$

(5) (bi)(anti)(per)symmetric $\{\mathbf{M}, \blacktriangle, \blacktriangledown, \sigma\}$

$$\theta\mathbf{L} = -\mathbf{L}, \quad \Rightarrow \quad [\mathbf{L}, \mathbf{H}] = [\mathbf{H}, -\mathbf{L}]$$

The spin σ is the only antipersymmetric operator in matrix logic.

THE UNIVERSAL INFERENCE RULES

The logic which operates with four truth values -**True, False, True and False, Neither True Nor False** - might have been developed earlier, if logicians had paid greater attention to the reasons for the existence of four fundamental inferences. Human intelligence makes use of the methods of affirming, denying, affirming **and** denying, and **neither** affirming **nor** denying. These are the well-known **Modus Ponens, Modus Tollens, Modus Tolendo Ponens**; and we may call the last method **Modus NAND.** These covariant inferences is such an organic part of our thinking that we hardly notice their basic structure which permits us, respectively, to infer

(1) from x **implies** y and x that y,

(2) from x **if** y and $\bar{x}$ that $\bar{y}$,

(3) from x **or** y and $\bar{x}$ that y,

(4) from **not both** x and y and x that $\bar{y}$.

The four fundamental inferences are particular realizations of the canonical tautology

$$\overline{\langle x|L|y\rangle\langle x|\bar{L}|y\rangle} = \overline{\langle x|L^{\blacklozenge}\bar{L}|y\rangle} = \overline{\langle x|0|y\rangle} \equiv 1,$$

taking the form of the following entanglement codons:

$$\overline{\langle x|{\to}|y\rangle\langle x|a^{*}|y\rangle} \equiv 1, \qquad \overline{\langle x|{\leftarrow}|y\rangle\langle x|a|y\rangle} \equiv 1,$$

$$\overline{\langle x|\vee|y\rangle\langle x|\uparrow|y\rangle} \equiv 1, \qquad \overline{\langle x|\downarrow|y\rangle\langle x|\wedge|y\rangle} \equiv 1,$$

from which one may see that **Modus Ponens** and the unitary transformation are sufficient for deductive intelligence; other inference rules follow. **Modus Tollens** is deduced from **Modus Ponens** by complete unitary transformation, **Modus Tolendo Ponens** and **Modus NAND** by partial unitary transformations:

$$\overline{\langle x|\theta{\to}|y\rangle\langle x|\theta a^{*}|y\rangle} = \overline{\langle x|{\leftarrow}|y\rangle\langle x|a|y\rangle},$$

$$\overline{\langle x|N{\to}|y\rangle\langle x|Na^{*}|y\rangle} = \overline{\langle x|\vee|y\rangle\langle x|\uparrow|y\rangle},$$

$$\overline{\langle x|{\to}N|y\rangle\langle x|a^{*}N|y\rangle} = \overline{\langle x|\downarrow|y\rangle\langle x|\wedge|y\rangle}.$$

Finally, the syllogism inference rule, which is central to matrix theory,

$$((x \to y) \wedge (y \to z)) \to (x \to z)$$

is the fundamental three-atomic bracket tautology

$$\overline{\langle x|{\to}|y\rangle\langle y|{\to}|z\rangle\langle x|a^{*}|z\rangle},$$

which one also reduces to the canonical constant

$$\overline{\langle x|0|z\rangle} \equiv 1,$$

however, not by operator transmultiplication, but by scalar nullification:

$$\overline{<x|a^*|z>(1-<y|0>-<y|1>)} = \overline{<x|a^*|z>0} = \overline{<x|0|z>} \equiv 1.$$

INDUCTION. VACUUM TERMS

Prior to the development of matrix logic, the unitary character of deductive thinking went unnoticed and undetected. The unitarity which distinguishes deductive inference, however, breaks down in creative intelligence. Induction is not unitary. The key algorithm for inductive inference is connecting a matrix string to a vacuum [Ref 91]. We explain the algorithm with an example of the inductive proof of the tautology

$$(x \to y) \to x = x,$$

where at the third step the vacuum term $<x|0><1|x><y|0> = 0$, introducing a new variable inductively , is added:

$$|x> = |x>$$
$$<x|0> = <x|0>$$
$$<x|0> - <x|0><1|x><y|0> = <x|0>$$
$$<x|0>(1 - <x|1><0|y>) = <x|0>$$
$$\overline{<x|a^*|y><x|0>} = <x|0>$$
$$<x|\to|y>\overline{<x|1>} = <x|0>$$
$$\overline{<x|\to|y>\overline{<x|1>}} = \overline{<x|0>}$$
$$(x \to y) \to x = x$$

IF-THEN-ELSE

The ternary operator **IF-THEN-ELSE** is essential for logic, and recursive nesting of **IF-THEN-ELSE** is an important feature of cognitive dynamics. Although in (2×2) logic there is no single logical connective which can alone carry this ternary inference, it is possible to obtain matrix logical fundamental reduction. If we write the ternary inference as

$$\textbf{IF x THEN y ELSE z} = \begin{cases} y & \text{if } x = 1 \\ z & \text{if } x = 0 \end{cases}$$

then with the aid of the canonical matrix form

$$<x|\to|y><\bar{x}|\to|z>$$

every logical operation can be executed by the **IF-THEN-ELSE** together with the constants 1 and 0. For the standard set of connectives we have

$\bar{x} \quad \Rightarrow \quad$ if x then 0 else 1

$x \wedge y \quad \Rightarrow \quad$ if x then y else 0

$x \vee y \quad \Rightarrow \quad$ if x then 1 else y

$x \rightarrow y \quad \Rightarrow \quad$ if x then y else 1

$x = y \quad \Rightarrow \quad$ if x then y else (if y then 0 else 1)

Next we give proofs for some of the connectives:

$\bar{x}$:

$$<x|\rightarrow|0><\bar{x}|\rightarrow|1> = <x|0><x|\vee|1> = <x|0><x|1> = <x|0> = \overline{<x|1>}$$

$x \wedge y$:

$$<x|\rightarrow|y><\bar{x}|\rightarrow|0> = <x|\rightarrow|y><x|\vee|0> = <x|\rightarrow|y><x|1> = <x|1> - <x|1><0|y> = <x|1>(1-<0|y>) = <x|\wedge|y>$$

$x = y$:

$$<x|\rightarrow|y>\overline{<x|0>\overline{<y|\rightarrow|0><\bar{y}|\rightarrow|1>}} = <x|\rightarrow|y>\overline{<x|0><\overline{<y|0><\bar{y}|s_+>}} = <x|\rightarrow|y>\overline{<x|0>\overline{<y|0>}} = <x|\rightarrow|y>\overline{<x|0><y|1>}$$

$$= <x|\rightarrow|y><x|\leftarrow|y> = <x|\leftrightarrow|y>$$

LOGIC WITH DIVISION. SOLVING LOGICAL RATIOS

We study the world by dividing things into parts and joining them back together, gaining new knowledge in the process. We understand the whole through its parts. Just as a surgeon first cuts and then recomposes, before we can learn a whole, we have to take things apart. In the physical laboratory we have descended to the level of quarks and leptons, and already begun to look deeper. But in the 'mental laboratory' of the mind we continue to struggle in search for the logical primitives of consciousness. In the attempt to dissect thoughts into constituent parts, logic has come a long way. We have understood many regularities of the intellect, the algorithms of computation and formal deduction. We know how to break a thought into subthoughts and with the aid of logical connectives to 'synthesize' it back. However, in all known systems of logic there seems to be a fundamental limit as to how far the logical whole can be divided into constituent parts. Logical connectives are considered to be indivisible and elementary. This commonly held view began to crumble with the advent of matrix logic. One of the first key discoveries concerned the

fundamental operation of negation, which we had become used to treating as a total, indivisible and elementary. Matrix logic succeeded in factorizing the single **NOT** of classical logic into the **40** different operator products **NOT=A•B**, where A and B, termed subnegators, only jointly accomplish the complete negation [Ref 88]. This critical result motivated the question: whether it is possible to divide the elementary logical connectives in general, getting to their 'interiors', just as we try to study the interior of nucleons made of quarks. But what are the parts, for example, of **AND** or **OR**? Do they have parts? Suppose we have two arrays corresponding to two logical functions. Can one array be 'divided' by the other to yield some meaningful logical array? At first such questions seemed bizarre, in particular for traditional logicians. However, the problem is solvable in matrix logic, where logical connectives are transformed into genuine mathematical objects.

The standard model of matrix logic is a noncommutive matrix algebra in which every logical operation is characterized by a **(2×2)** matrix. What one has to do is to form quotients, process which is one of basic operation throughout mathematics, often causing considerable difficulties. For example, quotients of topological spaces are often poorly behaved. They can fail to be **Hausdorf** or can easily have the trivial topology of two open sets. The process of forming moduli spaces is a quotient process. An important way of dealing with matrix logical quotients is the notion of a groupoid G. An equivalence relation on a set X, $G \subset X \times X$, gives rise to a groupoid G whose elements are the pairs $(x, y) \in G$. The multiplication $(x, y)\cdot(z, t)$ exists if $y = z$, and the product then (x, t), which again is in G by transitivity.

Until the development of matrix logic the important question of quotients was irrelevant for the calculus of classical logic, in which the division operation is absent. The novelty of matrix logic resides in its introducing logic with division. We consequently consider the logical ratios

$$\frac{\mathrm{A}}{\mathrm{B}}$$

where A and B are logical connectives, A is called the logical nominator, B the logical denominator. The ratio exists on the two conditions: $\mathrm{B} \neq 0$, and nonsingular, $\det \mathrm{B} \neq 0$. We also must take into account that logical division is not unique, and that the left and right divisions exist, which we distinguish by a corresponding dot. The ratios may or may not respond to changeover of left and right:

$$\bullet\frac{\wedge}{\vee} = a, \qquad \frac{\wedge}{\vee}\bullet = a^*$$

$$\bullet\frac{\mathrm{M}}{\mathrm{N}} = -\frac{\mathrm{M}}{\mathrm{N}}\bullet \qquad \text{but} \qquad \bullet\frac{\mathrm{H}}{\mathrm{N}} = \frac{\mathrm{H}}{\mathrm{N}}\bullet$$

The orthogonal rotation of the denominator can orthogonally transform (complement) the whole ratio:

$$\bullet\frac{1}{\rightarrow} = \text{TRUE}, \qquad \bullet\frac{1}{\leftarrow} = \text{FALSE}$$

A multiplicity is a distinct feature of intelligent thought which can be derived in many different ways. A given logical nominator can be divided by different denominators but yield the same result, for example,

$$\frac{\text{TRUE}}{\text{OR}}\bullet = \frac{\text{TRUE}}{\text{IF}}\bullet = \text{AND}, \qquad \bullet\frac{\text{AND}}{\text{NOT}} = \bullet\frac{\text{AND}}{\text{OR}} = \text{OFF}.$$

In particular cases a logical ratio falls into a steady state when the logical nominator remain unaltered with the application of the denominator,

$$\bullet\frac{\text{AND}}{\text{IF}} = \text{AND}, \qquad \frac{\text{AND}}{\text{IMPLY}}\bullet = \text{AND}.$$

If we add **OR** to **AND** in the nominator, the same denominators will change the ratios to the dual ratios:

$$\bullet\frac{\wedge + \vee}{\leftarrow} = \vee, \qquad \frac{\wedge + \vee}{\rightarrow}\bullet = \vee$$

For numbers a ratio is equal to the inverse ratio in a trivial case when nominator equals denominator. Not so for logical ratios. It is possible that

$$\frac{A}{B} = \frac{B}{A} \quad \text{for} \quad A \neq B.$$

Consider

$$\bullet\frac{\vee}{\leftarrow} = \bullet\frac{\leftarrow}{\vee}, \qquad \frac{\vee}{\rightarrow}\bullet = \frac{\rightarrow}{\vee}\bullet.$$

What is interesting is that these ratios are also unchanged with simultaneous orthogonal rotation of the nominator and denominator:

$$\bullet\frac{\vee}{\leftarrow} = \bullet\frac{\downarrow}{\rightarrow}, \qquad \bullet\frac{\leftarrow}{\vee} = \bullet\frac{\rightarrow}{\downarrow}$$

$$\frac{\vee}{\rightarrow}\bullet = \frac{\downarrow}{\leftarrow}\bullet, \qquad \frac{\rightarrow}{\vee}\bullet = \frac{\leftarrow}{\downarrow}\bullet$$

Finally, for the ordinary number ratio

$$\frac{A}{B} = \frac{C}{D}$$

we derive the cross-product:

$$AD = BC$$

where the order of multipliers is immaterial. This rule, however, does not extend to the logical ratios, where one must take into account left and right logics. It follows that for the left and right division different rules apply:

$$\bullet\frac{A}{B} = \frac{C}{D}\bullet \qquad \times \qquad AD = BC$$

$$\frac{A}{B^{\bullet}} = {}^{\bullet}\frac{C}{D} \qquad \text{DA} = \text{CB}$$

Division-based matrix logic forces us to revise the traditional concept of logic. According to the conventional view logical connectives are elementary and cannot be divided into more primitive elements. There is no part of **AND** or **OR** or **NOT**. However, just as physicists split atoms, previously considered to be indivisible, matrix logic has shown that what we considered to be the 'atoms' of thoughts have parts which we can study in rigorous technical fashion. There is a complex hidden universe within each and every logical connective.

DIFFERENTIAL FORMAL SYSTEMS

The closure of the computational gaps between logic and mathematics is most visible in differential matrix logic, which opened up new ways of examining logical phenomena, adding an important dynamical dimension to logical theory [Ref 89]. Traditionally, formal systems make use of the

(1) constants 0, 1
(2) variables x, y, z, ...
(3) connectives $\mathbb{N}$, $\wedge$, $\vee$, $\rightarrow$,...
(4) punctuation brackets: (,)
(5) quantifiers, e.g. $\forall$ and $\exists$
(6) inference rules, e.g. Modes Ponens
(7) axioms, e.g. the Law of Excluded Middle

The purpose of a formal system is to construct from primitives elements the well-formed formulas, to evaluate the validity of these formulas and to compare different formulas in order to determine their equivalence or otherwise. Well-formed formulas (**wff**) are defined recursively:

(a) if x is **wff** then $\mathbb{N}$x is **wff**.

(b) if x and y are **wffs**, then $x \wedge y$ and $x \vee y$ are **wffs**.

A differential formal system differs from existing formal systems in that to the standard set of primitives and rules we add the differentiation and integration operators and rules. Logic is commonly viewed as a branch of discrete mathematics which is without any apparent links with analysis and differential calculus. Differential formal systems, first developed by the author, pave the way for the analytic treatment of logical problems based on differential methods in which logical inferences result from the differentiation and integration of logical functions. This introduces a dynamical modality into logical analysis, and a new large class of theoretical and applied problems can be formulated in the framework of differential formal systems. What is important is that differentiation and integration acquire the meaning of logical

operations, opening a world of unexplored possibilities, and paving the way for incorporating logic fully into the framework of the exact sciences.

Implication

We begin with the the implication operator which is central to matrix theory. Differentiating the implication function and setting the result to zero, we obtain a condition for an extremum which reflects important logical properties. The derivative with respect to the premiss delivers the negated consequent, and with respect to the consequent the premiss:

$$\frac{d}{dp}<p|\rightarrow|q> = \bar{q}, \qquad \frac{d}{dq}<p|\rightarrow|q> = p,$$

In line with the contraposition rule

$$(q \rightarrow p) = (\bar{p} \rightarrow \bar{q})$$

the differentiation of the antecedent extracts the adjoint consequent:

$$\frac{d<p|\rightarrow|q>}{dq} = p, \qquad \frac{d<p|\rightarrow|q>}{d\bar{p}} = \bar{q}.$$

The laws of implication are universal tautologies, holding both in Boolean and fuzzy logics:

$$<0|\rightarrow|q> \equiv 1, \qquad <p|\rightarrow|1> \equiv 1.$$

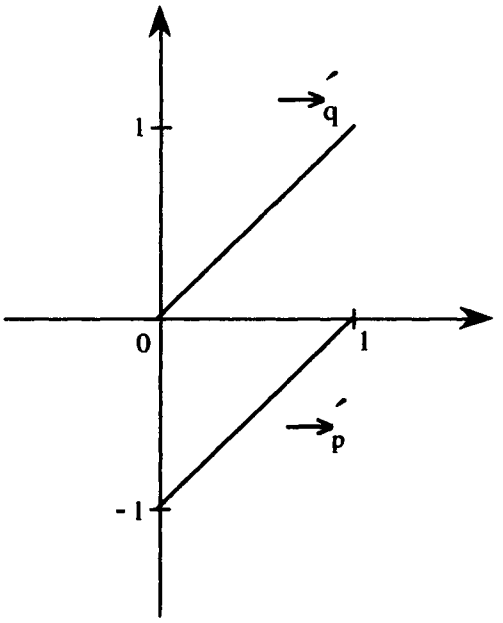
Matrix logic as a universal theory provides a uniform description. Solving the extrema, we in effect formulate differential conditions for these tautologies, **verum ex quolibet** and **ex falso quodlibet**:

$$\frac{d}{dp}<p|\rightarrow|q> = 0 \quad \Rightarrow \quad q=1$$

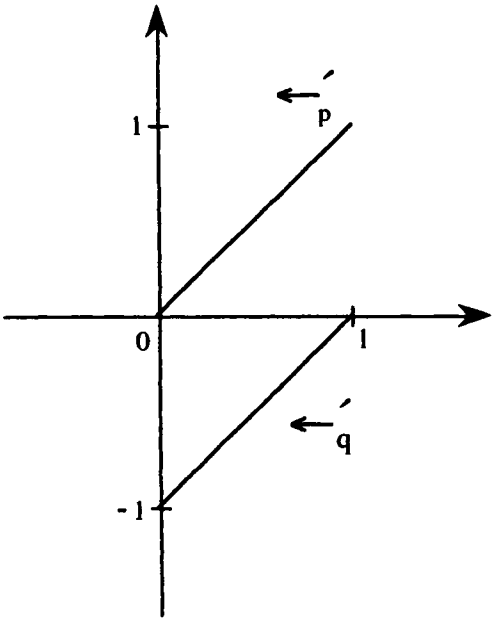
$$\frac{d}{dq}<p|\rightarrow|q> = 0 \quad \Rightarrow \quad p=0$$

It follows that an implication is at extremum with the consequent 1 or the premiss 0. This agrees with the logical rules: it is exactly at these logical points, the false premiss p = 0 or the true consequent q = 1, that implication is universally true, irrespective of the truth-status of the adjoint logical variable. Linking the logical concept of truth and the concept of extremum allows us to employ differential methods to solve the important problem of the optimization of logical functions.

Differential formal systems introduce into logic graphical analysis, which provides explicit visual support for solving logical problems. With the change of the variable of differentiation for the implication function we observe a parallel shift of the derivatives:



For the converse implication these relations are naturally reversed:



What is the logical meaning of differentiation? Differentiation can be put to different logical use. Suppose we want to negate a variable q, which is 'hidden' inside a logical function, for example <p|OR|q>, which by some

reason we cannot 'open up'. What we can do, however, is to differentiate $<p|\mathbf{OR}|q>$ and to determine the negated q in a quite unusual manner:

$$\frac{d}{dp}<p|\mathbf{OR}|q> = \bar{q}.$$

If we first differentiate and then convolute the scalar product, this example, as already discussed in the section devoted to the logical derivative, shows that the meaning of differentiation is the quantization effect, explicitly revealed in the action of the differentiation operator on the general logical state

$$-\frac{d}{dq}|q> = \begin{pmatrix} 1 \\ -1 \end{pmatrix} = |S_->$$

The average of the differentiation operator is the logical momentum, which gives a direct way to compute the momentum. There are also indirect ways. The derivatives of the **YES** and **NOT** functions are the relative truth-values, and hence also compute the signed logical momentum:

$$\frac{d}{dp}<p|\mathbf{YES}|q> = q - \bar{q}, \qquad \frac{d}{dp}<p|\mathbf{NOT}|q> = \bar{q} - q.$$

Both, direct and indirect computation result in the secret metric which links logic with the ideas of relativity theory.
While the logical momentum quantizes a logical state, integration is a reverse operation which undoes the effect of quantization.

Theorem

A logical state is or will converge to a fixed point of the integration operator.

Proof

(a) pure states $|1>$ and $|0>$:

$$\int_0^1 |1>dq = |1> \quad \text{and} \quad \int_0^1 |0>dq = |0>.$$

(b) superposed state $|s_\pm>$:

$$\int_0^1 |S_\pm>dq = \int_0^1 \begin{pmatrix} dq \\ \pm dq \end{pmatrix} = \begin{pmatrix} 1 \\ \pm 1 \end{pmatrix} = |S_\pm>.$$

(c) mixed state $|q>$:

$$\int_0^1 |q>dq = \int_0^1 \begin{pmatrix} \bar{q} \\ q \end{pmatrix} dq = \begin{pmatrix} 1 - \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} = |\tfrac{1}{2}>.$$

and

$$\int_0^1 \cdots \int_0^1 |q>dq \cdots dq = |\tfrac{1}{2}>.$$

□

The superposed and pure logical states are fixed points, steady under integration. A mixed normed state integrates to stable equilibrium or the undecided state |1/2> which is normed. Integration preserves the nature of a logical state, as opposed to differentiation, where for example a mixed state will be converted into a superposed state. To recover a logical state from a superposed state, one must take the fact into account that integration depends essentially on the appropriate integration limits. For a negative scalar the integration constant is 1, and for a positive one 0 [Ref 89]. This identifies the integration constants for basic logical computations: a logical system has to be reset to a pure state:

$$\int_0^q |s_>\, dq + |0> = |q>$$

Differential dualities

As was already stressed, the importance of logical derivatives resides in the fact that the differentiation and integration operators acquire the role of logical operators. This is most evident and significant in the differential duality laws. Because the differentiation of the complement changes the sign of a function, the DeMorgan dualities in differential logic take the form:

$$<p| \wedge |q>' = - <\bar{p}| \vee |\bar{q}>'$$

$$<p| \vee |q>' = - <\bar{p}| \wedge |\bar{q}>'$$

In order to restore the vanished complements it is necessary to cancel the unitary transformation of the operator, which then leads to the special differential duality relations where the differentiation operator acts under the complement:

$$<p|\wedge|q>' = \overline{<p|\vee|q>'}$$

$$<p|\vee|q>' = \overline{<p|\wedge|q>'}$$

These duality transformations hold exclusively for the **AND** and **OR** because these are the only duals with complementary derivatives. The special differential dualities can be put into self-referential format as follows:

$$<p|\wedge|q>' = \overline{\overline{<\bar{p}|\wedge|\bar{q}>'}}$$

$$<p|\vee|q>' = \overline{\overline{<\bar{p}|\vee|\bar{q}>'}}$$

For the canonical logical functions **AND** and **OR** the complement and differentiation operators naturally do not commute, but can applied in the

reversed order, provided the logical states and the atomic product are simultaneously negated.

Dynamics

Differential logic enables us to capture the intricate dynamics of the thought process. The operations of differentiation and integration, which previously had no logical meaning or applications whatsoever, become important new instruments of logical descriptions. We know that to design dynamical logic in a fundamental way we start from the vacuum whose square root polarization gives the logical momentum operator. But the action of the momentum operator is equivalent to the action of the differentiation operator:

$$M = -\frac{d}{dq}$$

giving the differential operator the status of a fundamental logical operation. Introducing this new logical operation, we gain simultaneously on several counts. First of all, logic explicitly and naturally becomes a dynamical theory, allowing for dynamical analysis of logical inferences. Secondly, the differentiation operator, acting on classical logical states, gives rise to the states of coherent superposition, the fundamental elements of the cognizing mechanism and quantum computation. The dynamical logic is a source of coherent superposition in the brain, which puts logical states into an entanglement.

VIRTUAL LOGIC

Logic means different things to different people. What is common to different classical systems of logic is that, from **YES** to **NOT** and from **ONE** to **NONE**, all logical operations are cognizable. In Boolean logic we have **4** unary and **16** binary, **20** in total, cognizable operations. Although children or intellectually retarded persons may not be able to cognize, for example, **NOR** or contraposition, we clearly have a direct access to any Boolean operation which is easy to explain and to grasp [Ref 101]. In contrast, matrix logic gives rise to numerous operations which are without direct logical meaning. There are **81** binary and **81** unary, **162** in total, operations. Beside this impressive gain new abstract operations become manifest. We have already discussed many such operations: differentiation, exponentiation, the tensor product, the trace, the quantum commutator, the power, the logical logarithm and so on. A fundamental feature of matrix logic, absent in other forms of logic, is the interaction of logical connective operators. This interaction takes various forms: the matrix product, the Lie commutator and anticommutator, and matrix powers. All these new noncognizable operations which do not lend

themselves to explicit linguistic cognitive description we join under the umbrella of 'subconscious logic'.

The operations of subconscious logic are not cognizable but have much greater powers in comparison to Boolean logic. For example, we can perceive **NOT** and **NOR**, but the subtraction **NOT - NOR** is not cognizable. In classical logic such an operation is meaningless, but in matrix logic it gives the inverse of **OR**, an operation also undefinable in Boolean logic:

$$\mathbf{OR(NOT - NOR) = YES.}$$

One has perhaps already realized where the problem lies. Although **NOT - NOR** can be written as a (2×2) matrix, it is in fact an algebraic difference of the two logical matrices, resulting in a mixed positive-negative matrix:

$$\vee^{-1} = \mathbf{NOT - NOR} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} - \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

Likewise, take **AND•OR = TRUE** or **AND•OR•AND = AND**. The results of these operations are cognizable, and so are their components, **AND** and **OR** separately, but not the interaction products. Whenever a logical operation involves the direct interaction of operators our cognitive mechanism fails. The operation is not analyzable into parts. And what we cannot analyse we cannot fully cognize. As in a good theater play, many levels of meaning exist in consciousness. Decoding them involves emotions, a pleasure for some, or displeasure for others. Love seems to be an unresolved coherent superposition of emotions with opposite meanings: those of both pleasure and pain. The question of emotions is outside the scope of this study, which focuses on the intellect only. We will study in greater details these important features of subconscious logic.

One major obstacle to understanding high-level logic is our belief that we could in principle describe abstract thought explicitly in terms of logical operations, if only we could get to the bottom of creativity and intuition. We are thinking here in terms of complexity, not in terms of fundamental limitations imposed on the cognizing mechanism. These limitations exist, and concern the division of logical operations into cognizable logic and subconscious logic. Boolean logic, with which we are all familiar and very much at ease is cognizable logic occurring at the conscious level. All Boolean operations are positive and cognizable. But in matrix operator logic we encounter alongside the 20 positive operations of cognizable logic another 142 negative or mixed operations. The majority of logical operations occur in the subconscious domain. Negative logic is a subconscious logic, the complete reading of which consciousness cannot provide. The paradox here is serious. The cognizing brain is prevented for some fundamental reason from cognizing a certain subset of its own operations. To be cognizable a logical operation must be positive, more correctly phase-congruent, since nothing prevents one from formulating Boolean logic in negative scaling, and considering the rest of positive matrix logical operations to be a subconscious logic. Another feature

of subconscious logic is the product interaction of logical operators, first identified in matrix logic. For example we have no problem cognizing the statement <x|IS|y>, but compiling it in the quantum operation of subconscious logic:

$$\langle x|IS|y\rangle = \langle x|[AND, TIME]|y\rangle$$
$$= \langle x|AND \bullet TIME|y\rangle - \langle x|TIME \bullet AND|y\rangle,$$

we have no cognitive access to the product operations.

These examples illustrate the general rule. With few exceptions, cognizing does not extend beyond the set of positive lofical operators. Neither the operator difference nor the product nor many other quantum operations of subconscious logic are cognizable, unless brought into the standard canonical form of a cognizable Boolean operator. The (2×2) matrix is not only a fundamental mathematical and physical for that matter, unit. It is also a fundamental cognitive unit of cognitive logic.

By including virtual subconscious elements in the thought process we have broadened our concept of logic. There are logical operations, involving cognitive actions, and nonlogical operations, involving subconscious logic. For example, the cognizable operation **NOT** can be available to consciousness directly but may also arise from nonlogical transformations which are not cognizable:

$$IMPLY \bullet OR^{-1} = NOT \qquad \text{or} \qquad OR - AND = NOT.$$

The quantum commutator plays a particularly prominent role in virtual logic. Anyone can cognize without difficulty the Boolean true but our consciousness cannot track down the commutator generation of the quantum **TRUE**,

$$[AND, OR] = TRUE,$$

although it has no difficulty in dealing with the result itself. One may speculate that in the future we may learn to think quantum-mechanically. Quite possibly. But at present many matrix logical operations are entirely out of reach of consciousness. At the same time, subconscious logic, devoid of any cognitive content, forges an important link between logic and physical theory, giving rise to a unified approach which organically combines and explores the formalisms of quantum physics, logic and information theory. Logic obtains the status of an exact science, making it possible to obtain a quantum-mechanical formulation of logical theory. Moreover, the intelligence code, underlying consciousness, can be treated as universal logic calculus in which particular logics, such as Boolean, fuzzy, quantum, differential logic and others are naturally derived as particular limits. Apart from this powerful unification matrix logic is distinguished by another remarkable and unique feature, the nonlogical operations. The intelligence code is a higher-order universal logic or metalogic in which operators can act in a standard way on variables but also on other operators. As a result, the important new operations which have no logical meaning in conventional logic, like differentiation, commutation, exponentiation etc, emerge. Metalogic is logic of high-level intelligence and consciousness, but in matrix theory it exists side by side with

ordinary logic. An operator can act not only on a logical state but also on another operator. The operations in metalogic, as opposed to lower-level logic, are abstract operations which cannot be projected to language. Subconscious logic is a relation of relations, not simply a relation of concepts. A matrix product is an important subconscious interaction but it is only one of many. The interaction can include additive or transmultiplicative fusion and decay, commutative and anticommutative operators products, they can be purely operator in nature or also involve the operands. Effective new inference rules become available, immensely extending the computational power of logic, further melting away the border between the physical and the mental.

Think twice

Subconscious logic can be put to practical use, demonstrating the need for subconscious operations from first principles. This is the case with the 'think-twice 'rule'', when one is advised not to rush into a decision but to evaluate it carefully. In matrix logical terms it could mean applying the operator twice, to square the operator, hoping to obtain a universally valid tautology. This is indeed the case with the expectation of the squared implication:

$$<\rightarrow^2> \equiv 1, \qquad <\leftarrow^2> \equiv 1.$$

The implication operators are the counters:

$$\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & n \\ 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}.$$

UNDERSTANDING SEPTONS

The question of emotion and aesthetic consciousness is the hardest one in the framework of subconscious logic. Between **NONE** and **ONE** there are **79** logical operators, a majority of which are not cognizable. To get better insight into this huge variety of logical operations, we have introduced five fundamental logics, determined by the corresponding potency laws. The greater the potency of logic, the more abstract it is and less cognizable. Matrix logic is a universal logical theory which has the capacity to account for all five fundamental logics: Boolean, tripotent, quatric, pentapotent and septonic. The latter has direct application to the questions of emotion and aesthetic consciousness.

The different potencies of the operators of matrix logic allow us to distinguish five fundamental logics as follows,

Classical (knot-free) logic:

$$\text{TRUE}^2 = \text{TRUE}, \text{FALSE}^2 = \text{FALSE},$$
$$\text{AND}^2 = \text{AND}$$

Tripotent (self-inverse) logic:

$$(\text{FALSE - AND})^3 = \text{FALSE - AND}$$
$$(\text{TRUE - NOR})^3 = \text{TRUE - NOR},$$

Quatric (nonnegative) logic:

$$(\text{OFF - TRUE})^4 = \text{OFF - TRUE},$$
$$(\text{ON - FALSE})^4 = \text{ON - FALSE}$$

Time (imaginary) logic:

$$\text{TIME}^5 = \text{TIME}$$

Septonic (nonassociative) logic:

$$(\text{TRUE - OFF})^7 = \text{TRUE - OFF},$$
$$(\text{FALSE - ON})^7 = \text{FALSE - ON}$$

Matrix logical phase space is saturated and stops growing with potency 7, reflecting the fact that consciousness does involve nonalternating knots which start only with 8-crossings. The cognitive knots interact according to specific topological laws. The logical verbalization of septonic elements is one of the hardest problem: simply speaking, how to express emotion in Boolean form. Next we prove three key theorems concerning septons which establish the rules for the conversion of the elements of subconscious logic into conscious ones. There are several ways of doing so: by modifying the logical amplitudes or phases. There is also the statistical way of averaging. Reformulating septons as the operator sum of time logic and classical logic, we determine that a septon, being averaged, returns us to the idempotent knot-free Boolean logic.

Theorem

The septonic average equals the average of its idempotent Boolean component:

$$\langle \blacktriangledown + \text{AND} \rangle = \langle \blacktriangledown \rangle + \langle \text{AND} \rangle = \langle \text{AND} \rangle.$$

The proof follows from the fact the the time average is zero.

Theorem

$$\blacktriangledown^5 + \text{AND}^2 = (\blacktriangledown + \text{AND})^{5+2}$$

Proof

Since

$$\blacktriangledown^5 = \blacktriangledown \quad \text{and} \quad \text{AND}^2 = \text{AND}$$

we can write

$$\blacktriangledown^5 + \mathbf{AND}^2 = \blacktriangledown + \mathbf{AND}$$

But

$$\blacktriangledown + \mathbf{AND} = \mathbf{TRUE} - \mathbf{OFF}$$

which is septonic:

$$(\mathbf{TRUE} - \mathbf{OFF})^7 = \mathbf{TRUE} - \mathbf{OFF}.$$

Hence ($\blacktriangledown$ + AND) is septonic:

$$(\blacktriangledown + \mathbf{AND})^7 = \blacktriangledown + \mathbf{AND}$$

and

$$\blacktriangledown^5 + \mathbf{AND}^2 = (\blacktriangledown + \mathbf{AND})^7 = (\blacktriangledown + \mathbf{AND})^{5+2}$$

□

(1) Amplitude

A septon is a compound of Boolean knot-free logic and imaginary time logic. Through amplitude change a septon can decay into a logical knot of lesser complexity which is equivalent to 'emitting' the 5-potent **TIME** or the idempotent **AND**:

$$(\mathbf{TRUE} - \mathbf{OFF}) - \blacktriangledown = \mathbf{AND},$$

$$(\mathbf{TRUE} - \mathbf{OFF}) - \mathbf{AND} = \blacktriangledown.$$

(2)Half-unitary transform

Theorem

The half-unitary logical transformation, which is a unilateral application of **NOT**, converts the septons into **TRUE-OFF** or **FALSE-ON**, which are the self-inversers relevant for tripotent logic:

$$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}\mathbf{N} = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}\mathbf{N} = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

where

$$\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

and

$$(\mathbf{TRUE} - \mathbf{NOR})^3 = \mathbf{TRUE} - \mathbf{NOR},$$

$$(\mathbf{FALSE} - \mathbf{AND})^3 = \mathbf{FALSE} - \mathbf{AND}$$

or explicitly

$$\begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}^3 = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^3 = \begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}$$

(3)Phase

By a total phase change a septon is converted to a quatric matrix logic of **true, false, neither true nor false** and **both true and false** which is the quatron: (00), (01), (10), (11), running on two parallel Boolean logics.

Theorem

$$e^{i\pi} \bullet \text{septon} = \text{quatron}$$

Proof

We can change to a quatric logic by emitting a tripotent logical knot. This is equivalent to the global phase reversal, changing a septon into a quatron:

$$\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}^7 = \begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}^7 = \begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}$$

but

$$\left(e^{i\pi}\begin{pmatrix} 0 & -1 \\ 1 & 1 \end{pmatrix}\right)^4 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}^4 = \begin{pmatrix} 0 & 1 \\ -1 & -1 \end{pmatrix}, \qquad \left(e^{i\pi}\begin{pmatrix} 1 & 1 \\ -1 & 0 \end{pmatrix}\right)^4 = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}^4 = \begin{pmatrix} -1 & -1 \\ 1 & 0 \end{pmatrix}$$

□

The change of phase does not always have repercussions. There are logical operators that are not phase-sensitive. Consider, for example, the case in which the global phase change has no effect on operator identity

$$\begin{pmatrix} 1 & 1 \\ 0 & -1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & -1 \\ 0 & 1 \end{pmatrix}^2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

and the case in which it leads to a two-fold jump of power, from 3 to 6:

$$\begin{pmatrix} 0 & -1 \\ 1 & -1 \end{pmatrix}^3 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \quad \text{while} \quad \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}^6 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

The inclusion of the subconscious in logical theory is important for the advanced study of the intelligent brain. A fundamental theory of the brain cannot be built on formal principles alone, but necessarily has to include subconsciousness, emotions, aesthetic feeling, art and beauty. The intellect works with logics of low valuedness, the lower the better for the mind's deduction engine. However, we cannot describe or understand the intricate structure of say Mozart's music in simple terms of true and false. Only logics of higher order can tell us more about the underlying laws of the subconscious and intuition. Septons seem to provide necessary mathematical language for broadening the concept of logic and by unifying the subconscious with the primary language of the cognizable Boolean and near-Boolean logics. **In the human mind there are more logical relations than there are logical descriptions.**

MIND'S DYNAMICAL EQUATIONS

Whereas for a logician a thought is a 'static' implication, matrix logic unexpectedly reveals that implications are in fact dynamic operator waves. The fundamental operator equation of the logical mind-brain is

$$IF^{IF} = e^{a^*}$$

where

$$||IF|| = \text{YES}.$$

In beautifully concise form the wave equation codifies everything one needs to know about the behavior of cognitive nibbles in the brain. It, however, does not tells us much, if anything, about the dynamics of logical states, the question we will tackle next.

Consciousness is a coherent state which has to be actively maintained and any challenge to coherent superposition must be effectively cancelled out. Taking full step in mathematics and physics, matrix logic has extended the power of computation to the domain of dynamical logical valuations. The logical equations of motion should tell us how coherent logical states can be formed, maintained and dynamically manipulated. Suppose one could write a system of equations of motion for all particles comprising the brain. Has one included all the unknowns? How many unknowns are there? To solve a system in n unknowns one generally require a system of n equations. But, since consciousness is a fundamental unknown too, we would have more unknowns than equations unless we account for this extra unknown of consciousness by including logic in the mind's dynamical equations.

The equations of motion have established themselves as one of the basic concepts in physics, both classical and quantum. With the development of quantum theory a radical change has occurred in what we understand by an equation of motion. Whereas in classical physics the functions that figure in the equations stand for actual physical quantities, the equations of quantum mechanics 'no longer deal with fundamental particles themselves but with our knowledge about them'. With its bold ideas quantum mechanics has inadvertently and in the most profound way contributed to cognitive science.

In this section we derive logical analogs of the Schrödinger and the Heisenberg equations of motion, which unify the concept of logic with the fundamental language of physics and enable us to capture the underlying physics and to understand the intricate mechanics of the thought processes and consciousness.

In virtue of its definition and in essence, "I" is a dynamical process. Thinking dynamically alternates between classical states and the states of coherent superposition, and like any dynamical process it must follow the Hamilton optimization or the least action principle. Out of all possible paths by which a logical system could travel from its state at time t_1 to its state at time t_2, it will actually travel along that path for which the integral

$$\int_{t_1}^{t_2} L dt$$

is an extremum, whether a minimum or maximum.

In matrix logic **the least action** principle takes the form of **the least choice** principle. We may evaluate a in multivalued regime of **n = 3, 4, 5, 7** potent logics, but in the end typically decide and communicate in the classical limit of the **2**-potent logic. Intermediate steps of thinking need not be in a form that is cognizable; only the ultimate result counts. The mind does not merely **acquire** information, it dynamically **creates** information. The method of matrix logic is physical in spirit and foreign to traditional logic. A logician proves theorems, building bottom-up or top-down a hierarchy of logical statements, linking them by **Modus Ponens** or other inference rules. A physicist thinks differently, formulating the equations of motion, relying on data or an educated guess. When such an equation is established, with the brain's power or the power of a computer, the physicist seeks to extract information hidden inside the equation.

The concepts we have presented are crucial for the introduction of the dynamical logical equations of motion. Just as the equations of motion are central to physical analysis, motion is an essential feature of the cognizing mechanism. If cognitive motion stops, interiorization and awareness are very quickly terminated. For example, the brain becomes effectively 'blind' to visual information when an image is presented to the brain by a device which is fixed on the eyeball. For short period of time one cognizes the image, but then it vanishes, becoming 'invisible' for consciousness. Cognitive motion takes various forms: information may enter the brain in the fermionic code of thoughts and in the bosonic code of feelings. Deductive "I" is fermionic and naturally lends itself to description with quantized matrix logic. Emotions are bosonic and easier to describe with continuous fuzzy logic. When a perceptual signal enters the brain, this does not automatically mean that it will get cognized. For that the machinery of differential logic must be switched on, giving the mental states logical momentum. The classical input information in a sense disappears, getting distributively hidden in the quantum correlations.

As we mentioned earlier, the formulation of the equations of motion is a central feature of physics. We have an easy and intuitive concept of a physical motion, and we understand what the motion of particles and waves means. To understand a logical motion is much harder. Throughout this study, we have developed the theme that the laws of thinking and consciousness appear simple and unified in matrix dimensions. For a dynamical thinking process we therefore consider the matrix logical equations of motion.

It must be emphasized that "I" exercises control over the thought process, but only to a certain extent, because **consciousness fluctuates** and is in permanent flux. We observe these fluctuations directly in the perceptual rivalry experiments and in the paradox of the Liar. "I" itself is a logical motion. To get a sense of these fluctuations, consider a dynamical cognitive effect, induced by a perfectly stable object, presented in Fig. 39.

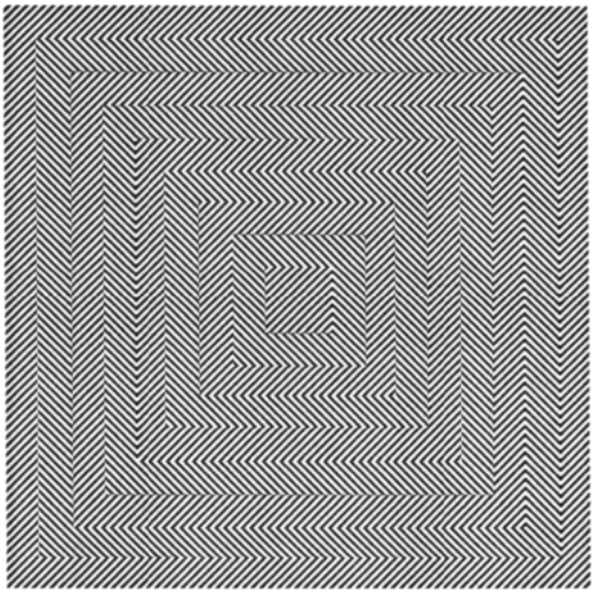


Fig. 39
A dynamical effect in perception. As you look at this picture, the image begins to move and pulsate. A conflict between physical fact and mental effect persists without a resolution. The picture is physically motionless and the fluctuations are not in the picture itself but produced in your consciousness. The effect builds up and cannot be stopped by voluntary commands of the brain.

A conflict between a static picture and the 'fictitious' cognitive motion cannot be resolved, even though the effect is not in the picture but added-in by your consciousness. Although the figure is physically motionless, the discrepancy between physical fact and mental effect sets it in perpetual flux. What initially appeared to be an innocent enough array of alternating lines gradually begins to move with pulsation that is felt almost physically. After a while the assault on the brain becomes intolerable, but the waves of the fictitious motion are not physical but in your consciousness. The mind works by alternating between classical and superposed states which are essentially quantized. This is reflected in language, which is not smooth but broken down into discrete words. Consciousness is discontinuous, with the possible exception of musical consciousness, which comes close to uninterrupted form, but still comprises discrete notes; no matter how short the breaks may be, they exist.

When a quantum system is specified in terms of its state in Hilbert space, the problem is then to determine the Hamiltonian operator. Once the Hamiltonian is found, the physics of the problem opens up. One diagonalizes the Hamiltonian by rotating the space into an optimal principal axis alignment and calculates the energy eigenstates of interest. For the logical equations of motion the situation is quite similar. In many important respects logical motion is analogous to quantum motion, casting light on how the fundamental level dictates the overall behaviour of the brain.

In the section devoted to the logical energy function we dealt extensively with logical eigenstates, and determined that these are the states of coherent superposition. The logical **Lagrangian** changes a logical state with **complementary** components to a logical state with components of **opposite** momentum, apparently subtracting the complementary state from a given state:

$$\mathbf{H}|q\rangle = \begin{pmatrix} \bar{q} - q \\ q - \bar{q} \end{pmatrix} = |q\rangle - |\bar{q}\rangle$$

If the logical states are Boolean, the energy operator can act as a signed momentum operator, generating superposed states:

$$\mathbf{M}|x\rangle = \pm\, \mathbf{H}|x\rangle.$$

In the brain this operation can be naturally accomplished by comparison of the two consecutive states, the active and the refractive. The logical momentum propels the system into quantized states, temporarily disrupting the initial normalization. Cognitive states exist in coherent superposition of the true and false until a problem is solved and the brain can revert to the deterministic classical mode of operation.

A strict coherent logical superposition has to be maintained if the cognitive process is to be of any practical use. The goal of the logical equations of motion therefore is to describe how coherent superpositions are formed and retained. The perceptual information or information from memory enters consciousness in the classical form of the normed logical states $|q\rangle$. When the cognitive interiorization takes place the state $|q\rangle$ is converted into a state of

coherent superposition $|S_{\pm}>$, and the canonical way to obtain such a conversion is to employ the logical derivative:

$$\{T, \perp\}\,|q> = |S_{\pm}>.$$

The dynamic logical equations that we study involve derivatives with respect to truth-value and not with respect to time, which enters in the equation of motion not as a variable but as an operator:

$$-\frac{d|q>}{dq} = \mathbf{H}\blacktriangle\,|q>,$$

where the **Lagrangian** $\mathbf{H} = -\ln \mathbf{NOT}$ is suprasymmetric:

$$\mathbf{H}^2 = 2\mathbf{H},$$

and

$$\mathbf{N} = e^{-\mathbf{H}},$$

$$[\mathbf{H}, \mathbf{N}] = 0, \quad \mathbf{NHN} = \mathbf{H}, \quad \mathbf{NH} = -\mathbf{H},$$

where $\mathbf{N} = \mathbf{N}^* = \mathbf{N}^{-1}$ is the logical evolution operator, conserving the scalar product

$$<q|\mathbf{N}\mathbf{N}|q> = <q|q>.$$

When logical motion is controlled by the evolution operator **NOT**, we deduce the equation of motion with the pair of canonical operators (see below):

$$\frac{d|q>}{dq} = \mathbf{H}\wedge\vee|q>.$$

But the time operator can also act as the evolution operator:

$$\blacktriangle = e^{-\dot{H}}$$

where the Hadamardian $\dot{H}$ is the unbroken topological phase,

$$\dot{H} = -\ln \blacktriangle, \quad \dot{H}^2 = 2\blacktriangledown,$$

and

$$\blacktriangle^* = \blacktriangle^{-1} = \tilde{\blacktriangle} = -\blacktriangle = \blacktriangledown,$$

$$<q|\blacktriangle\blacktriangledown|q> = <q|q>,$$

$$[\dot{H}, \blacktriangledown] = 0, \quad [\dot{H}, \blacktriangle] = 0,$$

$$\blacktriangle\dot{H}\blacktriangledown = \dot{H}, \quad \blacktriangle\dot{H} = .H,$$

$$[\dot{H}, .H] = 0.$$

The corresponding equation of motion is:

$$\frac{d|q>}{dq} = \dot{H}\wedge\vee|q>,$$

from which we see that the **Lagrangian H** and the Hadarmardian $\dot{H}$ are exchangeable. Likewise for the reversed time

$$\blacktriangledown = e^{-.H}$$

where the Hadarmardian is transposed,

$$.H = -\ln \blacktriangledown, \qquad .H^2 = 2\blacktriangle,$$

and

$$\blacktriangledown^* = \blacktriangledown^{-1} = \tilde{\blacktriangledown} = -\blacktriangledown = \blacktriangle,$$
$$\langle q|\blacktriangle\blacktriangledown|q\rangle = \langle q|q\rangle,$$
$$[.H, \blacktriangledown] = 0, \quad [.H, \blacktriangle] = 0,$$
$$\blacktriangledown.H\blacktriangle = .H, \qquad \blacktriangledown.H = \dot{H},$$
$$[.H, \dot{H}] = 0.$$

The corresponding equation of motion

$$-\frac{d|q\rangle}{dq} = .H\overline{\wedge\vee}|q\rangle$$

acquires a negative sign on the left hand side, concomitantly with the complementation of the matrix postfactor on the right hand side.

CANONICAL EQUATIONS OF MOTION

Now we will be concerned with the suprasymmetric aspect of the logical equations of motion, which are dictated by the underlying Matrix Statistics. The Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

describes the development of a quantum system in time. Although the thought process also takes place in time, it is **logical** order, rather than **time** order, which is of primary importance. Truth is not dependent on time. As with motion in a potential field, it is the **information distance** between the initial and final states of knowledge, and not the character of the 'trajectory' from one cognitive state to the other which determines the outcome of a logical evaluation. It is this line of reasoning which led us to replace in the logical equation of motion the derivative with respect to time with the derivative with respect to truth-value [Ref 91]. In contrast to the Schrödinger equation, the logical equation of motion is not a postulate but a mathematical fact. Since the derivative with respect to truth-value is the logical momentum operator, it is straightforward to realize that the equation of motion is simply an operator definition of the logical momentum:

$$-\frac{d}{dq}|q\rangle = M|q\rangle,$$

where

$$M = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}.$$

Instead of the Hamiltonian H, found in the Schrödinger equation, we have the momentum operator **M**. Moreover, the Hamiltonian H is Hermitian, hence a quantum-mechanical observable, in contrast to the logical operator **M** which is non-Hermitian.

Nonetheless, just as in quantum mechanics, the logical momentum **M** and the **Lagrangian H** are intertwined. A changeover between the operators is mediated by the spin operator σ:

$$\mathbf{M}\sigma = \mathbf{H},$$

and, since σ is self-inverse:

$$\mathbf{M} = \mathbf{H}\sigma.$$

Just as the quantum momentum and energy operators are quadratically related:

$$\mathbf{p}^2 \sim \mathbf{E},$$

so are the logical momentum and energy:

$$\langle \mathbf{M} \rangle^2 = \langle \mathbf{H} \rangle$$

Both operators **M** and **H** induce coherent superpositions:

$$\mathbf{M}|q\rangle = |s_-\rangle$$

and

$$\mathbf{H}|q\rangle = \alpha|s_-\rangle, \qquad -1 \leq \alpha \leq 1.$$

However, **M** is self-dual: $\mathbf{M}^2 = \mathbf{0}$, with a degenerate eigenstate, while **H** has a complete set of orthogonal eigenstates: $\{|s_-\rangle, |s_+\rangle\}$, also shared by **NOT**, the unitary evolution operator, and the tautology **1**. The eigenstates are the states of coherent superposition, whose outer products generate the energy and momentum operators themselves:

$$\mathbf{H} = |s_-\rangle\langle s_-|, \qquad \mathbf{M} = |s_-\rangle\langle s_+|.$$

As was indicated earlier, in the Schrödinger equation the physics of the problem is known once the Hamiltonian operator H is found. But in a logical equation of motion the **Lagrangian** is known in advance. What has to be determined is some unknown interaction **X**, which couples **H** to a logical state $|q\rangle$ in such a way that the equality

$$\mathbf{M}|q\rangle = \mathbf{H}\mathbf{X}|q\rangle$$

holds. Consequently, we are presented with the problem of finding a solution to the matrix equation

$$\mathbf{M} = \mathbf{H}\mathbf{X},$$

where **X** is an unknown 'interaction' operator, intelligently preparing $|q\rangle$ for action of **H** or vice versa. The solution of this matrix equation is not unique. Elsewhere we proved that there are infinitely many operators **X**, transforming **H** into **M**. Any operator of the form

$$\mathbf{X} = \begin{pmatrix} n & m+1 \\ n-1 & m \end{pmatrix},$$

where n, m = 0, ±1,± 2, ... , including n = m, satisfies the equation.

Since the antisymmetric time ▲ and the antipersymmetric spin σ are the two fundamental cases of **X**:

$$\blacktriangle = X(n = 0,\ m = 0), \qquad \sigma = X(n = 1,\ m = -1),$$

in the logical equation of motion we can employ two equivalent forms of the logical momentum operator:

$$H\blacktriangle = H\sigma = M.$$

In particular

$$-\frac{d}{dq}|q> = H\sigma|q>.$$

We determined the canonical equation of motion

$$(H\blacktriangle + \frac{d}{dq})|q> = |0>$$

which expresses the condition for obtaining the cognitive superposition $|s_->$.

To derive the adjoint equation for the orthogonal superposition $|s_+>$, we make use of the fact that the **Lagrangian H** and **Hamiltonian 1** are spectral twins, **[1, H] = 0**, with matrix time unitarily transforming one into the other:

$$\blacktriangle H\blacktriangledown = 1, \qquad \blacktriangledown 1\blacktriangle = H.$$

This fact assists us in deriving the adjoint equation of motion for the cognitive state $|s_+>$, which is obtained from the original equation by acting on it with reverse time:

$$\blacktriangledown(H\blacktriangle + \frac{d}{dq})|q> = |0>$$

$$(\blacktriangledown H\blacktriangle + \blacktriangledown\frac{d}{dq})|q> = |0>$$

and reversing the direction of time, we get:

$$(1 - \blacktriangle\frac{d}{dq})|q> = |0>.$$

Dealing with the unknown of consciousness, we introduced the **TIME** and **SPIN** operators explicitly into the logical equations of motion. Because both can be naturally expressed in terms of the annihilation and creation field operators:

$$\blacktriangle = (a - a^*), \qquad \sigma = [a,\ a^*],$$

this approach laid the foundation for a challenging research programme whose goal is to determine the points of contact between the mechanism of thought and quantum relativistic equations. The canonical pair of the logical equations of motion determines the superpositions $|s_+>$ and $|s_->$ which are at the heart of the nonorientable topology of consciousness. There is a close link to the Lorentz-invariant Weyl equations for zero-mass particles, disobeying the parity symmetry, describing the neutrino fields:

$$(\sigma\nabla - \frac{\partial}{\partial t})|\psi> = 0 \qquad |\psi> = \frac{1}{\sqrt{2}}(\varphi + \chi)$$

$$(\sigma\nabla + \frac{\partial}{\partial t})|\phi> = 0 \qquad |\phi> = \frac{1}{\sqrt{2}}(\varphi - \chi)$$

This inspires one to consider one system of equations comprising the neutrino fields and the logical equations of motion. The topologies of neutrinos and consciousness, effectively isolating themselves from the outside universe, are two sides of a coin.

SUPERCOMMUTATION THEOREM

Cognitive dynamics (truth) is controlled by the logical order rather then by the time order. Restricting ourselves to the logical framework, we can express the antisymmetric time operator ▲ as the commutator of the symmetric logical operators, **AND** and **OR**, and the symmetric spin operator σ as the commutator of the asymmetric logical operators **IF** and **IMPLY**. This allows us to rewrite

$$\mathbf{H}\blacktriangle = \mathbf{H}\sigma$$

equivalently as

$$\mathbf{H}[\vee, \wedge] = \mathbf{H}[\rightarrow, \leftarrow].$$

The postfactors on the left and right hand sides of this equality obey different laws of symmetry. In analogy to the logical subtraction rule:

$$A - B = \bar{B} - \bar{A}$$

for the spin commutator the implications, if complemented, recombine:

$$[\rightarrow, \leftarrow] = [\overline{\leftarrow}, \overline{\rightarrow}],$$

but for the time commutator the changeover is not required; the equality is maintained under complementation:

$$[\vee, \wedge] = [\overline{\vee}, \overline{\wedge}].$$

and

$$[\vee, \mathbf{H}] = \blacktriangle, \qquad [\mathbf{H}, \wedge] = \blacktriangledown.$$

Representing the logical momentum by symmetric or antisymmetric operators, we detect that it supersymmetrically combines bosonic and fermionic properties.

Theorem

The action of the differentiation operator on a normed logical state $|q>$ is equivalent to the action of the product of the **Lagrangian H** with the field contraction product $\overbrace{\wedge\vee}$

$$\frac{d|q>}{dq} = \mathbf{H}\overbrace{\wedge\vee}|q>,$$

which is

$$\frac{d|q>}{dq} = \mathbf{H}\{\wedge,\vee\}|q>,$$

where the supercommutator brackets {,} signify either anticommutator or commutator.

Proof

The result of the differentiation of any normed state $|q>$ equals the **Lagrangian H** acting on the true state $|1>$:

$$\frac{d|q>}{dq} = \mathbf{H}|1>.$$

In order to have the same vector variable on both sides of the equation, we replace $|1>$ with the equivalent term

$$\mathbf{AND{\cdot}OR}|q> = |1>,$$

thus obtaining $|q>$ on both sides of the equation of motion:

$$\frac{d|q>}{dq} = \mathbf{H}\wedge\vee|q>.$$

We have achieved our first objective: the **Lagrangian H** will act on $|q>$ as the differentiation operator, provided that $|q>$ is premultiplied by the matrix product $\wedge\vee$, formed from the canonical pair of the logical operators, **AND** and **OR**. Next we prove that $\wedge\vee$ can be replaced with the field contraction product $\overbrace{\wedge\vee}$. To show that such a replacement is legitimate, we proceed in the following steps. Since the action of the vacuum on a logical state is again a vacuum, nothing prevents us from adding an empty operator **0** to the equation:

$$\frac{d|q>}{dq} = \mathbf{H}(\wedge\vee + \mathbf{0})|q>.$$

But since

$$\mathbf{H}\vee\wedge \equiv \mathbf{0},$$

the equation can now be conveniently rewritten as

$$\frac{d|q>}{dq} = \mathbf{H}(\wedge\vee \pm \vee\wedge)|q>,$$

where $(\wedge\vee \pm \vee\wedge)$, residing between the **Lagrangian H** and $|q>$, can be collected into the contraction product $\overbrace{\wedge\vee}$. We therefore had proven the theorem:

$$\frac{d|q>}{dq} = \mathbf{H}\,\overbrace{\wedge\vee}\,|q>$$

Depending on whether an interaction is bosonic or fermionic, the contraction product $\overbrace{\wedge\vee}$ will assume either the antisymmetric mode:

$$\overbrace{\wedge\vee} = \wedge\vee - \vee\wedge$$

or the symmetric mode:

$$\overbrace{\wedge\vee} = \wedge\vee + \vee\wedge.$$

Even though it is clear that

$$\wedge\vee - \vee\wedge \neq \wedge\vee + \vee\wedge,$$

their products with the **Lagrangian H** are equal:

$$\mathbf{H}(\wedge\vee - \vee\wedge) = \mathbf{H}(\wedge\vee + \vee\wedge),$$

explicitly:

$$\mathbf{H}\begin{pmatrix}0 & -1\\ 1 & 0\end{pmatrix} = \mathbf{H}\begin{pmatrix}0 & 1\\ 1 & 2\end{pmatrix} = \begin{pmatrix}-1 & -1\\ 1 & 1\end{pmatrix}$$

The logical equation of motion can therefore be supersymmetrically written with the bosonic commutator:

$$\frac{d|q>}{dq} = \mathbf{H}[\wedge,\vee]|q>$$

or equivalently with the fermionic anticommutator:

$$\frac{d|q>}{dq} = \mathbf{H}\{\wedge,\vee\}|q>.$$

Consequently, the equation describing the formation of coherent logical states has the universal general form of a supercommutator which is degenerate with regard to anticommutation and commutation:

$$\frac{d|q>}{dq} = \mathbf{H}\{\wedge,\vee]|q>.$$

□

In quantum field theory a distinction between bosons and fermions is fundamental, determining whether the field contraction is a commutator or anticommutator. In contrast to the quantum field, logical motion displays a supersymmetry. In cognitive phase space the logical transformations can consequently be regarded as a matrix superfield, degenerate with regard to the fermionic and bosonic operator ordering.

SATELLITE EQUATIONS

The supersymmetric logical equation of motion gives rise to a large family of satellite equations, of which we consider several important examples:

(1)

$$\frac{d|q>}{dq} = H(\wedge + \vee)|q>$$

where $(\wedge+\vee)$ is an operator sum of the pair of canonical logical operators;

(2)

$$-\frac{d|q>}{dq} = H \rightarrow^2 |q>$$

where $\rightarrow^2$ is the implication operator raised to the power of 2;

(3)

$$-\frac{d|q>}{dq} = H(\wedge + \vee)\blacktriangledown(\wedge + \vee)|q>$$

where $\blacktriangledown$ is the time operator.

The equation (1) follows from the theorem

$$\frac{d|q>}{dq} = H\{\wedge, \vee\}|q>,$$

since the anticommutator of **AND** and **OR** reduces directly to the sum:

$$\{\wedge, \vee\} = \wedge + \vee,$$

To prove (2) we take the equation of motion (1) and insert the identity **YES = NOT**^{2} between the **Lagrangian H** and the operator sum of **AND** and **OR**:

$$\frac{d|q>}{dq} = H(\wedge + \vee)|q> = H \cdot NOT^2(\wedge + \vee)|q>$$

Acting with one **NOT** on the left, we get

$$H \cdot NOT = -H$$

$$-\frac{d|q>}{dq} = H \cdot NOT(\wedge + \vee)|q>$$

and with another **NOT** on the right, we get

$$NOT(\wedge + \vee) = \rightarrow^2,$$

which proves the equation of motion (2).

To obtain the proof of (3), we begin with the equality

$$H\blacktriangledown = H(\wedge + \vee).$$

Multiplying both sides of the equation from the right by the time inverse $\blacktriangle$, we get

$$H\blacktriangledown\blacktriangle = H(\wedge + \vee)\blacktriangle,$$

which reduces to

$$H = H(\wedge + \vee)\blacktriangle,$$

or equivalently, by transposition, to

$$\blacktriangledown(\wedge + \vee)H = H.$$

We see that the **Lagrangian H** is a matrix fixed point for the compound operator

TIME (AND + OR),

hence:

$$\mathbf{H}(\wedge+\vee)|q> = \mathbf{H}(\wedge+\vee)\blacktriangle(\wedge+\vee)|q>.$$

Multiplying by -1 and transferring the sign on the time operator, we obtain the proof for theorem (3):

$$-\frac{d|q>}{dq} = \mathbf{H}(\wedge+\vee)\blacktriangledown(\wedge+\vee)|q>.$$

Here the operator

$$+ = \mathbf{AND} + \mathbf{OR}$$

assumes the role of the unitary transformation of time $\blacktriangle$ and $\blacktriangledown$,

$$+\blacktriangle+ = \blacktriangledown, \qquad +\blacktriangledown+ = \blacktriangle,$$

acting exactly as the unitary evolution operator **NOT**:

$$\mathbf{N}\blacktriangle\mathbf{N} = \blacktriangledown, \qquad \mathbf{N}\blacktriangledown\mathbf{N} = \blacktriangle.$$

Theorem

$$-\frac{d|q>}{dq} = \mathbf{H}(1+\blacktriangle)|q>$$

Proof

A direct proof of this version of the logical equation of motion can be obtained by exploring the fact that $\mathbf{H}\bullet\mathbf{1} = \mathbf{0}$, which reduces the equation to the equation of motion we have already proven earlier. More meaningfully we can explore theorem (2), expressing the squared implication as the product:

$$\rightarrow^2 = \mathbf{NAND}\bullet\mathbf{OR}.$$

Next we recall that a logical operator can be expressed as a sum of symmetric and antisymmetric operators. Through symmetrization we obtain the tautology operator **1**, which is symmetric:

$$A = \tfrac{1}{2}\{\downarrow,\vee\} = \tfrac{1}{2}\left(\begin{pmatrix}1&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&1\end{pmatrix}+\begin{pmatrix}0&1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\1&0\end{pmatrix}\right) = \begin{pmatrix}1&1\\1&1\end{pmatrix} = \mathbf{1},$$

and through alternation the time operator $\blacktriangle$, which is antisymmetric:

$$B = \tfrac{1}{2}[\downarrow,\vee] = \tfrac{1}{2}\left(\begin{pmatrix}1&1\\1&0\end{pmatrix}\begin{pmatrix}0&1\\1&1\end{pmatrix}-\begin{pmatrix}0&1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\1&0\end{pmatrix}\right) = \begin{pmatrix}0&1\\-1&0\end{pmatrix} = \blacktriangle.$$

The sum $A + B$ is the required squared implication operator:

$$\mathbf{1} + \blacktriangle = \rightarrow^2.$$

□

SUPRASYMMETRY

Due to the fact that there are numerous ways of generating the logical momentum operator, cognitive motion accommodates the extraordinary variety and multiplicity of thoughts. Different matrix interactions can effectively lead to the same final outcome. Above we have proven that the logical **Lagrangian** supersymmetrically does not distinguish between the commutator and anticommutator of the operators **AND** and **OR**. In addition

to that, there is an important extra symmetry where the logical **Lagrangian** also does not discriminate between the product and the sum of these canonical operators:

$$\mathbf{H}(\wedge\bullet\vee) = \mathbf{H}(\wedge+\vee),$$

and even simply between the operators **AND** and **OR** in the commutator with the logical **Lagrangian:**

$$-\frac{d|q>}{dq} = \mathbf{H}[\wedge, \mathbf{H}]|q> \qquad \text{and} \qquad -\frac{d|q>}{dq} = \mathbf{H}[\vee, \mathbf{H}]|q>.$$

The logical equation of motion describes the conversion of a normed logical state into a state of coherent superposition. For a supersymmetric interaction the contraction terms, although different in the bosonic and in fermionic modes, nonetheless, jointly with the **Lagrangian** isomerically produce the differentiation momentum operator. This result links up with quantum field theory where the bosons and fermions differ in their commutation algebras. In marked contrast, Matrix Statistics, governing logical thought, is supersymmetric and does not discriminate between the bosonic and fermionic rules of composition.

The logical symmetries can be analysed from the viewpoint of operands, which involves the concept of antilogic (improbability). The associativity of matrix multiplication permits us, instead of recording how the contraction product interacts with the **Lagrangian** on the left, to investigate how it acts on a logical state to the right. Depending on whether the contraction is bosonic or fermionic, the logical state will be prepared differently:
Symmetric mode:

$$(\wedge\vee+\vee\wedge)|q> = \begin{pmatrix} q \\ 1+q \end{pmatrix}$$

Antisymmetric mode:

$$(\wedge\vee-\vee\wedge)|q> = \begin{pmatrix} -q \\ 1-q \end{pmatrix}$$

Comparing the resulting vectors, we observe that a changeover from anticommutator to commutator involves a changeover from positive logic to the negative antilogic:

$$\begin{pmatrix} q \\ 1+q \end{pmatrix} \xleftrightarrow[+q \leftrightarrow -q]{} \begin{pmatrix} -q \\ 1-q \end{pmatrix}.$$

When the contraction product itself is evaluated as a logical function, the change of sign is manifested in the addition or subtraction of truth-values, for the symmetric and antisymmetric mode respectively:

$$<p|\ \{\wedge, \vee\}\ |q> = p + q, \qquad <p|\ [\wedge, \vee]\ |q> = p - q.$$

There is a certain similarity between the antisymmetric mode and isospin. In the isospin model the particle and the antiparticle doublets retain the same group symmetry if in the antiparticle doublet the two isospin projections of the nucleon are reordered and the antineutron is given the minus sign. But that is precisely the transformation which occurs with the components of a logical state when it is premultiplied by the commutator $[\wedge, \vee]$. Just as is the case

with the antiparticle isospin doublets, the time operators swap the falsum and verum components of the logical state and label the falsum component with the minus sign. Only after a logical state has been prepared as described can the **Lagrangian** be applied to it, satisfying the logical equation of motion. Likewise the expression $\mathbf{H}\sigma|q\rangle$ can be understood as synergy **H** acting on a classical state $|q\rangle$, in which, prior to that, the phase of the lower component q has been inverted, $q \Rightarrow qe^{i\pi}$, by the spin operator. While the composition $H|\psi\rangle$ in the Schrödinger equation is simply the product, the composition of **H** and $|q\rangle$ is mediated by spin: $\mathbf{H}\sigma|q\rangle$. This replaces the synergy operator, which is a Hermitian operator, with the non-Hermitian momentum operator. In the Boolean limit where the states are separable $\langle x|y\rangle = \delta_{xy}$ and the logical density Ω is reduced to a projection operator $\Omega^2 = \Omega$ we get the equality:

$$\pm\langle q|\mathbf{H}|q\rangle = \langle q|\mathbf{M}|q\rangle.$$

THE INTEGRAL EQUATION OF MOTION

The basic theorem of analysis states that up to a constant differentiation and integration, executed jointly, is identity:

$$\int f'(x)dx = f(x)$$

Employing this theorem to describe the evolution of a logical state on the logical interval [0, q], we obtain the integro-differential equation of motion:

$$\int_0^q \frac{\partial|p\rangle}{\partial p}\,dq + |0\rangle = |q\rangle,$$

where the false state $|0\rangle$ is the logical integration constant [Ref 92]. When the integration domain is Boolean, q=1, then the integration is logical negation, converting the initial state $|0\rangle$, which is false, into the final state $|1\rangle$, which is true. In the integral equation of motion we differentiated the logical state $|p\rangle$ with respect to the variable p, and integrated the resulting function with respect to the new variable q. During this metamorphosis the logical system goes from the classical state $|p\rangle$ to the classical state $|q\rangle$, passing through a state of coherent superposition. A self-reference occurs when the variables of differentiation and integration coincide:

$$\int_0^q \frac{\partial|q\rangle}{\partial q}\,dq + |0\rangle = |q\rangle.$$

Here a classical state $|q\rangle$ is differentiated to evolve into the superposed state $-|S_-\rangle$, and the same state $|q\rangle$ is recovered through integration. These are valid but tautological transformations. However, we can begin to extract useful

information from the integral equations of motion when we replace the differentiation operator with the matrix logical operators, for example:

$$\int_0^q (\mathbf{YES} - \mathbf{NOT}) \blacktriangledown |q> dq + |0> = |q>,$$

where **YES - NOT** is the **Lagrangian H** written explicitly logically, and $\blacktriangledown$ is the commutator **[AND, OR]**. From this one directly derives the logical integral

$$\int_0^q (\mathbf{TIME} - \mathbf{SPIN}) |q> dq + |0> = |q>$$

or equivalently

$$\int_0^q (\mathbf{TRUE} - \mathbf{FALSE}) |q> dq + |0> = |q>,$$

and many other integral equations of motion. Differentiation is a single operation in analysis, but in the intelligent brain it can take numerous matrix logical forms, providing the basis for multiple realization of the thought process and for expressing a logical thought by the infinite variety of logical integrals.

The integral equation of motion demonstrates that in order to accomplish a proper logical evaluation the system has to be reset to or start from a false state $|0>$, the algorithm which we frequently employ for solving everyday logical problems by '**starting all over again**'. This algorithm is in direct relation to the **Modus Tollens** which is used to obtain a proof from the opposite. In order to prove that some statement x is true, one assumes that it is false, $\bar{x}$, and then derives that $\bar{x}$ implies another statement y, about which it is known in advance that it is false. Hence, $\bar{y}$ is true, and one concludes that x is true. To sum up

$$((\ \bar{x} \text{ implies } y) \text{ and } \bar{y}\ ) \text{ implies } x,$$

which is a tautology constant of the form:

$$\overline{<\bar{x}|\rightarrow|y><1|\bar{y}><0|x>} \equiv 1.$$

The reason why in the integral equation of motion the initialization state is a false state becomes somewhat clearer.

THE COMMUTATION EQUATION OF MOTION

The thought process inevitably has to alternate between the Boolean states $|1>$ and $|0>$ and superposed states $|S_+>$ and $|S_->$. Because the first are the eigenstates of **AND** and the second are eigenstates of **NOT**, this gives a

new physical motivation for taking {**NOT, AND**} as a functionally complete basis set. Significantly we have here a non-Abelian system: the canonical operators **AND** and **OR** do not share common eigenstates and therefore do not commute:

$$[\mathbf{AND}, \mathbf{OR}] \neq \mathbf{0}.$$

However, multiplying the operators by **NOT**, we recover the Abelian algebra:

$$[\mathbf{NOT \cdot AND}, \mathbf{NOT \cdot OR}] = \mathbf{0},$$

most importantly obtaining operators **NOT•AND** and **NOT•OR**, satisfying the logical commutation equations of motion, considered next.

Theorem

The implication operator satisfies the commutation equation of motion

$$\frac{d\langle q|\rightarrow|q\rangle}{dq} = \langle q|[\rightarrow, \mathbf{H}]|q\rangle.$$

Proof

We differentiate the left hand side of the equation and derive:

$$\frac{d\langle q|\rightarrow|q\rangle}{dq} = \frac{d\langle q|}{dq}|\rightarrow|q\rangle + \langle q|\rightarrow|\frac{d|q\rangle}{dq}$$

Substituting for the differentiation operator the matrix operator of the logical momentum:

$$\frac{d\langle q|}{dq} = \langle q|\begin{pmatrix}-1 & 1\\ -1 & 1\end{pmatrix},$$

$$\frac{d|q\rangle}{dq} = \begin{pmatrix}-1 & -1\\ 1 & 1\end{pmatrix}|q\rangle.$$

Continue:

$$\langle q|\begin{pmatrix}-1 & 1\\ -1 & 1\end{pmatrix}\rightarrow|q\rangle + \langle q|\rightarrow\begin{pmatrix}-1 & -1\\ 1 & 1\end{pmatrix}|q\rangle$$

$$= \langle q|\begin{pmatrix}-1 & 0\\ -1 & 0\end{pmatrix}|q\rangle + \langle q|\begin{pmatrix}0 & 0\\ 1 & 1\end{pmatrix}|q\rangle.$$

Rearranging matrix elements:

$$= \langle q|\begin{pmatrix}0 & 0\\ -1 & 1\end{pmatrix} - \begin{pmatrix}1 & 0\\ -1 & 0\end{pmatrix}|q\rangle,$$

Expressing the operators as products:

$$= \langle q|\rightarrow\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix} - \begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}\rightarrow|q\rangle$$

where

$$\begin{pmatrix} 0 & 0 \\ -1 & 1 \end{pmatrix} = \rightarrow \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix},$$

$$\begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} \rightarrow$$

and collecting them into the commutator, we prove the theorem:

$$= \langle q|[\rightarrow, H]|q\rangle$$

□

Consequences

(A)

We have proved that the asymmetric logical operator **IMPLY** satisfies the commutation equation of motion. In fact the commutation equation of motion is satisfied by the whole quadruplet of asymmetric logical operators. We begin with **IF**, which is the transpose of **IMPLY**, and of course

$$\frac{d\langle q|\leftarrow|q\rangle}{dq} = \frac{d\langle q|\rightarrow|q\rangle}{dq}$$

Since for asymmetric logical operators

$$[\, L, H \,] = [\, H, \tilde{L} \,]$$

the commutation equation of motion for **IF** takes the form:

$$\frac{d\langle q|\leftarrow|q\rangle}{dq} = \langle q|\,[H, \leftarrow]|q\rangle.$$

Next, making use of the equations of motion for implication operators, we can prove that their complements, **ON** and **OFF**, also satisfy the equation. Substitution of the operator **ON** for **IMPLY** in the equation of motion leaves it invariant, changing the sign on both sides of the equation to its opposite. The left hand side inverts its sign due to the property of differentiation of the complement. Since $\mathbf{ON} = \overline{\rightarrow}$,

$$\frac{d\langle q|ON|q\rangle}{dq} = -\frac{d\langle q|\rightarrow|q\rangle}{dq}.$$

The right hand side changes sign due to the vanishing products $\mathbf{1 \cdot H = 0}$:

$$\begin{aligned} [\, ON, H \,] &= ON \cdot H - H \cdot ON \\ &= (1 - \rightarrow) \cdot H - H \cdot (1 - \rightarrow) \\ &= - \rightarrow \cdot H + H \cdot \rightarrow \\ &= [\, H, \rightarrow \,] = - [\, \rightarrow, H \,] \end{aligned}$$

This proves the commutation equation of motion:

$$\frac{d\langle q|ON|q\rangle}{dq} = \langle q|\,[ON, H]|q\rangle.$$

Likewise, one proves:

$$\frac{d\langle q|OFF|q\rangle}{dq} = \langle q|\,[H, OFF]|q\rangle.$$

This completes the proof of our assertion that the quadruplet of asymmetric logical operators obey the commutation equation of motion.

(B)

The commutation equations of motion have important links with the logical momentum operator, which in turn connects them in fundamental way with spin. Thought tells the brain how to spin; spin tells the brain how to think. It is possible to obtain the logical commutation equations of motion as consequences of the singularization theorem

$$<q|M|q> = < q|\sigma|q>.$$

The derivatives of the expectation value of the asymmetric logical operators equal by modulo the expectation value of the logical momentum:

$$\frac{d<q|L_{asym}|q>}{dq} = \pm<q|M|q>$$

where $L_{asym} \in \{\mathbf{ON, OFF, IMPLY, IF}\}$.

In particular:

$$\frac{d<q|\rightarrow|q>}{dq} = -<q|M|q>$$

We calculate:

$$\frac{d<q|\rightarrow|q>}{dq} = (\overline{\overline{qq}})' = 2q-1$$

$$<q|M|q> = 1-2q$$

Thus for the **IMPLY** and **IF** the minus sign is needed to establish the equality. Considering the right hand side of the commutation equations, we observe that the commutators [**L, H**] and [**H,** $\tilde{\mathbf{L}}$] are reduced to the spin operator $\pm\sigma$:

$$[\rightarrow, \mathbf{H}] = [\mathbf{H}, \leftarrow] = -\sigma$$

$$[\mathbf{ON, H}] = [\mathbf{H, OFF}] = \sigma$$

Whenever the minus sign appears on the differential left hand side of the equation, it pops up congruently in the commutator on the right hand side:

$$\pm<q|M|q> = \pm<q|\sigma|q>.$$

Consequently, the commutations equation of motion and singularization interface are fundamentally intertwined. From the singularization theorem one inductively derives the commutation equations of motion, and from the commutation equations one deductively derives the singularization theorem.

The commutation equations of motion for implications agree with the conversion theorem, which is most clearly seen in their **Lagrangian**-free formulation:

$$\frac{d<q|\rightarrow|q>}{dq} = \frac{d<q|\leftarrow|q>}{dq} = <q|[\leftarrow,\rightarrow]|q>$$

which is the finite exponentiation of

$$\frac{d<q|a|q>}{dq} = \frac{d<q|a^*|q>}{dq} = <q[a, a^*]q>$$

where a^* and a are the creation and annihilation operators. Here we benefit from the duality of the second-quantized field and matrix logic:

$$[a, a^*] = [\rightarrow, \leftarrow].$$

There are also other hidden dualities which reveal themselves in conjunction with the logical **Lagrangian**:

$$\mathbf{H}[a, a^*] = \mathbf{H}[\vee, \wedge],$$

where the canonical pair of field operators can be swapped with the pair of canonical logical operators, even though

$$[a, a^*] \neq [\vee, \wedge].$$

One has freedom to choose between a quantum or logical design of the momentum operator, $\mathbf{M} = \mathbf{H}[a, a^*]$ or $\mathbf{M} = \mathbf{H}[\vee, \wedge]$, where the commutator $[a, a^*]$ is Hermitian but the logical commutator $[\wedge, \vee]$ is non-Hermitian. Clearly

$$\frac{d<q|\rightarrow|q>}{dq} = <q|[\theta\rightarrow, \rightarrow]|q>$$

and

$$\frac{d<q|\rightarrow|q>}{dq} = <q|[\vee, \theta\vee]|q>.$$

(C)

Finally, it is possible to join the supersymmetric and commutation equations of motion, replacing the spin operator with the commutator $[\mathbf{H}, \rightarrow]$:

$$-\frac{d|q>}{dq} = \mathbf{H}[\mathbf{H}, \rightarrow]|q>.$$

RELATION TO THE HEISENBERG EQUATION

The dynamical logical equations of motion which were derived above are fractal copies of the Heisenberg quantum-mechanical equation of motion

$$i\hbar\frac{<\psi|A|\psi>}{dt} = <\psi|[A, H]|\psi>,$$

where A is a quantum-mechanical operator and H is the quantum Hamiltonian. Indeed, replacing in the logical equations the logical state by a wavefunction and the differentiation versus truth-value by a differentiation versus time, instead of the logical state space $\mathcal{C}$ we will find ourselves in the Hilbert space of quantum mechanics.

Significantly, the commutation equations of motion are satisfied exclusively by non-Hermitian logical operators, which are observable in $\mathcal{C}$, but are not observable quantum-mechanically. This fact once again draws our attention to the critical line separating the unique physics of thoughts from

quantum mechanics, where observables are necessarily self-adjoint. Another important implication, stemming from the results we obtained, is a link emerging between physical time and logical truth-value. Starting from the singularization theorem, one derives the commutation equation of motion, which in turn is connected to the Heisenberg equation, effectively deriving quantum equations from the formalism of matrix logic. This is a major conceptual shift, because we are typically used to thinking that it is logic which should follow from quantum mechanics and not the other way round. The logical twins of the Heisenberg equation and the Schrödinger equation, which we established earlier, powerfully tell us otherwise. **Quantum mechanics comes from logic, no less than logic can be read off from Hilbert space.** In the true spirit of the chicken-and-egg problem, consciousness explores the circular causation between the quantum laws and the dynamical laws of matrix logic. This fact allows us introduce a mixed quantum-logical description in which cognitive dynamics is controlled on the one hand by logical operators and on the other by the quantum spin operator. Just as to the quantum evolution operator U conserves the scalar product in Hilbert space:

$$< \psi|\psi> = <\psi|U^{*}U|\psi>,$$

the fundamental logical evolution operators, negation N and time ▲, conserve the logical inner space product:

$$<q|q> = <q|\,NN\,|q>, \qquad <q|q> = <q|\blacktriangle\blacktriangledown|q>.$$

The fact that the product of the two unitary evolution operators is the quantum spin unitary operator:

$$N\blacktriangledown = \sigma_z,$$

entails the equations of motion, involving the triplet constellation of the fundamental operators:

$$-\frac{d|q>}{dq} = HN\blacktriangledown|q>$$

and

$$-\frac{d|q>}{dq} = H\sigma_z N|q>.$$

Theorem

$$(\frac{d}{dq}+\blacktriangle)\,|q> = \sigma_z|q>.$$

Proof

$$-\frac{d|q>}{dq} = H\blacktriangle\,|q> = (I-N)\blacktriangle\,|q> = (\blacktriangle-\sigma_z)|q>$$

$$-\frac{d|q>}{dq} - \blacktriangle\,|q> = -\sigma_z|q>$$

$$(\frac{d}{dq}+\blacktriangle)\,|q> = \sigma_z|q>$$

□

The logical equations of motion and the quantum equations have a strikingly similar structure. However, a differentiation with respect to time in the Schrödinger and Heisenberg equations has to be replaced by a differentiation with respect to truth-value. The truth development of a logical state **leads to** the creation of a coherent logical superposition, as opposed to a wavefunction, which **is** a coherent superposition. The logical equations of motion make the best of both worlds: while being quantum-mechanical in form, they are at the same time deterministic and classical in essence. While quantum-mechanical measurement destroys coherences, in logical motion coherences emerge out of a decoherent statistical mixture. The two dynamical processes exist side by side, conditioning each other, and it is no longer possible to divide the system into its constituent parts, which are two sides of a coin. Whereas the quantum equations represent **statements** about the **states** of a physical system, the fundamental theory of the brain is based on a formalization in which the **statements** of the system are simultaneously the **states** of the system. The opposing views of Bohr and Einstein should not necessarily be mutually exclusive in the descriptions of the cognitive brain which combines knowledge about reality and reality itself. Information is physical, physics is logical. In this framework consciousness can be treated as a dynamical process creating the conditions under which a logical state is guided to a chosen point in cognitive phase space while a corresponding quantum state converges to a dual point in Hilbert space. Matrix theory is revealing the unexpected links between logic and quantum theories. The logical equations of motion can be expressed directly in the quantum-theoretic language of the creation and annihilation operators:

$$-\frac{d|q>}{dq} = H(a - a^*)|q>, \qquad \frac{d<q|\rightarrow|q>}{dq} = <q|[a, a^*]|q>.$$

By joining the logical and field concepts we are unravelling fundamental aspects of 'thinking matter'. A flaw of existing theories of the brain is that they tend to distinguish between physical and cognitive states, which is the wrong way to go. The physical processes in our brains give rise to logic, the same logic which we consequently employ to describe the physical processes.

COGNITIVE CONSTANTS OF MOTION

A quantum observable A is an integral or a constant of motion when it commutes with the Hamiltonian:

$$[A, H] = 0.$$

According to the Heisenberg equation

$$i\hbar\frac{<\psi|A|\psi>}{dt} = <\psi|[A, H]|\psi> = 0,$$

where A is without an explicit **time** dependence. An important example of a constant of motion is the full energy of a system:

$$[\,E, H\,] = 0.$$

Exploring a similar line of reasoning, we can define a cognitive constant of motion, associated with a logical operator **L**. In accordance with the logical equation of motion, we require that a corresponding cognitive observable has no explicit **truth** dependence and commutes with the **Lagrangian H**:

$$[\,\mathbf{L}, \mathbf{H}\,] = 0.$$

and

$$\frac{\partial <q|\mathbf{L}|q>}{\partial q} = <q|[\mathbf{L},\mathbf{H}]|q> = 0\ .$$

Excluding the trivial case of contradiction, represented by the empty operator **0**, these two requirements uniquely identify the logical operator of tautology **1** as a fundamental cognitive constant of motion:

$$\frac{d <q|\mathbf{ONE}|q>}{dq} = <q|[\,\mathbf{ONE}, \mathbf{H}\,]|q> \equiv 0.$$

The right side of the equation is nil since the tautology **1** and the **Lagrangian H**, sharing the pair of common eigenstates $|S+>$ and $|S->$, commutes: $[\mathbf{1}, \mathbf{H}\,] = 0$, and orthogonal: $\mathbf{1} \bullet \mathbf{H} = \mathbf{0}$. The left side is nil because universally

$$<\mathbf{ONE}> \equiv 1,$$

and the derivative of a constant is 0.

Tautology and its complement (contradiction) are the only completely effective cognitive constants of motion. Indeed, commutation with the **Lagrangian** is necessary but not sufficient for a logical operator to be a cognitive constant of motion. For example, the **Lagrangian H** shares simultaneous eigenstates and consequently commutes with the bisymmetric **NOT** and its complement **YES**:

$$[\,\mathbf{NOT}, \mathbf{H}] = \mathbf{0}, \qquad [\,\mathbf{YES}, \mathbf{H}] = \mathbf{0},$$

hence, the right side of the commutation equation of motion will vanish. But the left side of the equation vanishes only at the point $q = \frac{1}{2}$, corresponding to the undecided state of renormalized coherent superposition $\frac{1}{2}|S_+>$. Since, due to commutation, the right side of the equation is zero, while the left side is differentiated, the problem of a constant of motion is effectively reduced to the problem of determining an extremum of a logical function:

$$\frac{d <q|\mathbf{NOT}|q>}{dq} = 2 - 4q = 0, \qquad \frac{d <q|\mathbf{YES}|q>}{dq} = -2 + 4q = 0,$$

$$q = \frac{1}{2}.$$

The identity and negation are at extremum at the same point but for identity it is a minimum, and for negation it is a maximum.

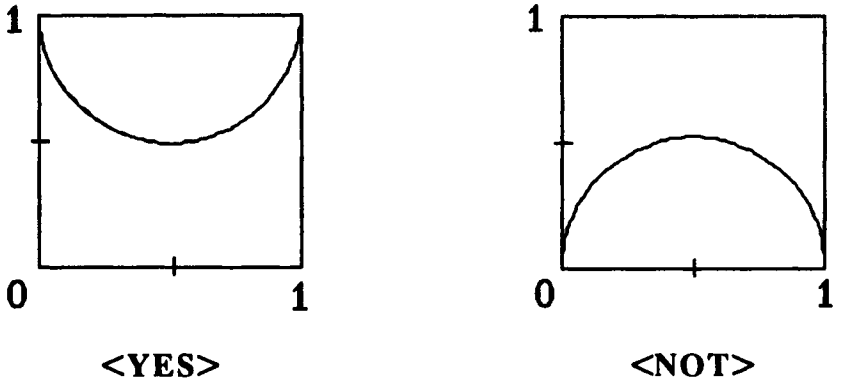


Fig. 40
The identity and negation logical functions.

The negation and affirmation is the only pair of complementary operators, both in Ω, which commute: [**NOT**, **YES**] = **0**, and since generally

$$[L, \bar{L}] = [L, 1],$$

replacing the complement by the logical constant of motion, we derive:

$$[\mathbf{NOT}, \overline{\mathbf{NOT}}] = [\mathbf{NOT}, \mathbf{ONE}].$$

The formalism of the cognitive constant of motion uncovers the important physical properties of the thought process. The problem of extrema is also one of the most important problems in differential logic [Ref 92]. We know that the bisymmetric **YES** and **NOT** and the asymmetric **IMPLY** and **IF** all have their extremum at $q = \frac{1}{2}$, the same point at which the expectations of the annihilation a and creation a^* operators are at their maximum. Although for asymmetric operators the commutation equation of motion is universally satisfied, they never commute with the **Lagrangian** and never are cognitive constants of motion. There are also the biasymmetric **TRUE** and **FALSE** which are distinguished by neither satisfying the commutation equation of motion nor having cognitive constants of motion. However, due to the singularization theorem, **TRUE** and **FALSE** satisfactorily replace the asymmetric operators **IMPLY** and **IF** in the commutation equations of motion:

$$\frac{d\langle q|\rightarrow|q\rangle}{dq} = \langle q|[\mathbf{FALSE}, \mathbf{H}]|q\rangle, \quad \frac{d\langle q|\leftarrow|q\rangle}{dq} = \langle q|[\mathbf{H}, \mathbf{TRUE}]|q\rangle.$$

The commutation equation for symmetric operators is a special case of the extremum equations. Although the symmetric operators **AND** and **OR** do not commute with **H**,

$$[\wedge, \mathbf{H}] \neq \mathbf{0}, \qquad [\vee, \mathbf{H}] \neq \mathbf{0},$$

the expectation value of these commutators is universally nil:

$$<q|\,[\wedge,\mathbf{H}]|q> \;=\; <q\left(\begin{matrix}0&0\\-1&1\end{matrix}\right)-\left(\begin{matrix}0&-1\\0&1\end{matrix}\right)q> \;=\; <q|\mathbf{TIME}|q> \;=\; 0,$$

$$<q|\,[\vee,\mathbf{H}]|q> \;=\; <q\left(\begin{matrix}-1&1\\0&0\end{matrix}\right)-\left(\begin{matrix}-1&0\\1&0\end{matrix}\right)q> \;=\; <q|\mathbf{TIME}|q> \;=\; 0.$$

Since the right side of the corresponding equations of motion is zero, they fix the extremum conditions for the left side of the equation:

$$\frac{d<q|\wedge|q>}{dq} \;=\; 2q \;=\; 0, \qquad q \;=\; 0,$$

$$\frac{d<q|\vee|q>}{dq} \;=\; 2-2q \;=\; 0, \qquad q \;=\; 1,$$

which are Boolean limits, consistent with the definition of **AND** and **OR** as $\min(p, q)$ and $\max(p, q)$ respectively.

A thinking system, left without a goal, should settle at equilibrium. According to the commutation equation of motion, a logical constant of motion must commute with the **Lagrangian**. Because in such an event the derivative of a logical function must be zero, a logical constant of motion simply identifies a condition for an extremum of the logical function, be it a minimum or a maximum. A distinction between constants of motion in logic and in physics is reflected in the fact that the problem of a logical constant of motion is part of a greater problem of the optimization of a logical function. An intelligent logical system tends to attain a stable (dis)equilibrium-state of tautology, choosing it out of all possible truth states. Tautology is an optimal covariant extremum, which clarifies the meaning of the logical constant of motions. It commutes both with **YES** and **NOT**, providing for their simultaneous observation

$$[\,\mathbf{1}, \mathbf{NOT}] = [\,\mathbf{1}, \mathbf{YES}] = \mathbf{0}.$$

SPECTRAL THEOREM

The spectral representation of the logical momentum for thinking with a continuous spectrum is given by

$$\tilde{\mathbf{M}} \;=\; \mathbf{Const.}\int_{0}^{1} dq|q>(-\frac{d}{dq})<q|$$

where the matrix constant is the integral inverse of the logical density Ω and the isomery of **YES** and **NOT**:

$$\mathbf{Const.} \;=\; \Big(\int_{0}^{1} dq\Omega\Big)^{-1} \;=\; 2(2\mathbf{YES}-\mathbf{NOT}).$$

Proof

Multiplying on the right by $<q|$ the equation of motion for the logical momentum

$$-\frac{d}{dq}|q> = M|q>$$

we get

$$M|q><q| = -\frac{d}{dq}|q><q|.$$

Collecting one of the outer products into a logical density matrix Ω, and shifting the differentiation operator to the centre to act on the bra, instead of the ket, we have

$$\tilde{M}\Omega = |q>(-\frac{d}{dq})<q|.$$

The transposition on the left hand side is due to the shift of the differentiation operator, which in fact is equivalent to the transposition of the right hand side. Since Ω is symmetric,

$$\Omega\tilde{M} = |q>(-\frac{d}{dq})<q|.$$

We want the momentum operator alone on the left hand side, but Ω, as we know, is singular,

$$\det\Omega = \det\begin{pmatrix} \bar{q}^2 & \bar{q}q \\ q\bar{q} & q^2 \end{pmatrix} \equiv 0,$$

and cannot be eliminated by dividing the equation by Ω. In order to expurgate the momentum we change to the integral form, integrating both sides of the equation on the logical interval:

$$\int_0^1 dq\,\Omega\tilde{M} = \int_0^1 dq|q>(-\frac{d}{dq})<q|.$$

The integral of the density matrix is also a density matrix with fixed entries which are the iqon and quark logical charges:

$$\int_0^1 dq\,\Omega = \int_0^1 dq\begin{pmatrix} \bar{q}^2 & \bar{q}q \\ q\bar{q} & q^2 \end{pmatrix} = \begin{pmatrix} q - q^2 + \frac{q}{3} & \frac{q}{2} - \frac{q}{3} \\ \frac{q}{2} - \frac{q}{3} & \frac{q}{3} \end{pmatrix}\Bigg|_0^1 = \begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{pmatrix}.$$

It is easily verified that the overall trace of both the density matrix Ω and its logical integral are normalized:

$$\Sigma\Omega \equiv \Sigma\int_0^{\dagger} dq\Omega \equiv 1,$$

but while Ω is singular, its logical integral is not:

$$\det\int_0^1 dq\,\Omega \;=\; \det\begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{pmatrix} \;=\; \frac{1}{12}$$

Hence, the inverse can be obtained, and is calculated to be the difference of the isomeries of the affirmation and the negation:

$$\begin{pmatrix} \frac{1}{3} & \frac{1}{6} \\ \frac{1}{6} & \frac{1}{3} \end{pmatrix}^{-1} \;=\; 2(2\mathbf{YES} - \mathbf{NOT}).$$

Thus the spectral representation of the logical momentum is

$$\tilde{\mathbf{M}} \;=\; 2(2\mathbf{YES} - \mathbf{NOT})\int_0^1 dq|q><(-\frac{d}{dq})<q|$$

which is confirmed by direct calculation:

$$\tilde{\mathbf{M}} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix} = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}\int_0^1 dq|q><(-\frac{d}{dq})<q| = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}\int_0^1 dq\begin{pmatrix} \bar{q} \\ q \end{pmatrix}(1 \quad -1)$$

$$= \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} q - \frac{q^2}{2} & q - \frac{q^2}{2} \\ \frac{q^2}{2} & -\frac{q^2}{2} \end{pmatrix}\Big|_0^1 = \begin{pmatrix} 4 & -2 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}.$$

□

Consequence

The logical momentum operator is a matrix fixed point for the algebraic sum of the isomery of tautology $\mathbf{n} = \mathbf{n}1$ and the unitary logical operator of affirmation/negation:

$$\left(\begin{pmatrix} \mathbf{n} & \mathbf{n} \\ \mathbf{n} & \mathbf{n} \end{pmatrix} + \mathbf{YES}\right) \cdot \mathbf{M} \;=\; \mathbf{M}$$

and

$$\left(\begin{pmatrix} \mathbf{n} & \mathbf{n} \\ \mathbf{n} & \mathbf{n} \end{pmatrix} - \mathbf{NOT}\right) \cdot \mathbf{M} \;=\; \mathbf{M},$$

where $|n| = 0, 1, 2, \ldots$

Consequence

In the sense of generalized eigenfunctions the logical momentum is a matrix eigenstate of the logical **Lagrangian.**

Proof

Adding the two equations above and deleting the vanishing terms, we deduce:

$$\left(\begin{pmatrix} n & n \\ n & n \end{pmatrix} + \mathbf{YES}\right) \bullet \mathbf{M} + \left(\begin{pmatrix} n & n \\ n & n \end{pmatrix} - \mathbf{NOT}\right) \bullet \mathbf{M} = 2\mathbf{M}$$

$$\left(2\begin{pmatrix} n & n \\ n & n \end{pmatrix} + \mathbf{YES} - \mathbf{NOT}\right) \bullet \mathbf{M} = 2\mathbf{M}$$

$$(\mathbf{YES} - \mathbf{NOT}) \bullet \mathbf{M} = 2\mathbf{M}$$

$$\mathbf{H} \bullet \mathbf{M} = 2\mathbf{M}.$$

Synergy increases the logical momenta in the multiple of 2 quanta, which is a fundamental eigenvalue of the **Lagrangian H**, the 2 matrix. This important result is less surprising if we observe the fact that the logical momentum is a Cartesian product of two copies of the eigenstate of **H**.

LOGICAL POLYMERS

Matrix logical fixed points are important for the understanding of cognitive phenomena. In this section our objective is to gain new insight into the relation between logical fixed points and duality. The matrix fixed point which we consider here is the logical **Lagrangian H**, appearing in the logical equations of motion. Since

$$\mathbf{H} \bullet \mathbf{AND} \bullet \mathbf{H} = \mathbf{H}$$

instead of the equation of motion

$$-\frac{d|q>}{dq} = \mathbf{H} \blacktriangle |q>$$

we can write

$$-\frac{d|q>}{dq} = \mathbf{H} \wedge \mathbf{H} \blacktriangle |q>$$

and generally by iteration

$$-\frac{d|q>}{dq} = \ldots H \wedge H \wedge H \wedge H \wedge H \blacktriangle |q>$$

where

$$\ldots H \wedge H \wedge H \wedge H \wedge H$$

is an example of a logical **polymer**, comprising recurring elemental units, similar to the macromolecular polymers in organic chemistry.

Theorem

In the logical equation of motion, a logical polymer for the given direction of time is equivalent to the dual logical polymer for the inverse direction of time.

Proof

It will be sufficient for our purposes if we consider the elemental unit of the conjunctive polymer in the equation of motion

$$-\frac{d|q>}{dq} = H \wedge H \blacktriangle |q>$$

The transformation of the conjunctive logical polymer to the dual disjunctive polymer requires that we replace **AND** with the dual **OR**.
Then, since

$$H \vee H = -H,$$

we obtain the logical equation of motion

$$-\frac{d|q>}{dq} = -H \vee H \blacktriangle |q>$$

which with the inversion of the direction of time reduces to

$$-\frac{d|q>}{dq} = H \vee H \blacktriangledown |q>$$

The resulting equation is equivalent to the original one, provided we replace logical connective operators by their duals, simultaneously changing the direction of time to the opposite.

PHASE TRANSITION

A two-dimensional system of Ising spins on a square lattice is known to be equivalent to a dual spin system on an inverse lattice. If the original system is at temperature T, the dual system has temperature 1/T. This symmetry allows high and low temperatures to be exchanged, with a phase transition occurring at the critical temperature, T=1. The Ising duality has nonlinear analogues in field theory where instead of a classical spacetime we have the corresponding two-dimensional field theory. By mirror symmetry two apparently distinct spacetimes turn out to be equivalent, corresponding to equivalent two-dimensional field theories. A duality, generalizing the duality

of the Ising lattice, exchanges the physics of the large with that of the small, suggesting that one cannot compress a string loop below a certain fundamental length scale, 10^{-33}cm. As one tries to compress the loop to probe small distances, the string expands instead and the winding and vibrating modes are swapped. To an outside observer, however, the different physical origins of the dual winding and vibration states are not apparent, both states yielding ultimately the same energy levels, which we interpret as particles.

A counterpart of the Ising symmetries in logic concerns the dual normal forms. The generalized DeMorgan theorem of classical logic states that any logical formula can be reduced to a disjunctive or conjunctive normal form. If we swap the truth-values 0 and 1 of logical variables and replace connectives by their dual we obtain an equivalent logical formula:

$$\bigvee_{i=1}^{n}\left(\bigwedge_{j=1}^{m} x_{ij}\right) = \overline{\bigwedge_{i=1}^{n}\left(\bigvee_{j=1}^{m} \bar{x}_{ij}\right)}$$

The dual expression is equivalent to the original one, and an external observer will detect no differences between the two logics. New dualities, generalizing this DeMorgan duality of classical logic, are manifest in matrix logical polymers. A phase transition from a logical polymer to an equivalent dual polymer occurs when the number of connective operators, **AND** or **OR**, is even:

$$\mathbf{H \wedge H \wedge H \equiv H \vee H \vee H}$$

We gain new insight into this invariance by treating it as a fixed point. The **Lagrangian H** is a matrix fixed point, if acted upon by the condensed pairs $(H\wedge H\wedge)$ or $(H\vee H\vee)$:

$$\mathbf{(H\wedge H\wedge)\cdot H = (H\vee H\vee)\cdot H = H}$$

reduced to

$$\mathbf{(AND\text{ - }OFF)\cdot H = (NOR\text{ - }ON)\cdot H}$$

which is

$$\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\mathbf{H} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}\mathbf{H}$$

The condensed pairs are unitary logical transforms of each other

$$\theta\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix} = \mathbf{N}\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\mathbf{N} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}$$

and the fixed point is invariant under the unitary transformation:

$$\theta\begin{pmatrix} 0 & -1 \\ 0 & 1 \end{pmatrix}\mathbf{H} = \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}\mathbf{H}$$

We determined earlier that in matrix logic the classical DeMorgan duality transform is the unitary transformation with complement:

$$\mathbf{L} = \overline{\theta \mathbf{L}^*}$$

where $\mathbf{L}$ is a logical operator and $\mathbf{L}^*$ is its dual, in particular:

$$\wedge = \overline{\mathbf{N}\vee\mathbf{N}}, \qquad \vee = \overline{\mathbf{N}\wedge\mathbf{N}}$$

Further, in m(atrix) theory we identified a quantum-mechanical version of the DeMorgan dualities which took the form of the commutation relation

$$[N, \wedge] = [N, \vee]$$

or more generally

$$[N, L] = [N, L^*].$$

The commutation DeMorgan duality is straightforwardly translated to include the energy operator:

$$[\wedge, H] = [\vee, H].$$

The unitary rotation of the dual operators changes logical time to its opposite but keeps the equalities intact:

$$[\theta\wedge, H] = [\theta\vee, H].$$

One surprising duality symmetry involves spin. The spin orthogonal transform carries out the duality exchange of the difference of the canonical pair of operators, swapping **AND** and **OR**:

$$\sigma(\wedge - \vee)\sigma = \vee - \wedge.$$

We also learned about the differential duality relation with their partial and signed symmetry:

$$\langle \bar{p}|\wedge|q\rangle' = - \langle \bar{p}|\vee|q\rangle'$$

The impressive career of the DeMorgan laws is reaching new heights. One realises that these laws are not simply a mathematical abstraction but fundamental in origin, with direct physical implications. We derived the duality law which includes time:

$$H \wedge H\blacktriangle = H \vee H\blacktriangledown$$

and the universal time-independent duality exchange:

$$H\wedge H\wedge H = H\vee H\vee H$$

In an actual logical system, such as the brain, the realization of the phase transition due to the duality exchange fundamentally depends on the evenness or oddness of a logical polymer. Another prediction concerns the isoenergetic character of the thinking process. While for ordinary physical systems energy is proportional and grows with the increase in the number of the constituent particles, the energy fixed point confirms that logic is isoenergetic, retaining an invariant energy level, independently of the number of operational units or computational particles involved. For the serial logical polymer we calculate a single solution

$$E = \langle H \rangle = \langle \prod_{i=0}^{n} (H\wedge)_i H \rangle$$

and for the parallel logical polymer the two solutions, positive

$$E = \langle H \rangle = \langle \prod_{i=0}^{2n} (H\vee)_i H \rangle$$

and negative

$$E = -\langle H \rangle = \langle \prod_{i=0}^{n} (H\vee)_i H \rangle$$

where n = 0, 1, 2, . . .
The **AND** polymer is stable and remains invariant if a single elemental unit $H\wedge$ is detached from a polymer

$$H\wedge H\wedge H = H\wedge H = H$$

If a logical problem contracts, the **AND** polymer remains unaffected whether it shrinks by single elemental units or by their pairs. But the **OR** polymer is stable only against the detachment of the two elemental units $H\vee H\vee$, and if a single unit is lost, the polymer's **Lagrangian** will undergo a global phase reversal:

$$(H\vee H\vee)\cdot H = H$$

but

$$(H\vee)\cdot H = -H$$

For the invariancy of the **OR** polymer the expansion or condensation in pairs must be preserved. Logical polymers can be uniformly and invariantly changed, provided that matrix particles are added or subtracted in pairs. Parity is essential, just as the Cooper pairs are essential for supercurrent.
The polymers are stable against the duality exchange of logical operators, provided their number is even. Since **AND** is serial and **OR** is parallel, this result generalizes our earlier result concerning the serial-parallel symmetry of the energy operators:

$$H \cdot H = H + H$$

Since

$$H\wedge H\wedge H = H\vee H\vee H = H,$$

discarding one **H** and joining the **AND** polymer and **OR** polymer we get:

$$H\wedge H\wedge H\vee H\vee H = H$$

Observing that

$$\wedge H\vee = \backslash, \qquad \vee H\wedge = /,$$

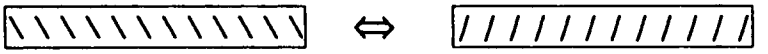
and

$$H\backslash H = -H, \qquad H/H = -H,$$

maintaining the parity of pairs, we can construct two equivalent logical polymers

$$H\backslash H\backslash H = H/H/H = H.$$

The changeover from **ON** to **OFF**, just like the changeover from **AND** to the dual **OR**, yields a different but equivalent polymer:



When **ON** and **OFF** are recast as the quantum field creation a^* and annihilation a operators, the phase transition from a^* to a has clear logical consequences. The fundamental transitive tautology:

$$((x \to y) \wedge (y \to z)) \to (x \to z)$$

entails the syllogism implication

$$x \to z.$$

Reducing the syllogism to the matrix string, we find the creation operator a^* inside the string:

$$\overline{<x|\to|y><x|a^*|z><y|\to|z>}.$$

If the phase transition from a^* to a occurs, the syllogism implication changes into the converse implication

$$z \to x.$$

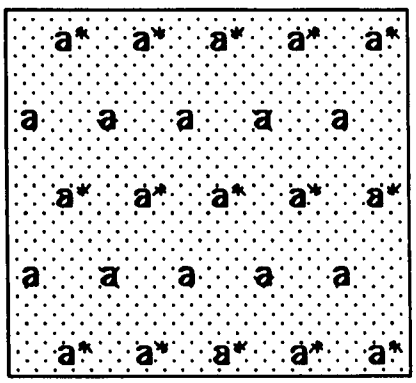
Explicitly:

$$\overline{<x|\to|y><x|a|z><y|\to|z>}$$
$$= \overline{\overline{<x|1><0|y>}<x|0><1|z>\overline{<y|1><0|z>}}$$
$$= \overline{(<x|0><1|z> - <x|1><0|y><x|0><1|z>)(1-<y|1><0|z>)}$$
$$= \overline{<x|0><1|z>(1-<y|1><0|z>)} = \overline{<x|0><1|z>} = <z|\to|x>.$$

The converse implication which we obtained is not a tautology, but becomes such, provided all creations are changed into annihilations. Then we derive

$$((z \to y) \wedge (y \to x)) \to (z \to x).$$

Given a logical polymer or more generally a logical lattice, and swapping the creation a^* and annihilation a operators, we get a dual lattice:



Apparently, what in psychology is called 'backward reasoning' in matrix logic can be naturally treated as a phase transition between dual logical lattices.

COGNITIVE DIAGRAMS

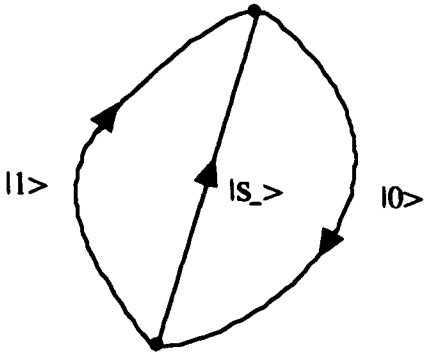
CONNECTING LOGIC TO THE VACUUM

In this section we introduce the method of cognitive diagrams which establishes a correspondence between the mathematical expressions of matrix logic and the thought process, connecting logic to the vacuum. The method of cognitive diagrams is close in spirit to the method of spacetime diagrams, suggesting a connection between logical inferences and quantum field models. A cognitive diagram describes the birth and annihilation of interacting matrix logical particles, just as spacetime field diagrams describe the creation and annihilation of actual quantum particles.

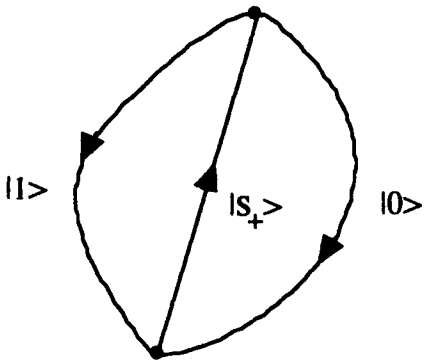
We begin with the fundamental case of the Liar paradox:

$$\langle x|\to|\bar{x}\rangle\langle|\bar{x}|\to|x\rangle \equiv 0$$

whose self-referential structure is described by the closed loop diagram:



or equivalently,



In the first diagram the true state $|1\rangle$ joining with the coherent superposition $|S_-\rangle$ results in the false state $|0\rangle$:

$$\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}1\\-1\end{pmatrix}=\begin{pmatrix}1\\0\end{pmatrix}$$

which feeds back, closing the vacuum loop. In the second diagram the superposed state $|S_+\rangle$ dissolves into the true and false states which are fed back, giving rise to $|S_+\rangle$:

$$\begin{pmatrix}0\\1\end{pmatrix}+\begin{pmatrix}1\\0\end{pmatrix}=\begin{pmatrix}1\\1\end{pmatrix}$$

The changeover from $|S_-\rangle$ to $|S_+\rangle$ reverses the orientation of the state $|1\rangle$ to its opposite. These logical diagrams resemble the diagram for the interaction of nucleons n and p, mediated by mesons in the prequark model of strong interactions. The neutron n and proton p can be put in correspondence to logical states $|0\rangle$ and $|1\rangle$, and the meson triplet $\{\pi^-, \pi^+, \pi^0\}$ to the superbits triplet $\{|S_-\rangle, |S_+\rangle, |S_0\rangle\}$ respectively. The Liar reads naturally as a logical vacuum polarization which is an important mechanism of logical inference generally. In inductive thinking vacuum singularities are commonly present, covariantly introducing new logical variables. For example, the reduction of the tautologies

$$((x \to y) \to x) \to x \equiv 1, \qquad (x \to (y \to x)) \to x \equiv 1,$$

to the canonical form of a fundamental product:

$$\overline{\overline{\langle x|0\rangle - \langle x|1\rangle\langle y|0\rangle\langle x|0\rangle}\langle x|0\rangle} \equiv 1$$

$$\overline{\overline{\langle x|1\rangle\langle y|1\rangle\langle x|0\rangle}\langle x|0\rangle} \equiv 1$$

explicitly reveals the presence of the vacuum terms:

$$\langle x|1\rangle\langle y|0\rangle\langle x|0\rangle \equiv 0$$

and

$$\langle x|1\rangle\langle y|1\rangle\langle x|0\rangle \equiv 0$$

where y is a new arbitrary logical variable, confined to the vacuum loop, and therefore having no impact on the resulting value of the logical expression:

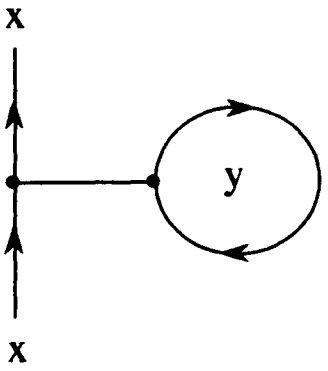


Fig. 41
Connecting a new vacuum variable to a logical string.

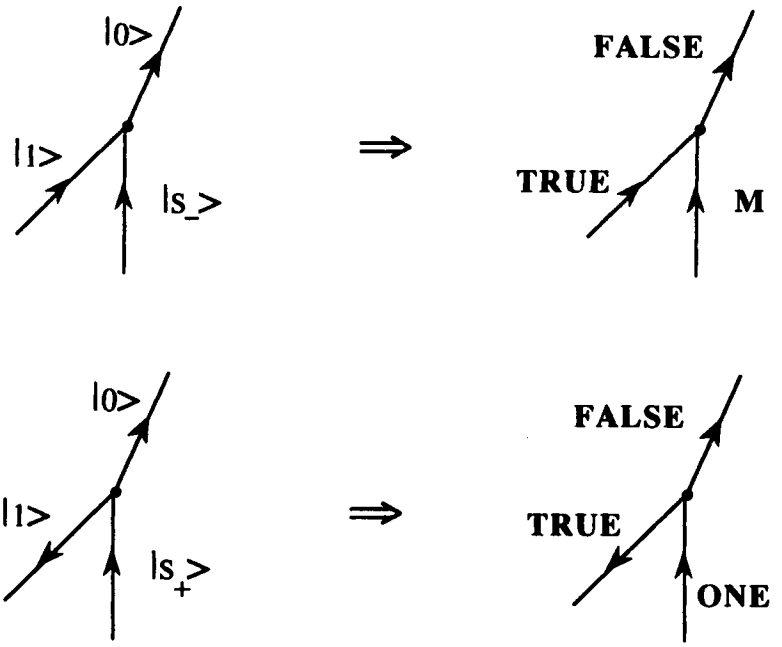
MATRIX NETS AND THE SELF-SOLVING BRAIN

The thinking brain is distinguished by its self-solving capability. Faced with a difficult intellectual or mathematical problem, we are sometimes wise to put it aside and 'forget' about it by engaging in some other unrelated activity or even going to sleep. After a while, suddenly, the brain may 'wake up' on its own, and a solution of the problem, which seemed to be difficult and unattainable, is there, found by itself without any controlled and deliberate effort. In order to gain insight into this self-solving capability of the brain or more generally into the mechanism of intuition, we reformulate the theory of cognitive diagrams in terms of operators. In this interpretation of cognitive diagrams we are not concerned with the logical states, which can be absolutely random and fuzzy, but with operators residing on links in diagrams. With each logical link we now associate a corresponding **knowledge** operator, whose eigenstate is the respective logical state. A cognitive diagram described in this manner is a **matrix net**, controlled by four singular operators:

{TRUE, FALSE, MOVE, ONE}.

Whatever the random impregnation of a diagram with normed states, known or unknown, may be, the knowledge operators will terminate the random walk

send the diagram into a required eigenstate. **Every logical thought is an eigenstate of a corresponding matrix net.** For example, from the **state** diagrams shown below we pass to the corresponding **operator** diagrams:



RENORMALIZATION

The matrix net of the brain is a complex many-particle system comprising billions of interacting nibbles. When a diagram includes a sufficiently large number of particles, we have a suitable quantum system for realization of the thought process as dictated by the cognitive delta theorem. To invariantly expand a cognitive diagram to include an effectively infinite number of interacting nibbles, we rely on the renormalization process. A nibble is subjected to various multiplicative, additive, commutative and anticommutative renormalizations. For example, we can renormalize **TRUE** and **FALSE** multiplicatively to include implications:

$$\mathbf{TRUE} = \mathbf{ON \cdot IMPLY}, \qquad \mathbf{FALSE} = \mathbf{OFF \cdot IF}$$

which in turn can be factorized into operators of the minor basis set:

$$\rightarrow = \mathbf{N \cdot OR}, \qquad \leftarrow = \mathbf{OR \cdot N}$$

or extended as the anticommutators in the major basis set $\{\mathbf{N}, \wedge, \vee\}$, where negation is transferred from **OR** on **AND**:

$$\rightarrow = \{\mathbf{N \cdot AND}, \mathbf{OR}\}, \qquad \leftarrow = \{\mathbf{AND \cdot N}, \mathbf{OR}\}$$

This reduces to

$$\rightarrow = \{\mathbf{OFF}, \vee\}, \qquad \leftarrow = \{\mathbf{ON}, \vee\}$$

which are the two key anticommutators among the set of **40** anticommutators which renormalize the implications.

The logical renormalization schemes are full of beautiful and complex pictures. Numerous possibilities present themselves. We can nest implications into a higher-order expression, making use of the above-mentioned anticommutators. We can also further increase the virtual "cloud" around a nibble, realizing **ON** and **OFF** as commutators:

$$\mathbf{ON} = [\mathbf{TRUE}, \leftarrow], \qquad \mathbf{OFF} = [\mathbf{FALSE}, \rightarrow]$$

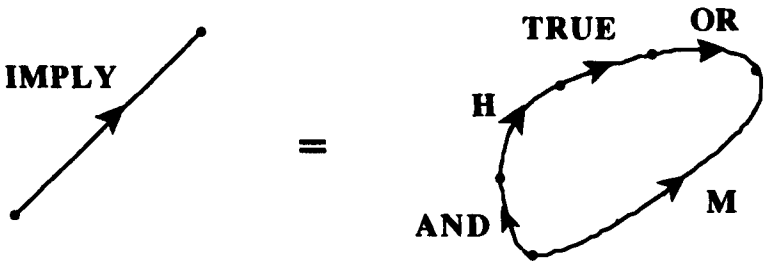
Further still, we can extend **TRUE** and **FALSE** as the products

$$\mathbf{TRUE} = \wedge\vee = \mathbf{AND}\bullet\mathbf{OR}, \qquad \mathbf{FALSE} = \uparrow\downarrow = \mathbf{NOR}\bullet\mathbf{NAND}$$

The diagram grows rapidly, including more and more renormalizations:

$$\rightarrow = \{\mathbf{OFF}, \mathbf{OR}\} = \{[\mathbf{FALSE}, \rightarrow], \mathbf{OR}\} = \{[\uparrow\downarrow, \rightarrow], \mathbf{OR}\} = \ldots$$

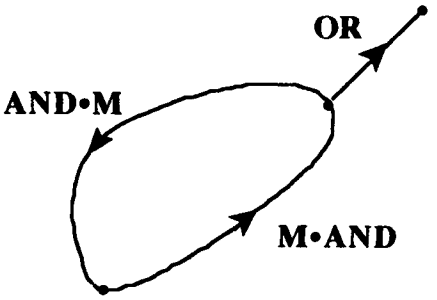
One among many possible diagrams for implication is given below:



which is due to the following renormalization process:

$$\begin{aligned}
\rightarrow &= \mathbf{N}\vee = [\,\mathbf{M}, \wedge\,]\vee = [\,[\,\wedge\vee, \mathbf{H}\,], \wedge]\vee \\
&= ((\wedge\vee\mathbf{H} - \mathbf{H}\wedge\vee)\wedge - \wedge(\wedge\vee\mathbf{H} - \mathbf{H}\wedge\vee))\vee \\
&= \wedge\vee\mathbf{H}\wedge\vee - \mathbf{H}\wedge\vee\wedge\vee - \wedge\wedge\vee\mathbf{H}\vee + \wedge\mathbf{H}\wedge\vee\vee \\
&= \perp\mathbf{H}\perp - \mathbf{H}\perp - \perp\mathbf{H}\vee + \wedge\mathbf{H}\perp\vee \\
&= \mathbf{0} - \mathbf{H}\perp - \mathbf{0} + \wedge\mathbf{H}\perp\vee \\
&= -\mathbf{H}\perp + \wedge\mathbf{H}\perp\vee \\
&= \wedge\mathbf{H}\perp\vee + \mathbf{M}
\end{aligned}$$

Quite legitimately we could have proceeded differently from the second row, arriving at the following different but equivalent diagram, which ends on **OR**:



This diagram is due to the following renormalization

$$
\begin{aligned}
\rightarrow \; &= \mathbf{N} \vee = [\mathbf{M}, \wedge] \vee = [\,[\wedge \vee, \mathbf{H}\,], \wedge] \vee \\
&= ((\wedge \vee \mathbf{H} - \mathbf{H} \wedge \vee) \wedge - \wedge(\wedge \vee \mathbf{H} - \mathbf{H} \wedge \vee)) \vee \\
&= (\wedge \vee \mathbf{H} \wedge - \mathbf{H} \wedge \vee \wedge - \wedge \wedge \vee \mathbf{H} + \wedge \mathbf{H} \wedge \vee) \vee \\
&= (\mathbf{0} - \mathbf{H} \wedge \vee \wedge - \mathbf{0} + \wedge \mathbf{H} \wedge \vee) \vee \\
&= [\wedge, \mathbf{H} \wedge \vee\,] \vee \\
&= [\,\mathbf{M}, \wedge\,] \vee .
\end{aligned}
$$

OPEN ENDED DIAGRAMS AND VACUUM TRAPS

The world is built up of mutual recursion. But at some stage an **evolving** dynamical system must be able to break out of the loop. Before we show that certain cognitive diagrams terminate, resisting further renormalization, it is useful to reflect on a recursive grammar of relativistic quantum mechanics. A quantum particle is defined through reference to all other particles, whose definitions in turn depend on the first particles. This is a recipe of the Feynman diagrams which is well known. To calculate a scattering amplitude and to evaluate the propagator one integrates over all possible paths of particles branching and rejoining and then sums over all possible sequences of interaction by which a given particle can evolve into a final particle, including itself. Many, in fact an infinite number of diagrams contribute to the same physical process. What this means is that a renormalized particle is in fact surrounded by a cloud of virtual particles, each of which is also associated with its own virtual cloud, and so on to infinity. This complexity gives rise to quite complicated interactions which allow deep nesting of fundamental structures inside each other. This is very much like the recursive grammar of human languages, which suggests a correspondence between cognitive diagrams and

Feynman diagrams. The knowledge operators that act in the cognitive diagrams can be renormalized with the aid of **80** logical nibbles which include the knowledge operators themselves. Nibbles are nested inside each other according to the fundamental 'grammar' of matrix logic. And just like a quantum particle a logical nibble is defined through reference to other nibbles, whose definitions in turn depend on the first particles. When nibbles are extended in the quantum basis {a, a*}, the cognitive diagram can be renormalized with commutative and anticommutative loops. In the figure below we show a renormalization for the logical momentum.

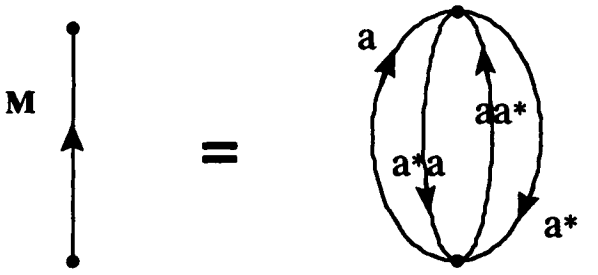


Fig. 42
Momentum operator extended in the quantum basis { a, a*}.

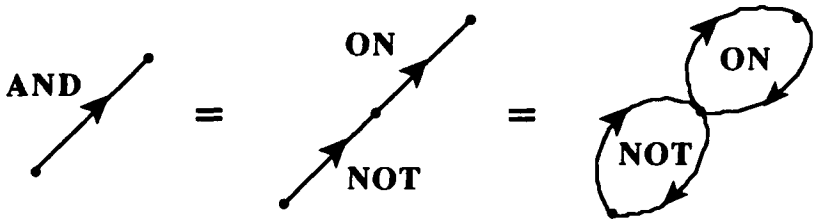
Each of the operators in the quantum basis set has **100** matrix logical products, **40** anticommutator and **136** commutators knots: $a^* = [A, B]$ and $a = [\tilde{B}, \tilde{A}]$. There are more knots than products. The a and a* links in the cognitive diagrams are elementary acts of creation and annihilation, and any diagram is a composition of the basic ones. But a and a* themselves can be encoded as anticommutators and commutators of compound cognitive nibbles, which again can be re-encoded in terms of a and a*, giving rise to complex self-referential concatenation. Computer simulations have shown that certain expansions, fall into the vacuum trap loops, recursively repeating themselves.

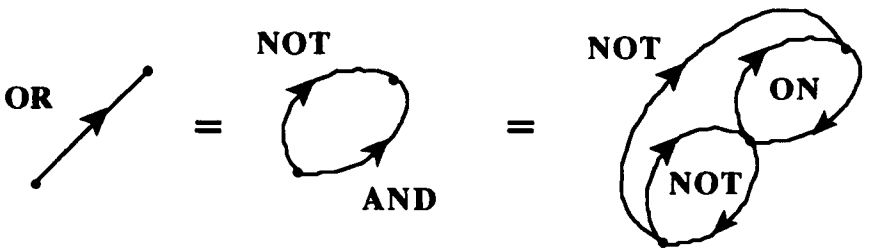
Consciousness is a global process, in which one typically does not descend to the level of the basic codes but thinks with complex cognitive nibbles. Some of the global nibbles are knots but there nibbles which cannot be renormalized as loops. Neither anticommutator or commutator knots exist for the canonical operators **AND** and **OR**, nor for **TRUE** and **FALSE**. This is contrasted with the momentum operator, which has **80** commutator and **40** anticommutator renormalizations, and **ONE**, which has **24** anticommutators. One of the central problems of the cognitive diagram method is to determine whether a diagram will halt without a loop. **AND** and **OR** and **TRUE** and

FALSE give rise to open-ended diagrams, the ones we use for speaking or thinking out loud. Although there are no knots for **AND** and **OR**, if these are factorized or algebraically re-expressed in terms of nibbles which are knottable, for example,

$$\mathbf{AND = NOT \cdot ON}, \qquad \mathbf{OR = NOT + AND}$$

then the renormalization process may continue, as shown in following diagrams:





Because **AND** and **OR** cannot be renormalized as anticommutators or commutators, one can distinguish renormalizable and nonrenormalizable basis sets. The basis sets {**N**, **AND**} and {**N**, **OR**} are commutatively nonrenormalizable because there are no commutators for **AND** and **OR**. But the basis sets {**N**, **ON**} and {**N**, **OFF**} are commutatively renormalizable because either of the operators in the sets is knottable. The basis set {**NOT**,**IF**} is renormalizable commutatively with regard to **NOT** and anticommutatively with regard to **IF**.

Understanding the code of individual nibbles and their small constellation in cognitive diagrams is only a first step towards understanding the complex collective behaviour of large matrix nets. Incorporating individual operations into a global description of the intelligence code is a challenging problem, the 'genome' project for the brain which has only started.

QUANTUM-LOGICAL COMPUTER

The laws of physics present no barrier to reducing the size of computers until bits are the size of atoms.

Richard Feynman

The development of logical theories is often driven by problems arising in applications, most importantly in cognitive studies and computer science. Matrix logic was created as an attempt to conquer the complexities of the thinking brain but in a surprising turn of events it has found application in quantum computing. In the same way as classical Boolean logic constitutes the basis of the conventional digital computer, matrix logic represents the theoretical foundation of the quantum logical computer. A proposal for a quantum computer was first made by the author in 1970 [Ref 78] and in 1982 [Ref 82] and by Feynman in 1985 [Ref 21]. In 1983 I sought a recommendation from Feynman towards the publication of my article in Foundations of Physics, providing him with my previous works on quantum computing and Nigtevegt's article [Ref 52]. A few years later Feynman himself published an article in Foundations of Physics. The idea of a quantum computer emerged ahead of its time, when neither technologists were ready to accept it nor quantum theorists prepared to accommodate this new paradigm. It was either ignored or passionately rejected, both by professionals and dilettantes. The suggestion that we might make computational use of the unruly quantum particles and individual atoms and eventually could fabricate a quantum computing device was received with much disbelief and scepticism. The conclusion that just one gram of quantum matter, the size of a grain of sand, could achieve and even exceed the computational power of a brain or powerful computer were treated as science fiction. Connecting a new theory by simple logical steps to existing knowledge is a challenge. Typically new ideas are adamantly rejected; the thinking animal, possibly due its genetic make-up, is scared of change. Truth is the object of science but not always of scientists. The main objection to the idea of a quantum computer in the 1970s and 1980s was that a computation cannot be effective in conditions of quantum uncertainty. Dogma held that a computer is a classical device that

must be flawless and has nothing to gain by calling or relying on the action quantum or the Schrödinger equation. Quantum effects were seen not as an advantage but as an obstacle which would render information processing useless and data storage impractical. Paradigm shifts take place slowly. As late as 1997 some experts called matrix theory an 'artifact' and quantum computer a 'fantasy'. But step by step the 'bizarre' idea of reducing computer bits down to the atomic level began to make headway. The scepticism slowly began to fade, and many of critics themselves jumped on the bandwagon. A success has a thousand fathers. The identification of superposed bits or superbits in matrix logic [Ref 85, 88], followed by the realization that matrix logic can offer a polynomial regime of computation [Ref 69], have led to a major shift in attitude. Quantum superposition and entanglement, which at the beginning of the computer era were regarded as a detrimental and undesirable feature, not only turned out to offer new promise for computer technology, but also shed light on the foundations of quantum physics itself. The initial fear of the sceptics that rapid decoherence and the unavoidable quantum errors would render information processing in the quantum computer impractical has greatly diminished. With the development of self-correcting codes, we now know that it is possible to protect computation against errors in a quantum but not too quantum medium [Ref 94]. In the long run, fault-tolerance in quantum computers will be achieved intrinsically with gauge-invariant topological hardware insensitive to local change [Ref 87]. In the decades to come computers will undergo a revolutionary leap in performance, intelligence and size. Manipulating quantum information, computers will simulate many natural processes, most importantly encompassing the intelligent brain, which has too many variables for any classical computer to handle. A change from today's silicon technology to new quantum chips will then be compared to a change from the Charles Babbage analytical machine to the modern supercomputer. In the same way as at present we take a Personal Computer for granted, in the rapidly approaching postsilicon era we will take for granted a Personal Supercomputer, a PS the size of desktop PC.

SUPERBITS

We all have bits that we cherish

John Preskill

Quantum computing is a recent logical theory which was born within the framework of matrix logic, although many still erroneously think of it as a brainchild of quantum mechanics. In a classical computer the fundamental units of information is the Boolean bit which can take two values: 0 or 1. Any computation can be encoded in bits and reduced to the basic operations that invert bits depending on the values of other bits. This scalar approach has been profoundly shaken up in matrix theory, in which a different kind of

information bits, superbits, has been unexpectedly discovered [Ref 74]. The important development which prepared the ground for this discovery was the tensorization of logical values, whereby in addition to **truth-value** we introduced **false-value**, and instead of scalar logic employed a two-dimensional formalism in which logical operations are executed on the matrix vector gates. At first glance the addition of a false-value to a truth-value, which are complementary, seems redundant. It indeed would be such if we treated truth-value and false-value one at a time. But in matrix logic, as we have seen, these are quantities in their own rights and join together according to the mapping from Boolean bits that are simply complementary to Boolean states that are also orthogonal:

$$1 \quad \Rightarrow \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad 0 \quad \Rightarrow \quad |0\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

While Boolean logic deals exclusively with scalar quantities, matrix logic is essentially and fundamentally a tensor, which simultaneously accounts both for the true and the false, neither being dispensable, very much as space and time in relativity theory are treated jointly as an undivided spacetime. A mapping from scalars to vectors led us immediately to a quite extraordinary and initially baffling result. At the intermediary steps of computation the logical operators commonly evolve the Boolean states $|0\rangle$ and $|1\rangle$ into superposed states, which seem meaningless from perspective of the probability theory but are essential for computation. Instead of evaluating some logical string serially step-by-step, for example

$$\langle x|a^*|y\rangle + \langle x|a|y\rangle + \langle x|a^*a|y\rangle,$$

we can compactify the string and computes globally

$$\langle x|L|y\rangle,$$

where

$$L = a^* + a + a^*a.$$

Such global computation is advantageous but would typically introduce coherent superpositions. For example

$$\langle 0|\mathbf{OR}|1\rangle = (1\ 0)\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (1\ 0)\begin{pmatrix} 1 \\ 1 \end{pmatrix} = 1$$

$$\langle 0|\mathbf{OR}^{-1}|1\rangle = (1\ 0)\begin{pmatrix} -1 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (-1\ 1)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

$$\langle 0|\mathbf{AND}|1\rangle = (1\ 0)\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = (0\ 0)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = 1$$

where the nonnormalized superposed states

$$\begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad -\begin{pmatrix} 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

appear at the intermediary steps of computation.

The intellect works in strange way. What is so clear today, was quite confusing and incomprehensible not such a long time ago. I recall how in 1963

Kolmogorov, a guru on probability theory, who was generally positive towards matrix operator logic, nonetheless advised me to "get rid" of the strange states without normalization. Breaking up the rules of classical probability, matrix logical computations give rise to the superbits which have no interpretation in probability theory. Matrix logic has revealed that information may exist not only in the form of the orthogonal Boolean true and false but also in the form of the coherent superpositions, the superbits

$$|s_{\pm}\rangle = |0\rangle \pm |1\rangle.$$

Although the existence of coherent bits was first proved theoretically by the author in matrix logic many years ago, it is only recently that computer scientists and theoretical physicists have begun to realize their full potential for high-tech applications in context of quantum mechanics. In explicitly deriving coherent superposition the computational reform accomplished in matrix logic achieves its utmost significance. Superbits provide a computer with a new, powerful mode of operation and a computational speed-up which are unattainable on a classical machine.

Using an internal degree of freedom it is possible to drive a quantum particle into a superposed state. This is quite different from a classical set-up in which we try to put two macroscopically distinct states into quantum superposition, a Schrödinger cat state. Because the ability to manipulate superposed bits from either side of the quantum-classical divide is essential for the I/O computer interface, both procedures must be considered for implementing quantum computers. The quantum-mechanical approach to computing takes its clue and inspiration from the superposition postulate of quantum mechanics. Basically, the idea of coherent superposition is borrowed from the quantum formalism and transferred, with a lesser or greater degree of justification, to the domain of computer science. This postulatory approach is typical of many works on quantum computation which go from quantum mechanics to logic. The matrix logical approach is deductive in nature: it begins within a traditional framework and Boolean logic alone, that is with no nonclassical premisses. No *ad hoc* coherent superposition is introduced. No reference to, or use of the formalism of quantum mechanics is made initially, nor required. However, when the scalar Boolean bits are bijectively mapped to the two-dimensional matrix logical phase space and then are acted upon by logical operators, they naturally evolve into superbits. The logical states that are classical in the initial conditions and separable are transformed into catlike states, which then can be dealt with equally well in logical terms or quantum-mechanically in terms of wavefunction. Quantum information is realized through nonorthogonal states which cannot be completely separated into orthogonal subspaces. To prepare an ensemble of such nonorthogonal states more information is required than could be retrieved from the ensemble through quantum measurement. The capacitor in a computer circuit cannot be in a coherent superposition of charged and uncharged states, but a quantum particle can reside in a coherent

state, for example, of spin-up and spin-down. A spin-1/2, existing in the orthogonal states $|1\rangle$ and $|0\rangle$, will also be found in the coherent superposition

$$|\psi\rangle = \alpha\,|1\rangle \pm \beta\,|0\rangle,$$

where

$$|\alpha|^2 + |\beta|^2 = 1.$$

Much as with spin, matrix logic demands that an intelligent complex system which resides in the logical states $|0\rangle$ and $|1\rangle$ will generally be forced into the superpositions $|S_\pm\rangle = |0\rangle \pm |1\rangle$ and into the empty state $|S_0\rangle$. In quantum-theoretic terms this entails the crossover from the classical to the quantized regime, where information degrees of freedom can no longer be treated as classical variables but must be represented by operators. The logical coordinates are identified by matrices; when the matrices commute their simultaneous eigenvalues are discrete Boolean bits in the traditional sense. This is why in matrix logic we have replaced the ordinary logical coordinates by corresponding operators, much like what happened to the coordinates of classical Hamilton phase space when quantum mechanics emerged.

It is important to re-emphasize that matrix logic did not borrow the idea of coherent superposition from quantum mechanics. The superbits emerged within the logical theory itself and are different from the notion of quantum bits or qubits, resulting from unitary evolution. Qubits refer to microphysical quantum-mechanical systems in which complex amplitudes are essential for the description. The matrix superbits, on the other hand, make use only of real numbers. Nonetheless the quantum bits and superbits can be joined congruently into a single matrix theory with duality, naturally connecting classical and quantum information. In this study we employ terms 'superbits' and 'quantum bits' interchangeably. Preference, however, must be given to superbits, as opposed to the commonly used term 'quantum bits'. Strangely enough, some physicists are unaware that 'bit' is short-cut for a 'binary digit' and therefore 'quantum bit', which by no means binary, is a contradiction in terms.

THE COMPUTER AS A QUANTUM PARTICLE

The universal computer is a finite-state machine executing algorithms or programs and there are various ways to describe a computer. In classical computing theory the characterization of a computer is accomplished by identifying its input. Computation is a succession of logical operations, making use of the basic Boolean gates, **NOT**, **AND** and **OR**, from which elaborate

computations can be constructed. The conventional computer is a physical realization of Boolean algebra, and the limitations of computers are due to the limitations of two-valued Boolean logic, which forms the core of the computer operations and is deployed both in programming and in the design of computer circuits. The reliance on Boolean logic is responsible for another drawback of the digital computer. The operations on digital computers do not obey the laws of arithmetics. The governing criteria in the design of the circuits are speed of execution and minimization of circuit complexity. Faithfulness to the laws of arithmetics is not even a consideration, let alone a design criterion. In fact, each concrete computer is known to violate these familiar laws, even for the integers. A computer is far more likely to obey the laws of Boolean algebra at the logic-design level than the laws of arithmetics at the instruction-execution level. Classical computers can never become mathematical machines because they are constrained to operate on representations. In contrast the quantum computer is envisioned as a fundamental machine which will operates on real objects (quantum particles).

Because the majority of Boolean operations are irreversible, the classical computer is an irreversible machine. In virtue of unitary evolution, the quantum computer is based on reversible logic, and therefore can be characterized as a group in which the inverse operation is universally defined. A reversible unitary machine can be played backwards, retracing the steps of computation in reverse order. Each operation induces a transformation in the machine, for which there always exists a reverse unitary operation, and if the two are performed one after the other the machine eventually returns to the initial state. Thus the identity operation is everywhere defined, and every pair of operations which are inverses of each other cancels out. The description of a computer as a group has interesting consequences. In fundamental terms the classical computer and the quantum computer can be distinguished as Abelian and non-Abelian machines respectively [Ref 88]. The Abelian computer makes use of Boolean logic which is irreversible because the inverse operation seldom applies. The non-Abelian computer makes use of matrix logic in which the order of operations is important and we can conceive and implement reversible unitary operations. The fact that the quantum computer is unitary deserves special attention. The model for quantum computation involves the unitarization of matrix logic. In quantum computing we are confronted with the need to make a selection among ill-defined alternatives or to draw inferences from the incomplete data. This may require nonmonotonic logic, which changes abruptly rather than continuously. Therefore, the effective quantum problem-solvers must incorporate an adaptive reversibility which enables them to restart computation from new premisses, when the need arises. From this point of view it is the processors capable of accommodating reversible gates that are of interest for the quantum computer. However, an entirely reversible machine is clearly of little use: it must be interfaced with a deterministic macroframe and, therefore, must include both Abelian and non-

Abelian components. Matrix logic, which combines Abelian and non-Abelian computations, is ideally suited for that.

Until recently computer theory was developing independently of the fundamental principles of physics. Matrix logic has had a major unifying effect, linking logic and quantum mechanics, which appear as two interconnected parts of one general matrix theory. A state of a quantum computer is a fundamental product of matrix logic, recast as a tensor product in Hilbert space. There is a correspondence map between the formalisms of quantum mechanics and matrix logic, which enables one freely to change over from logical computations that are tractable to quantum computations that are not, and vice versa. In this manner matrix logic provides a theoretical foundation for the quantum computer, linking logic directly with fundamental concepts at the heart of quantum mechanics.

MATRIX LOGIC	QUANTUM MECHANICS
$\|q\rangle$	$\|\psi\rangle$
$\|q\rangle = \Sigma q_i \|e_i\rangle, \ \bar{q} + q = 1$	$\|\psi\rangle = \Sigma c_i \|\psi_i\rangle, \ \Sigma \|c_i\|^2 = 1$
$\langle L\rangle = \langle q\|L\|q\rangle$	$\langle Q\rangle = \langle\psi\|Q\|\psi\rangle$
NOT, $\langle q\|NN\|q\rangle = \langle q\|q\rangle$	U, $\langle\psi\|U^*U\|\psi\rangle = \langle\psi\|\psi\rangle$
$H = -\ln N, \ [H, N] = 0$	$i Ht = -\ln U, \ [H, U] = 0$
$[q, M]\|q\rangle = \|q\rangle$	$[x, p]\|\psi\rangle = i\hbar\|\psi\rangle$
$\langle q\| M \|q\rangle$	$\langle\psi\|\sigma_z\|\psi\rangle$
$-\frac{d\|q\rangle}{dq} = M\|q\rangle, \ M^+ \neq M$	$-i\hbar\frac{d\|\psi\rangle}{dx} = p\|\psi\rangle, \ p^+ = p$
$L\|q\rangle = \lambda\|q\rangle$	$H\|\psi\rangle = E\|\psi\rangle$
$\langle M\rangle^2 = \langle H\rangle$	$p^2 \sim E$
$-\frac{d\langle q\|\rightarrow\|q\rangle}{dq} = \langle q\|[\rightarrow, H]\|q\rangle$	$i\hbar\frac{d\langle\psi\|Q\|\psi\rangle}{dt} = \langle\psi\|[Q, H]\|\psi\rangle$
Ω, $\mathrm{Tr}\,\Omega L = \langle q\|L\|q\rangle$	ρ, $\mathrm{Tr}\,\rho Q = \langle\psi\|Q\|\psi\rangle$

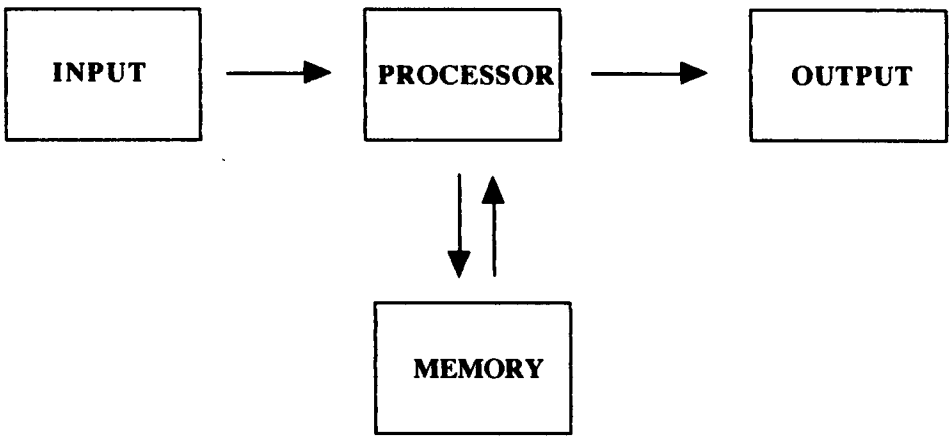
Table 4
A correspondence map linking matrix logic and quantum mechanics

The quantum-mechanical and matrix-logical approaches to computing, although different in their first principles, both lead to the concept of the quantum computer. As already discussed early, there exists a fundamental link between the Hilbert space of quantum mechanics and matrix-logical phase space: the first is the complex square root of the second, and the second is the inner product of the first:

$$\sqrt[i]{Ç} = H \qquad \text{and} \qquad <H|H> = Ç.$$

Matrix logic naturally motivates a quantum-mechanical approach to the computer, reaching out to deep areas of information theory and quantum mechanics. Quantum computation derives its new powers from exploring coherent superposition, the quantum bits. The discovery of quantized superbits and the fact that the laws of physics present no barrier to miniaturizing computers until bits are the size of atoms or particles have far-reaching implications for computer science. It becomes possible to treat a computer meaningfully as a quantum particle, 'moving' in a Hilbert phase space from some initial to some final state [Ref 84]. A solution is a particular state in which the 'particle' ends up, very much as an actual particle in quantum mechanics ends up at some energy level. A computation is then a path along a quantum 'trajectory' and a computing problem a 'potential barrier' the particle has to cross over or to tunnel through, which only a quantum particle can do. We can get the particle into a desired state not only by supplying it with additional momentum or rotating its phase, but also by lowering the potential barrier. Since wavefunction of the quantum particle is a coherent superposition of all possible states, the particle can reach a solution state instantly by quantum measurement, which forces the particle into a particular eigenstate.

The model of a computer as a quantum particle can be obtained through a limiting process. In the classical Von Neumann computer architecture there is the processor (**P**) or central processing unit (CPU), the memory unit (**M**), and the input/output (**I/O**) units; all are physically separate systems, wired up to each other in an organized fashion:



The processor and memory are linked by a communication channel. The input data flow to the CPU, which may call for additional data from memory and save them if the need arises. When the computation is completed, the result goes to the output or is stored in memory or both. Supercomputers make use of many processors acting in parallel, which allows several problems to be handled simultaneously. The use of parallelism has given rise to different parallel architectures, each with its own advantages and limitations. Taking a clue from the massive parallelism in the brain, performing millions of operations simultaneously, the ultimate goal is a decentralized distributive architecture, made up of a very large number of comparatively simple processors. The real and nontrivial problem consists in connecting the processors. While there is essentially only one way to construct a serial machine, there are infinitely many ways to build a parallel one. For once we open the door to more than one processor, we admit numerous possibilities for connecting processors and the problem of synchronization and coordination between the different units becomes acute and often intractable. In recent years various innovative ideas have surfaced. One original scheme is Daniel Hillis's Connection Machine, a precursor of the quantum machine, in which every processing element is attached to its own memory element. Still, the memory unit and the logical unit are separate.

With the limits of processing speed and miniaturization already in sight the next major leaps in computer power are expected to involve quantum computer architecture. When we compare existing computers with the brain, the fundamental differences become manifest. The brain operates as an undivided whole. As opposed to the computer, its memory and processing functions are not confined to separate units but are entangled, merging into a single information system [Ref 85]. Because classical physics cannot provide a basis for such a unified design of memory and processing, it is only natural to turn for help to quantum mechanics. The brain is a natural prototype for the quantum computer [Ref 74, 83]. The 'weird' laws of quantum mechanics allow us to place separate systems, in our case the processor and the memory, in a coherent superposition.

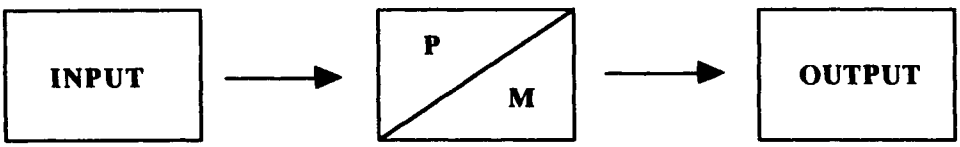
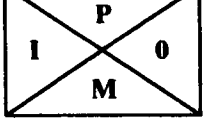


Fig. 43
Quantum computer architecture in which the processor and the memory are joined into a single P&M unit.

While in classical machines the processor and the memory are two separate systems, in quantum computer these are entangled into a single apparatus, the quantum processor [Ref 85]. Quantum entanglement gives the computer startling computing power and clearly distinguishes the quantum processor from a classical machine. The entanglement means that the state of the machine, which is an element of a tensor product of a finite dimensional Hilbert space, cannot be factored into the product of the states of the memory and the processor. Quantum operations are simultaneously processing operations and memory operations, disentangled only at the output end, which for all practical purposes operates as a quantum-measuring device. The bracket closure is a projection measurement which extracts information on the state of the machine [Ref 73]. The execution of programs on a quantum machine is attractive from the computational point of view, providing triple flexibility: the machine evaluates, stores and/or retrieves information, all at the same time. In this way quantum computer architecture inadvertently resolves the bottleneck problem which slows down the rate at which data can be stored or retrieved from memory. Since processor and memory are now unified into a single processor+memory unit, not requiring communication wiring, the computation in a sense represents a transformation of the memory of the machine. The question of a bottleneck is simply irrelevant because there is no other unit to which retrieved data must go, but a single quantum processor+memory unit(s). For more technical details see the patent [Ref 85]. Such an intelligent mode of operations is natural in the quantum domain but is in principle not attainable on a conventional computer. The architecture of a quantum computer is similar to the organization of the brain in which a total topological closure occurs and each logical unit is not only a memory unit but also an **I/O** unit. The logical elements of the brain are singularities closed into the 'information envelope':



In topological terms this is analogous to dimensional compactification, which can be depicted as gluing the opposite edges of a manifold, then shrinking the manifold, then gluing and shrinking again until we get a fundamental computational particle, as shown in Fig 44. Directing this topological transformation toward the microphysical limit, we would end up with an actual quantum spin which can serve as an elementary two-state unit for quantum computations [Ref 82, 52]. Many two-state systems are known to quantum physics. The attraction of spin as the basic unit lies in its unique double-group identity, which allows for a natural link to nonorientable topology.

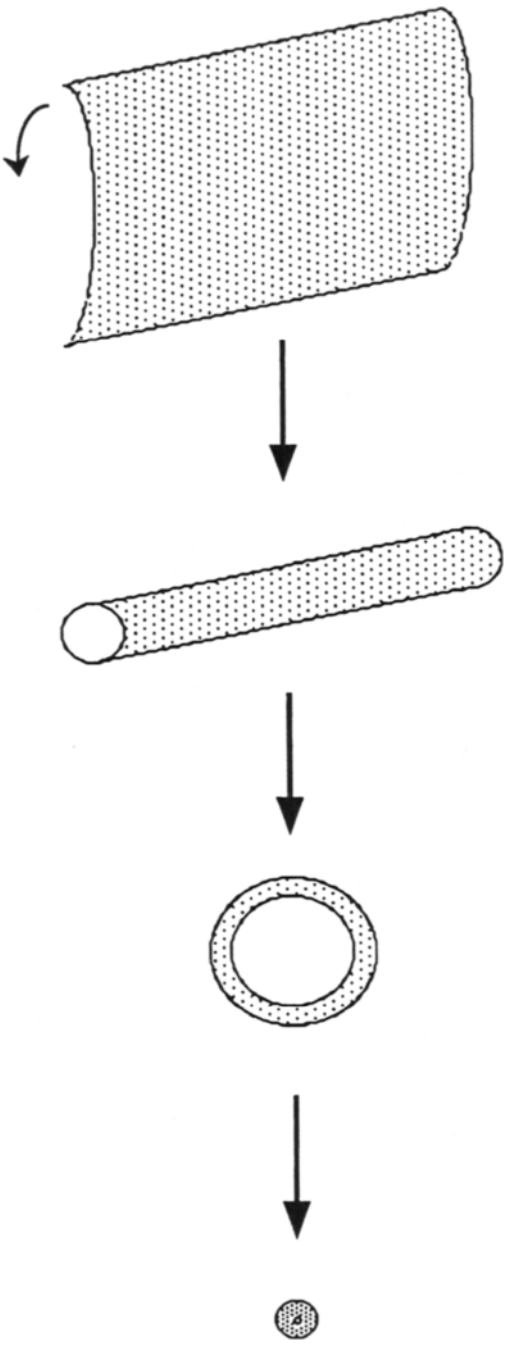


Fig. 44
Compactification of a manifold to a quantum particle.

QUANTUM TURING MACHINE

An abstract model of the computer is the Turing machine, the tale of which has been told many times. Any effectively computable process could be carried out on a Turing machine, roughly speaking, anything that can be computed by means of an algorithm can be computed using a Turing machine. The machine consists of the **HEAD** and an infinite **TAPE**, and the computation is realized through the simple **HEAD-TAPE** interactions.

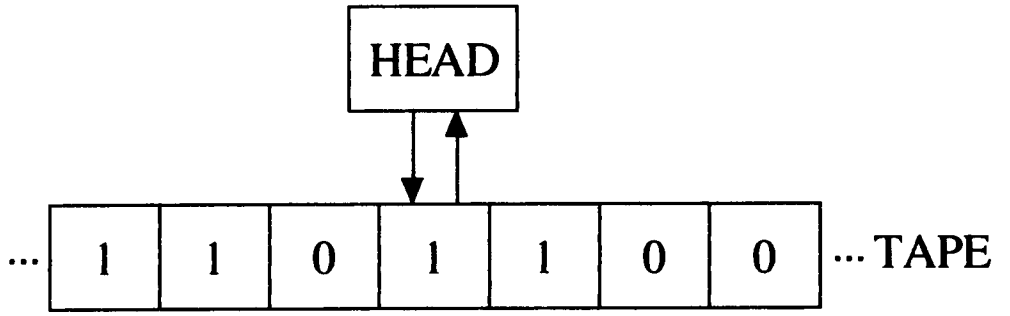


Fig. 45
Turing machine.

These interactions are embodied in the following actions of the machine. The **HEAD**, in some initial state, (1) **reads** the content of a cell, (2) **writes 0** or **1**, (3) **moves** to another cell (4) **changes** to a new state. The whole cycle repeats over and over again, if and until the machine halts.

In spite of its name, the Turing machine need not be an actual machine. It could be a piece of paper or an abstract algorithm. According to Turing, computation is independent of the physical nature of the system used to perform the computation. However, this might be true only for machines operating on the principles of classical physics. Although a classical machine can faithfully simulate a quantum one, a quantum computer can generate more powerful computational protocols that are unattainable on classical Turing machines. The notion of a Turing machine can be generalized into notion of a quantum Turing machine. A quantum Turing machine is the quantization of the classical machine, where the **HEAD** and **TAPE** are superposed. The **TAPE** now is a quantum tape, and the states of the machine are quantum states in Hilbert space spanned by the orthonormal basis of eigenstates. In a topological framework one can think of the **TAPE** of a quantum Turing machine as an infinite 'unilateral tape' which allows us to represent the superposed bits. A computation is a unitary transformation whose end result is determined by quantum measurement, reducing the 'unilateral tape' in coherent superposition to a classical bilateral tape with separable orthogonal eigenstates.

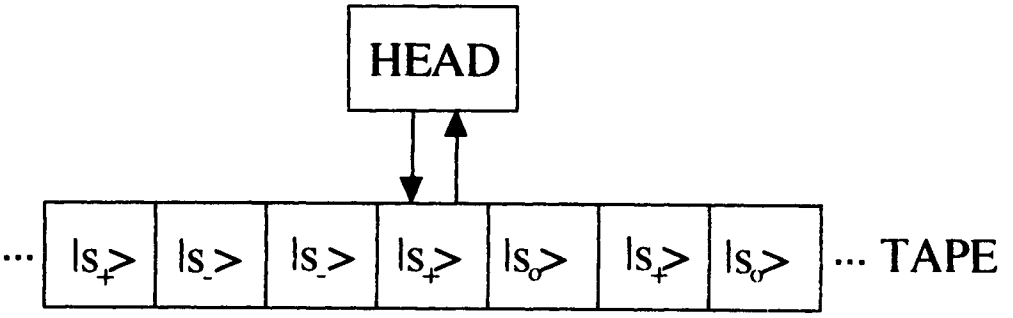


Fig. 46
Quantum **TAPE**

The bits in a classical Turing machine has only two Boolean states. The quantum **HEAD** has the capacity of evolving Boolean bits into coherent superposition. The operators residing in the **HEAD** of the machine are the matrix logical operators, among which there are the two-dimensional unitary operators, like **NOT**, which universally preserve the normalization of bits, and nonunitary operators, like **AND**, which preserve it weakly. Depending on the argument bit, **AND** may or may not destroy normalization. Nonunitary operators evolve classical bits into superbits, which, however, is not a sufficient condition for destroying a classical bit. The nonunitary operators, like **OR** or **IF**, may or may not evolve a classical bit into a coherent superposition, depending on the truth status of the input bit:

$$\mathbf{OR}|1\rangle = |s_+\rangle \quad \text{but} \quad \mathbf{OR}|0\rangle = |1\rangle$$

$$\mathbf{IF}|0\rangle = |s_+\rangle \quad \text{but} \quad \mathbf{IF}|1\rangle = |1\rangle$$

The requirement of unitarity for the preservation of the normalization of bits is unnecessarily restrictive. In matrix logic there are singular nonunitary operators, the edge operators **TRUE** and **FALSE**, which universally preserve the normalization of bits, mimicking some properties of the quantum creation and annihilation operators. The **TRUE** operator converts $|0\rangle$ to $|1\rangle$ but leaves $|1\rangle$ unchanged; the **FALSE** operator converts $|1\rangle$ to $|0\rangle$ but leaves $|0\rangle$ unchanged. Moreover, the nonunitary **TRUE** and **FALSE** operators evolve any fuzzy state into a definite Boolean state:

$$\mathbf{TRUE}|q\rangle = |1\rangle \quad \text{and} \quad \mathbf{FALSE}|q\rangle = |0\rangle.$$

The dynamical evolution of a quantum **TAPE** is fully described by the following five operations, remarkably all singular:

(1) **ACCEPT**: create a true state

$$\mathbf{TRUE}|x\rangle = |1\rangle$$

(2) **REJECT**: create a false state

$$\mathbf{FALSE}|x\rangle = |0\rangle$$

(3) **QUANTIZE**: create a superposed state

$$M|x\rangle = |S_-\rangle,$$

$$1|x\rangle = |S_+\rangle$$

(4) **PROJECT**: create a pure state

$$AND|S_+\rangle = |1\rangle,$$

$$NOR|S_+\rangle = |0\rangle$$

(5) **ERASE**: create an empty state

$$M^2|x\rangle = |S_0\rangle$$

$$0|x\rangle = |S_0\rangle$$

The Boolean zero 0 on the Turing tape corresponds to $|0\rangle$ on the quantum tape. But the quantum tape has also a different special kind of zero, the empty state $|S_0\rangle$, which has no counterpart on a classical machine.

The quantum Turing machine is important for the theory of the quantum computer but also for weighting the pros and cons of the claim that the brain is a computer. At the heart of the computer metaphor for the brain is the idea that thought processes can be formalized as algorithms and that inference is simply a computation. But can we actually model ourselves as algorithms? Closely related to this problem is the question, whether it is possible for an algorithm to create new information by itself. Due to spontaneous quantum gain a thinking system can output more information than existed at the start of computation. When a clear distinction is made between a quantum Turing machine, which supports the computer metaphor for the intelligent brain, and a classical Turing machine, where it breaks down, the brain emerges as a useful prototype for the quantum computer [Ref 78].

ATOMIC REGISTERS

A classical computer is a dynamical system which follows a path in a phase space of physical states. The content of the storage register, a binary string of Boolean bits such as

$$11100101000011\ldots$$

determines the state of the machine and changes into a new state each clock cycle in discrete time. When the computation comes to a halt, a certain binary number in the register gives the result. The transition map showing which state follows which is determined by the program and the inputs supplied at the start of the computation. Because the quantum computer manipulates quantum bits, the description of computer's register must be modified. The time evolution of the register is affected by the unitary operator **U** that takes input variables from state $|\psi_{in}\rangle$ to a state

$$|\psi_{out}\rangle = \mathbf{U}|\psi_{in}\rangle,$$

satisfying the group property

$$\mathbf{U}(t, t_0) = \mathbf{U}(t, t')\mathbf{U}(t', t_0)$$

for any triad of time t_0, t', t.
A quantum computer executes unitary transformations of the quantum states of the machine. For a system of **n** quantum bits, the operator describing the evolution of the computer will be represented by a $2^n \times 2^n$ unitary matrix. An important question is the factorization of the unitary matrix into basic components or quantum gates which then can be arranged in various ways to form quantum circuits.

The quantum registers can be modelled by a system of spins that has the form of an Ising interaction. For spin-1/2 we have a quantum system with four energy eigenstates:

$$|00\rangle, \quad |01\rangle, \quad |10\rangle, \quad |11\rangle$$

which are exactly the four discrete states of matrix logic. By coupling the spins and making use of spin-resonance techniques it is possible to tip spin-down to spin-up or vice versa. In the spin-spin interaction the spacing between every pair of energy levels in the four-level spectrum is distinct, and a tipping-pulse protocol allows specific resonances to be selected. Since the operations of quantum computers depend on maintaining strict phase coherence among the different bits, spins need to be kept isolated from the macroframe so that a coherent relation between phases of all degrees of freedom is maintained. Matrix logic provides the theoretical foundation of the quantum computer. There exists a direct mapping from the canonical reduction of matrix logic to atomic quantum registers. Every logical atomic bracket $\langle x|L|y\rangle$ can be put in correspondence to the following octet of spin-spin interactions:

$$|00\rangle, \quad |01\rangle, \quad |11\rangle, \quad |0S_+\rangle, |1S_+\rangle, \quad |0S_-\rangle, |1S_-\rangle, \quad |S_+S_-\rangle,$$

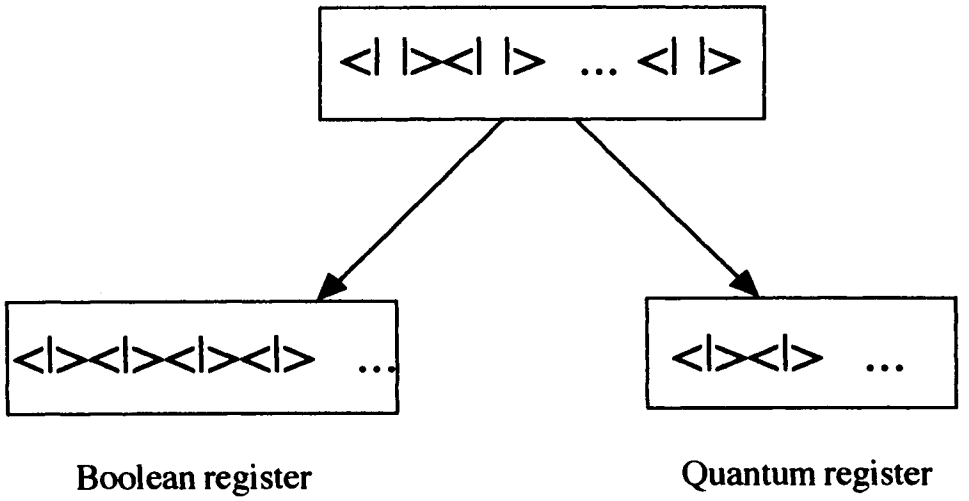
where $|0\rangle$ and $|1\rangle$ represent basis spin states and $|S_+\rangle$ and $|S_-\rangle$ the coherent superpositions. The above-mentioned spin-spin interactions are the basic elements of the quantum registers corresponding to the factorized terms of the fundamental product of matrix logic. Generally a fundamental product is the entanglement of logical codons, and so is a quantum register. While in a Boolean computer the bits in the register are classical elements which do not interfere with each other, in a quantum register this restriction is taken away. A bit can be in a state of coherent superposition and so can memory and the processor of the machine. In addition to that the quantum bits in the register can become entangled [Ref 74]. The coherent superposition and entanglement change the structure of the computational process, from a serial travel on a Boolean tree to a simultaneous transformation of the register, involving back and forth quantum tunnelling. While the initial and final states of the machine are identified classically as binary numbers, the description of the intermediary states requires quantum mathematics, where the orthogonal Boolean bits form coherent superpositions.

Generally, a register can be reduced to an entangled system of Boolean registers (**BR**) or quantum registers (**QR**). Factorized summands in a fundamental product of matrix logic correspond to these two types of registers. Switching between the operator-free mode and operator mode, the

computer switches between Boolean computation, where all steps are serially tractable, and quantum computation, which is collective superposition. A fundamental product, which is an output of the machine, can be written as an algebraic sum of registers:

$$\alpha = \sum^{n} BR_n \quad \text{or} \quad \beta = \sum^{m} QR_m$$

The number of operational registers, whether in Boolean or in quantum mode, is the same: n = m. However, the number of atomic products decreases if we pass from the classical to the quantum machine. In fact the number is cut in half: for **2k** Boolean products there will be **k** or less quantized products; hence a Boolean register requires twice as many gates as a corresponding quantum register:



During a computation process a matrix string densely compresses to a single inner product,

$$\langle|\rangle\langle|\rangle\langle|\rangle\langle|\rangle \Rightarrow \langle|\rangle\langle|\rangle \Rightarrow \langle|\rangle.$$

There are two different paths to achieve this compression: the inner product and the outer product spin-spin interactions, which, due to the associativity of the matrix product, must yield the same final result. The classical and nonclassical reductions of a register are obtained differently but the outcome is identical. For example:

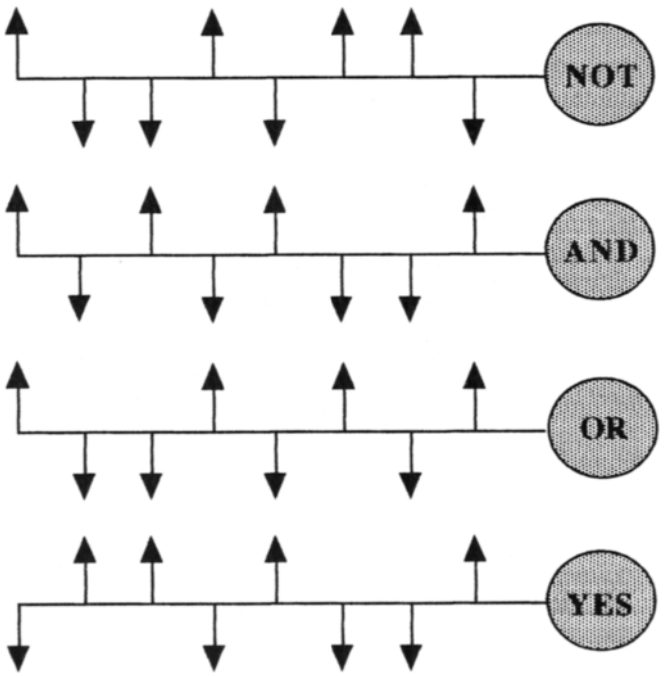
(a) quantum mode:

$$\langle 0|\rightarrow|1\rangle = (1\ 0)\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = (1\ 0)\begin{pmatrix}1\\1\end{pmatrix} = 1.$$

(b) classical mode:

$$<0|\rightarrow|1> \;=\; \overline{(1\ 0)\begin{pmatrix}0&0\\1&0\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix}} \;=\; \overline{(1\ 0)\begin{pmatrix}0\\1\end{pmatrix}(1\ 0)\begin{pmatrix}0\\1\end{pmatrix}} \;=\; \bar{0} \;=\; 1.$$

In the quantum mode a section of the register is in a coherent superposition of opposite truth-values, a state that has no classical counterparts. Not every quantum system is suitable for the performance of matrix computations. For example, zero-spin stable atoms are not available for spin-tipping interaction. However, the spins of a stable atom can be used as quantum memory carriers. The doubly-even self-dual code of the zero-spin oxygen atom is of particular interest because the octet of oxygen's spins is suitable for the codons of matrix logic [Ref 91, 25]. By ordering spins in a paticular fashion we can realize different logical operators, as shown below:



A codon in a register is a word with **n**=8 quantum bits, each bit being a spin with the two basis states labelled as |0> and |1>. The 2^n-dimensional Hilbert space of the register is an **n** term tensor product of H_2 pertaining to each quantum bit. A quantum state of the codon can be described by a density operator of the form

$$\rho(t) = \sum_{i,j=0}^{2^8-1} \rho_{ij}(t)|i><j|$$

where the computational basis $|i>$ is defined as a tensor product of quantum bit basis states:

$$|i>=|i_1>\otimes|i_2>\otimes|i_3>\otimes\ldots|i_8>$$

When mixed states are excluded and bits do not interact, we have a quantum system whose wavefunction is the product of individual wavefunctions

$$\Psi = \prod_{i=1}^{n}\psi(x_i) = \psi(x_1)\psi(x_2)\ldots\psi(x_n)$$

However, if interaction is switched on, the wavefunction of the system is no longer factorable,

$$\Psi \neq \prod_{i=1}^{n}\psi(x_i)$$

but becomes the entanglement of individual spins:

$$\Psi \neq \sum^{m}\prod^{k}\psi(x_k)$$

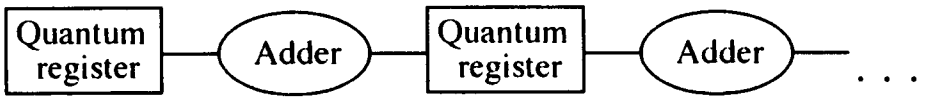
This is exactly the structure of a fundamental product in matrix logic:

$$\alpha \neq \sum^{m}\prod^{k}<x_i|L_n|y_j>_k$$

where the entanglement among the logical states prevents, in general, the channel matrix from being factorized:

$$L(x_1, x_2,\ldots, x_n, y_1, y_2,\ldots, y_n) \neq \prod_{i=1}^{n}L(x_i, y_i)$$

We take the factorized terms to be quantum registers, added classically. In this respect a quantum logical computer involves quantum computations entangled classically by the adder:



It follows that the fundamental product of matrix logic is the classical sum of the reduced quantum registers, added classically. The general conclusion that stems from this discussion can be summarized as follows. Whereas conventional computers are built around an adder that sums up Boolean bits, matrix quantum computers add quantum registers which are factorized multiplicative strings. The additive and the multiplicative machines can then be related to each other logarithmically.

DIFFERENTIAL QUANTUM ALGORITHMS

Many computational problems require an amount of computer power or time that grows exponentially with the size of the problem, and large programs often exhibit rather erratic behaviour. They start at a reasonable pace, but then suddenly collapse, either running out of storage or failing to compute in a reasonable amount of time. The inability of classical algorithms to compute such problems as prime factoring or the travelling salesman problem (TSP) in polynomial time make them quite impractical. Quantum algorithms offer a spectacular resolution to exponential explosions. The subject of quantum algorithms is currently receiving much attention. When a computation is described by a wavefunction, the different terms of the function are natural parallel computations. The quantum machine consequently computes in massive coherent superposition. It accepts information which is in superposition of many different inputs and subsequently evolves it into the corresponding superposition of outputs. Computation is a sequence of unitary transformations which simultaneously affect all bits, generating massive parallel processing within a single piece of quantum hardware. As result quantum computers can solve problems which are intractable on a classical computer. For example, the prime factoring of a number with **n** digits on a classical machine requires time that grows exponentially with **n**, while the quantum computer computes in polynomial execution time $\sim \mathbf{n}^2$ [Ref 69]. What would take many years of computation on a Boolean machine will be done in a matter of days on a quantum processor. Quantum algorithms speed up exponentially the solution of the TSP problem [Ref 86] and provide for instant search in large data bases where the detection of a single zero multiplier in a string of any length uniquely determines the result of valuation, and makes further computations unnecessary [Ref 84, 88]. The Grover quantum algorithm speeds up the search time for an item in an unordered list of **n** items from linear in **n** to the order of the square root $\sqrt{\mathbf{n}}$. Quantum mechanics helps in searching 'for a needle in a haystack' [Ref 29]. Nesting one quantum search within another provides optimization [86].

The gates that evolve classical states into superposition are of great utility for the quantum computer. In a classical register **n** elementary operations can prepare one state of the register representing only one particular number. Using matrix logic, **n** elementary operations can generate a superposition state comprising $2^{\mathbf{n}}$ possible values of the register. If after preparing the superposed state of the register all subsequent operations are unitary and linear, a matrix processor is a quantum computer, where the computations are performed simultaneously on all numbers present in the superposition. This idea of quantum parallelism was first realized in matrix logic for a multispinor matrix processor, where a search for zero item is carried out on a register in coherent superposition of many spins [Ref 84, 88]:

$$\frac{1}{\sqrt{2^n}}(<1><1><1><1><1><1><1>\ldots) = \begin{cases} 0 \\ 1 \end{cases}$$

The product value of the register is 0 if at least one spin-spin interaction is zero. As opposed to classical search, when the spins are in quantum superposition, the search can be carried out instantly. All states are tested simultaneously, and the solution is obtained in a single query. This algorithm was inspired by the way the brain instantly retrieves information from a large knowledge base. With the aid of quantum algorithms we are beginning to understand how the brain avoids exponential explosions and bottlenecks.

Quantum algorithms which explore coherent quantum superposition were developed in 1984 at Intelligent System Technology, Inc [Ref 86] and in 1994 at AT&T [Ref 69]. The quantum algorithms solve the TSP and prime factoring problems much faster than any known classical algorithm. Whereas any classical search algorithm necessarily scales as $O(\mathbf{n})$, where $\mathbf{n}$ is the dimension of the search space, a quantum algorithm provides a square-root speed-up. Although there is no guarantee that an efficient classical algorithm for these problems will not be found in the future, quantum parallelism has turned the dream of massive parallelism into reality. While a serial machine does one computation at a time, quantum algorithms allow an astoundingly large number of computational tasks to be performed simultaneously, speeding up the computation of problems previously considered to be intractable.

Finally, we must reflect on the analytical aspect of quantum algorithms which make use of continuous differentiable wavefunctions. A computer is a discrete finite-state machine, but in physics we typically construct continuous mathematical models which range from the dynamical system of classical physics to the operator equations and path integral of quantum field theory. These models are based on real numbers, which are, of course, an abstraction. While a rational number requires only a finite number of bits, it takes an infinite number of bits to represent a single real number. But infinitely many bits are not available in practice. Mathematical physics uses the continuum of the real numbers because it is a powerful and useful construct. The continuous quantum algorithms bring back into focus the argument for using the power of analysis in computing, which often has a clear advantage over discrete models. In a search for a particular entry in a matrix-logical string the differential algorithm makes use of the logical derivative which induces quantization of the Boolean register:

$$\mathbf{BR}' = \mathbf{QR}.$$

When the variable of differentiation is the variable of the searched item, the quantization condition for the Boolean register is the condition for the item to be found in the register. If the result of differentiation is zero, then the register is not quantizable and the item is not in the register:

$$\mathbf{BR}' = 0$$

This is an effective algorithm, which either leads to a quantum superposition or clears the register to zero, independently of its length and complexity. The quantization provides an instant solution and the syndrome identifying the location of the item: the quantized state $|s_{\pm}>$ is found where the searched item resides:

$$_{q}(|x>|y>|z>|p>|q>|r>...) = |x>|y>|z>|p>|s_{\pm}>|r>...$$

Quantum algorithms as randomized algorithms may produce an incorrect solution; however, they have the property that the probability of failure can be made arbitrarily small. In a realistic set-up the register may decohere due to interaction with the macroframe. From the point of view of computational complexity it is irrelevant how weak this interaction is. When the interaction causes an exponential increase of errors, the randomized algorithm cannot be regarded as efficient any more. This unwelcome increase in errors shows that quantum algorithms are not robust to quantum noise and that further efforts are necessary to make quantum computation practical. One seeks inspiration in the intelligence code of the brain, which has a built-in mechanism for correcting errors. Because an error may not be easily recognizable, in quest of higher fault tolerance we considered a multispinor quantum processor [Ref 88]. The processor works in the **n/2-out-of-n** code, implying that the code has parity check. To preserve parity, the operators which operate the processor locally are the unitary **YES** and **NOT** and the singular **TRUE** and **FALSE**. Ideally an output of the processor is an eigenstate vector with integer real values, while intermediary states can be complex-valued. These are handled in terms of several real elements, depending in part on the degree of precision which is required from the randomized algorithm.

QUANTUM ENGINEERING: SPINTRONICS

Current computer technology is based on solid-state electronics, where a large number of electrons in computer circuits represent a single bit. This is in contrast to a quantum machine, in which a single atom or a spin is used to represent information. The information is stored on individual particles and atoms are scanned to retrieve information. In analogy to **electronics**, we call this new quantum technology **spintronics**. Classical bits are replaced with quantum bits, the two-state quantum systems, such as the spin, which are the fundamental carriers of information. A quantum computer built with **n** such elementary units could be in a superposition of 2^{n} distinct states, each splinter of the superposition performing its own computation in parallel with all the rest. This enables the quantum computer to explore a computational mode not attainable on a classical Boolean machine, which works with **TRUE** and **FALSE** only. Quantum technology, in contrast, can also naturally accommodate the quantum states **TRUE and FALSE** and **neither TRUE**

nor FALSE. The Schrödinger cats, a bizarre factoid of quantum mechanics, finds practical use in quantum computing. Spintronics is currently exploring the frontier of computing and quantum physics. The goal is to devise an atomic level circuitry that is better at handling classes of problems that do not lend themselves well to electronics and to develop a quantum machine that will outstrip the fastest supercomputers. Although simple matrix quantum gates and demonstration prototypes exploring quantum superposition were built as early as in 1984 [Ref 85], the spintronics technology of quantum computing devices is not yet a match for conventional silicon chips. But the race to build a quantum computer is on, and progress is rapid. There are various choices for engineering realizations. The possible implementations of quantum gates include nuclear spins, ion traps, quantum dots, cavity quantum electrodynamics, nuclear magnetic resonance, optical lattices, Bose condensates and superconducting junctions. A quantum computer with a couple of million spins would certainly be a very powerful and valuable machine. But a machine with a less daunting number of quantum gates may still be very useful. First of all, quantum computers can do other things besides instant search, travel scheduling and prime factoring, and some of these other tasks, in particular quantum simulation, can be accomplished with a less reliable device that meets far less demanding specifications. While control over many individual atoms or electrons may seem far-fetched and grandiose from the point of view of state-of-the-art technology, the quantum computer is coming of age. Although we have a long way to go, the pieces of the puzzle have begun to fall into place. Taking spins to be the elementary information bits, we now have a fair notion of how good the performance of a useful quantum computer will need to be, what error rate could be tolerated, and what accuracy threshold is needed for a quantum computer to work effectively. The quantum revolution in computing is just beginning. With more refined analysis and various improvements, the goals of spintronics are certainly still ambitious, but not beyond the scope of what might be achievable in the near future.

Why one would wish go to all the trouble of quantum computation while existing silicon circuitry is so remarkably reliable? Packing more data onto a chip requires making each circuit element smaller. Continuously shrinking such elements is becoming increasingly difficult. In the coming decades computational speed and memory will approach fundamental physical limits. A further substantial increase of computer power will be possible only by descending deeper to the atomic quantum level of computing. Advantageous as it may be, spintronics has to overcome many technological hurdles. In virtue of the decoherence of quantum systems, the key obstacle holding back the development of a practical quantum processor is the avalanche of quantum errors which destroy the strict quantum-mechanical coherence required for quantum computation.

The challenge is to build a quantum machine whose coherent quantum states can be controlled in a systematic and predictable fashion. To obtain a general description of such a machine, we begin with the Boolean computer

whose deterministic evolution we can formally describe with the aid of the Hamilton equation:

$$\frac{dL}{dt} = \{L, H\}$$

where L represents the state of the machine, H the computational energy, and $\{L, H\}$ are the Poisson brackets. This system has a microphysical analogue. To convert the classical machine to a quantum one, the Poisson brackets must be replaced by the Heisenberg commutator

$$\{A,B\} \quad \Rightarrow \quad -\frac{i}{\hbar}[A,B]$$

The quantum machine, then, is a dynamical system whose general description is provided by the Heisenberg equation of motion

$$i\hbar\frac{dL}{dt} = [L, H]$$

where the left-hand side identifies the change of state of the machine and the right-hand side its basic code [Ref 94]. The operator L represents a dynamical observable and H is the Hamiltonian of the quantum register. Earlier we derived a logical counterpart of the Heisenberg equation of motion

$$-\frac{d<\rightarrow>}{dq} = <[\rightarrow, H]>$$

which is matrix-logical version of

$$i\hbar\frac{d<\sigma_z>}{dt} = <[\sigma_z,H]>$$

and which explicitly leads to the spinor code as the underlying code of the quantum computer:

$$\frac{d<\rightarrow>}{dq} = <\sigma_z>.$$

What is particular makes a case for spintronics is that the spin operator shows up in all key matrix-logical equations, from the singularization theorem

$$<q|M|q>=<\psi|\sigma_z|\psi>$$

to the dynamical equation of motion, which generates coherent superpositions,

$$(H\sigma_z + \frac{d}{dq})|q> = 0.$$

With the aid of spin, one converts the energy operator into momentum:

$$H\sigma_z = M,$$

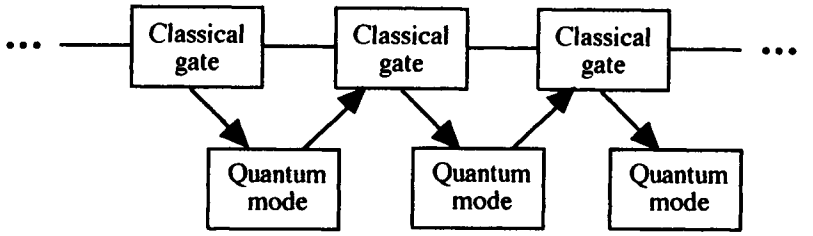
and with the unitary spin transformation changes the self-potent implication into the self-potent inverse:

$$\sigma_z \rightarrow \sigma_z = \rightarrow^{-1}$$

This provides for reversible computation, as opposed to the logical unitary transform which gives simply the converse:

$$\theta\rightarrow = N\rightarrow N = \leftarrow.$$

The links between matrix logic and spin provide a fundamental framework for designing quantum computer and for understanding quantum-theoretic machines in general. Making use of the singularization theorem we move freely backwards and forwards between classical and quantum computation, achieving computational speed-up in the quantum mode:



The isomorphism between the algebraic structures of matrix logic and quantum mechanics opens up an avenue to a general description which reduces either to the quantum commutator or to the Poisson brackets in the purely quantum and classical limits respectively. Matrix logic, alternating between the operator and the operator-free formats, is neither classical nor quantum but both. By exploring the operator format, a classical state can be transformed into a catlike state, which can be dealt with deterministically or in terms of wavefunction where coherent superposition is an essential intrinsic feature, 'hidden inside' classical computations. This entails that quantum superposition is a particular case of logical superposition, which places quantum theory in a subordinate position to fundamental logical theory. After all, isn't it our logic which gave rise to quantum theory? The quantum-logical computer is not just an engineering project but also a fundamental physical problem.

A quantum computer formulates problems classically but solves them quantum-mechanically. In quantum-logical computation, physical and logical, quantum and classical, analog and digital come together. This is in close affinity to the brain, which in a sense is an analog device which digitalizes the input it receives. For example, when you are asked a question, you are given the input in the classical analog form of a sound; when you think in order to give an answer, the input is converted in the brain to quantized form. Finally, the answer is again in a classical analog format. Likewise, when a painter draws your portrait, your image is presented to him in analog form. When the image is processed, it is quantized but only to be converted back to the analog output when the portrait is drawn:

Analog input	$\Rightarrow$	Digital processing	$\Rightarrow$	Analog output
Image		Brain		Drawing

The quantum computer takes in classical inputs and yields classical outputs but achieves quantum speed-up during the intermediate steps. In order to progress

from on-paper design to an actual quantum machine one important problem we must solve is the **I/O** interface problem. Whatever the advantages of quantum computation might be, the inability to control the interface between the quantum machine and its classical macroframe would render it useless. It is imperative that we be able to convert classical input information into quantum inner information and project quantum information into a classical format at output. Thus, the operations of a quantum computer embrace both sides of the classical-quantum divide, and link the technological problem of the **I/O** interface to the fundamental physical question of the classical-quantum interface. Since the computer is at core fundamentally quantized, while the input and output are classical, in order to write-in classical data and to read-out quantum data we must have a mechanism that enables information to pass freely through the quantum-classical divide, from Boolean bits to the Schrödinger cats, breeding in-between coherent 'kittens'.

We do not observe quantum cat states because they are very unstable, but we have learned how to prepare topological cat states. A topological cat can be dealt with equally well in terms of deterministic matrix logic or in terms of probabilistic wavefunction. In a reversal of the traditional approach which typically focuses on the impact a macroscopic system (a classical field operator) exerts on a microscopic one (a quantum state), we can legitimately ask what impact a microphysical system (a quantum-mechanical operator) could have on a macrophysical system (a classical state). Earlier we proved the singularization theorem which identifies the classical-quantum interface:

$$\langle q| M |q\rangle = \langle \psi|\sigma_z|\psi\rangle.$$

Considering this interface we dealt mostly with fermionic spins but the bosonic limit of Matrix Statistics is equally important. Recall that classical scalar logic is bosonic. Provided that the **I/O** interface is bosonic:

$$\psi(a)\psi(b) = \psi(b)\psi(a),$$

when we have a bosonic field or obtain it by bosonization, the exchange of operators between the Hamilton and Hilbert spaces is possible without disturbing the value of the atomic bracket:

$$\langle q|\sigma_z|q\rangle = \langle \psi| M |\psi\rangle$$

For all practical purposes the action of the quantum spin in classical logic equals the action of the matrix operator in quantum logic. The momentum operator acts top-down on quantum states in the same way as the quantum spin acts bottom-up on classical states, providing for an effective **I/O** computer interface. It is intriguing to speculate on the possibility of exploring the Cooper condensation in superconductors, to obtain a bosonic quantum computer interface. Since the **I/O** interface of a quantum computer is a classical-quantum interface, it makes much sense to put this result in a wider perspective. The quantum computer accepts Boolean bits and superbits while in classical and quantum regimes respectively. Joining Boolean and quantum logics into a unified framework, quantum computation sheds light on the controversies surrounding the fuzzy boundary between macro- and

microworlds. The ability of the computer to swap classical and quantized states allows us without impediment to choose between the classical treatment of information and the quantum evolution, modelling information bits by spins. At mesoscopic border where the classical and quantum realms overlap the quantum and classical degrees of freedom can be treated within one general formalism.

DIALING QUANTUM NUMBERS

The quantum computer is necessarily a many-particle quantum system. Each state of the computer we can describe by some number. But in contrast to a Boolean computer, whose states can also be characterized by numbers, this number is literally a **quantum number**. When we call a particular state of the machine, we have to 'dial up' a particular series of quantum numbers. A quantum computer is universal if we are able to generate on it any desired quantum number by enacting a unitary transformation on a set of quantum-mechanical variables. These quantum numbers are processed by quantum gates which form quantum circuits. The quantum gates are universal in sense of system independence, and a finite number of such gates, wired properly, make up a quantum computer. Unlike a communication line in a classical computer a quantum wire is itself a nontrivial device because it must take in some quantum number and produce its exact copy at the exit, which is not a trivial task because a quantum state cannot be copied or cloned [Ref 109]. A remarkable property of quantum numbers is that they cannot be simply duplicated in contrast to classical Boolean numbers. There exists no physical device that can produce perfect copies of a quantum system that is initially in an unknown quantum state. This no-cloning theorem is an immediate consequence of the linearity of quantum mechanics. In fact, if perfect cloning were possible, the Heisenberg uncertainty principle could be violated by measuring conjugate observables on many copies of a single quantum system.

Quantum gates, which process quantum numbers, are the building blocks of the quantum computer which allows new types of information processing. Reducing on a massive scale the size of computing gates to the size of an atom, or, if we are inventive enough, to the size of elementary particles, poses a great challenge. The actual construction of three-dimensional quantum circuits is a superb technological problem, requiring controlled manipulation of individual atoms and confinement of electrons to planes, one-dimensional lines and quantum dots. Because of the computational speed-up, quantum gates outperform classical gates but are much likely to fail or to give an incorrect answer when malfunctioning. If strict precision and not speed is the criterion of success, the quantum computer may make things worse instead of better. Executing many gates in succession, the quantum computer evaluates quantum numbers just as the classical computer evaluates Boolean numbers. We classify

computer gates as classical or quantum, depending on whether they process Boolean bits or superbits. The classical gates can be executed with great reliability. An atomic bracket in matrix logic converts Boolean states into quantized superpositions, and therefore it is essentially a quantum gate, if reversible. The question of reversibility is important because quantum processes must obey the law of unitary evolution. In conditional dynamics one gate undergoes an unitary evolution that depends on the quantum state of another gate. A reversible quantum gate, like the **KNOT** (controlled-**NOT**) acts on two bits: the control bit x and the target bit y. The value of y is flipped if x=1, otherwise y remains unchanged. In both cases the control bit x is unchanged:

$$\mathbf{KNOT}|x\rangle|y\rangle = |x\rangle|x \oplus y\rangle.$$

The **KNOT** is a unitary operation on the two-state quantum system which in the orthonormal basis reduces to the classical operation we can perform on a Boolean machine. A quantum-mechanical controlled rotation gate leaves its first input unchanged, and rotates the second input by an angle and multiplies it by a phase if and only if the first input is 1.

As opposed to existing two-level computer gates, quantum gates are multilevel, providing for dense encoding of information. Instead of just two scalar Boolean numbers 0 and 1, matrix logic accommodates the logical states:

$$(0\ 0),\quad (0\ 1),\quad (1\ 0),\quad (1\ 1).$$

This fact is important from the quantum engineering point of view. The use of the **4** levels of matrix logic, instead of the **2** levels of Boolean logic, will cut in half the number of memory cells needed to store a given amount of information. Because each memory cell can store two bits instead of one, halving the number of cells that are needed, this provides a substantial increase in memory capacity. Four-level matrix logic has recently been explored by Intel USA and NEC Japan for the design of a new generation computer memory chips, the dynamic RAM, which relies on four-level storage. In addition to the levels 0 and 1, corresponding to empty or full electric charge, each memory cell has levels in-between and can reside in four states, representing one of the four combinations 00, 01, 10, 11. Introducing the phase -1, one introduces more levels which can provide a further increase in memory capacity. The memory chip presently in use in conventional computers is the 16-megabit RAM. Matrix technology will allow us to develop a superdense quantum memory whose capacity exceeds many times the capacity of exiting memory chips.

Spintronics opens up many new possibilities in computer technology. A quantum computer is governed at the bottom by the laws of quantum mechanics and by the deterministic laws at the top. The asymptotically distinct quantum evolutions must be in strong correlation with different classical computations. The spins residing in a two-dimensional Hilbert space, however, can become entangled with the macroframe, and rotate in an unpredictable way. Moreover, the two-dimensional space may leak out into a larger Hilbert

space [Ref 56]. To control the leakage, we will need to project the spin and reverse the damage by a unitary transformation. It is helpful to know the location of the spin, since we can then streamline the repair by reversing the error at that known location. Exploring the physical contexts in which quantum information might be reliably stored and manipulated, we need to design fault-tolerant quantum gates that intrinsically protect themselves from the debilitating effects of decoherence. Instead of some ingenious but artificial schemes, which should prevent things from going wrong, we can delegate the control of computer operations to the actual laws of Nature. For example, if information is encoded in the energy values of spins, we can be absolutely certain that the conservation of energy will guarantee the absence of 'errors'. All we have to do is to isolate the system and prevent it from losing or gaining energy.

Quantum computers allow new types of information processing and solve certain problems far faster than any conceivable classical machine. The quantum principles can be embedded in the operations of the computer, bringing us closer to actual realization. However, quantum technology is still in its infancy, and to design quantum hardware we must not only have a clear idea about the components of the machine, but we must be able to put these components together. The devil is in the detail, and detail is very hard to control. One promising alternative is a topological design for the quantum computer, proposed by the author in 1980s [Ref 82, 87] and recently revived by Kitaev [Ref 41]. The topological design relieves us of the very difficult task of manipulating individual atoms and spins. Instead, we work with large macromolecular lattices, and with the aid of the topological ***CUT*** and ***ZIP*** operations construct the required computing architecture. The topological approach gives an additional boost to computer performance in that topological quantum gates are intrinsically fault-tolerant, not requiring intervention by the computer operator in case of errors. A fault-tolerant gate is a gate that works effectively even when its elementary components are imperfect. Topology naturally incorporates intrinsic fault tolerance into the design of quantum computing hardware.

As was indicated early, the main obstacle holding back the development of a quantum computer is the avalanche of quantum errors which breaks down the required strict quantum coherence. A quantum system inevitably interacts with the environment and the information stored in the computer will decay. The error rate grows exponentially regardless of the type of interaction with the macroframe. This is a serious technical limitation for any realistic model of quantum computation. Quantum computers are far more susceptible to making errors than Boolean computers, and some method of controlling and correcting those errors is needed to prevent crashes. Furthermore, to operate a quantum computer, we must do more than just store quantum information; we must process the information. In quantum gates, quantum bits interact with one another, and if an error occurs in one bit, it is likely to spread to the other. The gates must be designed in such a way that the propagation of error is

minimal. Decoherence is the most formidable, but not the only problem. Even if we were able to achieve excellent isolation from the macroframe, we could not expect to execute quantum gates with perfect accuracy. The quantum errors form a continuum. Small errors can accumulate over the course of a computation, eventually causing failure [Ref 58].

To sum up, a quantum computer, in order to be a useful and effective machine, must include efficient quantum error correction in its operations. Because the quantum states can be controlled only to the extent that the uncertainty principle will permit, quantum errors are inevitable. All we can hope for is to find the optimal codes which fight these errors with the highest but not absolute reliability. It is from this unpleasant but realistic platform that one must approach the question of designing a quantum computer. The efficient recovery from errors in itself is not sufficient to ensure that a quantum computer can perform reliably. To carry out error correction, we must encode and decode quantum information. But encoding and decoding are themselves complex quantum computations, and errors will inevitably occur while they are performed. Incorporating quantum error correction will surely complicate the operation of a quantum computer. To protect against errors, the number of gates will have to rise. Performing gates and error recovery will slow the computation down. Because of the increase in the complexity of the machine, it is not obvious that error correction will really improve its performance [Ref 58]. Quantum computers have a great potential, but this potential will never be realized if we are unable to protect the quantum machine from the destructive effects of decoherence. Solving the problem of quantum error correction, therefore, is the first critical step we must make. The prospects for quantum computing receive a major boost from the discovery that quantum error correction is possible.

QUANTUM CODES

We must send the Russians an ambiguous message which is clearly understood.

Alexander Haig

Those who think that quantum bits are simply an abstraction without actual use are mistaken. We often make use of quantum thinking in real-world situations, as the above statement of the ex-NATO chief vividly indicates. During the cold war years a deliberately ambiguous language was a rule of the game, keeping the opponents uncomfortably guessing.

The central problem of coding theory is to transmit encoded messages, typically strings of binary bits **0** and **1**, from a source to a receiver, protecting them as much as possible from destructive errors, which may occur

spontaneously or due to hostile interference. Originally, coding theory was viewed as purely mathematical theory, requiring no physical concepts. The need to account for the superposed bits $\mathbf{0\pm1}$, which are naturally treated as coherent quantum superpositions in Hilbert space, introduced fundamental physical principles into coding theory [Ref 88]. Matrix logic was instrumental in bringing an understanding that besides classical codes there must exist a distinct class of quantum codes [Ref 82, 94]. What previously been the exclusive domain of abstract information became the subject of theoretical physics. The codes, which are constructed with bits in coherent superposition, allow us to answer a number of important questions in quantum computing and in brain science into which they were, in fact, first introduced by the author as basis of the intelligence code [Ref 81].

When considering codes, we are concerned with the triumvirate: intelligence, security, errors. Quantum codes have opened up new prospects for computer security, a public system which struggles to respect the personal privacy of user, by definition running into a conflict. Quantum cryptography has offered new and unexpected solutions for secure coding and the prevention of errors. The question of errors will be our concern now. The laws governing the generation of errors determine the important features of a code: deterministic laws lead to classical codes, indeterministic laws to quantum codes. In classical codes the bits **0** and **1** are characterized by amplitude only, and can be changed over by simple **mod2** arithmetic. A nontrivial feature of quantum codes is that quantum probability amplitudes can be negative and complex-valued. In matrix logic the presence of the negative bits **-1** implies that in addition to the amplitude, the bits also have phases. Purely formally, instead of considering phases, it is also possible to extend matrix codes into the ternary alphabet $E_3 = \{\mathbf{0}, \mathbf{1}, \mathbf{-1}\}$, but in the **mod2** system the distinction between **+1** and **-1** vanishes. Therefore the phase is important; however, it is relevant only for nonzero bits. The change of phase is equivalent to the $e^{i\pi}$ rotation of the bit

$$\mathbf{1} \cdot e^{i\pi} = \mathbf{-1},$$

which of course will leave the zero bit unchanged. To perform quantum error correction, we will need to diagnose phase errors as well as amplitude errors. If we use negative logic it is possible to achieve effective quantum error correction through destructive interference and not by duplication of bits.

When a code is corrupted with errors, a proper codeword **X** turns into a faulty word. This can be defined as the direct problem of coding theory which is the one-to-many map:

$$\mathbf{X} \Rightarrow \{\mathbb{X}\},$$

where $\{\mathbb{X}\}$ denotes the set of errors. The inverse problem is a syndrome decoding which must diagnose the errors and recover a correct codeword from the error set. This is the many-to-one map:

$$\{\mathbb{X}\} \Rightarrow \mathbf{X}.$$

When one designs an effective code, the purpose is to optimize these two maps. We have a sort of a **minimax** problem. Optimization of the direct map entails an **error-prevention**, minimizing the number of elements in $\{\mathbb{X}\}$. The optimization of the inverse map, maximizing the number of elements in $\{\mathbb{X}\}$, determines the **error-correction** capability of the code which should map as many errors as possible, ideally all, to an unique proper single codeword.

We considered four types of local bit errors:

Amplitude error, denoted by A, occurs when a bit in a codeword is replaced by a complement bit:

$$A(\mathbf{0}) \Rightarrow \mathbf{1} \qquad \text{and} \qquad A(\mathbf{1}) \Rightarrow \mathbf{0}.$$

The amplitude errors are the additive errors, equivalent to the **mod2** addition of **1** to a bit, respectively: $\mathbf{0} \oplus \mathbf{1} = \mathbf{1}$ and $\mathbf{1} \oplus \mathbf{1} = \mathbf{0}$.

Phase error, denoted by P, occurs when a bit in a codeword is rotated by $e^{i\pi}$, acquiring the opposite sign:

$$P(\pm\mathbf{1}) \Rightarrow \mp\mathbf{1}$$

As we mentioned, the zero bit **0** is stable against phase change and therefore has no protection against phase errors.

Mixed errors occur when both the amplitude and phase of a bit are damaged:

$$AP(\mathbf{1}) \Rightarrow \mathbf{0}, \qquad PA(\mathbf{1}) \Rightarrow \mathbf{0},$$

and

$$AP(\mathbf{0}) \Rightarrow \mathbf{1}, \qquad PA(\mathbf{0}) \Rightarrow \mathbf{-1}.$$

In general the bit flip and phase flip do not commute $[A, P] \neq 0$; hence, the order of errors is relevant, resulting in different mixed errors.

Puncturing and insertion errors we denote by Λ and Λ^{-1} respectively. These involve the loss of bits or creation of new bits, which consequently contract or increase the length of a codeword. The role of puncturing and insertion error in coding theory is analogous to the role of creation and annihilation operations in quantum field theory. A puncturing error can fatally damage the encoded data. For example if a codeword is realized with the nucleons of an unstable atom, the decay process will irreversibly damage the data [Ref 81, 91].

In coding theory error-correction is typically achieved through redundancy. A common defence against errors is to add to the **data** bits of the codewords an ancilla of **check** bits. The ancilla is used both to implement codewords and to perform error correction. When the redundant bits are appended to a codeword there are no longer any differences between them and the rest. With a sufficiently large ancilla any message can be transmitted reliably. If one transmits the single digit codeword **1**, and the amplitude error flips the bit to **0**, there is no means to restore the original codeword. But if **1** is appended with redundant bits, say with a thousand copies of **1**, then the probability of all the bits suffering error is negligible, and by majority voting

on the receiving end one identifies the correct codeword with high statistical confidence. The protection of the code through duplication will typically work but is inefficient, because we have copied too much information. *Bad* codes use repetition many times; *good* codes only rarely, if at all. An arbitrarily long codeword can be decoded reliably, with the probability of mismatch falling to zero as the length increases. Though redundancy is a widely used method, it can be introduced only at the expense of the economy of the code. In real-world situations there should be a proper trade-off between the length of the codewords, which should not be too long, and the acceptable number of errors which the code can handle.

Error correction schemes are well developed for codes which suffer only one or few errors per codeword, which is clearly is an optimistic assumption. When a single bit is in error, the parity check enables us to diagnose the error and to correct it. When many bits in a codeword are in error, the task of error correction become difficult and often intractable. Suppose that **X** is an unknown codeword, and one of the bits, the i-th bit, flips. Then **X** becomes $\mathbf{X}+e_i$. To determine which bit flipped, one apples the parity check matrix H to the $\mathbf{X}+e_i$ and learns where the error occurred [Ref 47]. Such a strategy is typically employed in the classical and quantum error correction schemes that effectively recover from a single error in the code. Clearly, for quantum computation with its huge number of potential errors this is an impractical approach and better codes that can protect the from multiple errors must be constructed.

A quantum code makes use of quantum bits, the arbitrary states $\alpha|0\rangle+\beta|1\rangle$ in a two-dimensional Hilbert space, where the coefficients α and β are complex probability amplitudes. But in matrix logic they need not be complex numbers, they even can be operators. As opposed to a straightforward readout of classical codes, if we want to decode a quantum message, we must subject it to quantum measurement, projecting the encoded quantum states onto the orthogonal basis $\{|0\rangle, |1\rangle\}$ [Ref 78]. Quantum measurement can be a destructive measurement which disturbs the state that is being measured, but we also may need to perform a nondestructive measurement which preserves the code subspace; although it does of course destroy a coherent superposition. If we don't mind destroying the encoded information, the quantum state is reduced to either $|0\rangle$ with probability $|\alpha|^2$ or to $|1\rangle$ with probability $|\beta|^2$. Decoding through quantum measurement has an advantage for secure encryption, since information is neither in the cipher nor in the key but in their interaction, but it also has the disadvantage that quantum information is more prone to error. Moreover, checking for error can introduce further errors. All existing quantum error-correcting codes make use of the same fundamental strategy: a small subspace of the Hilbert space is designated as the code subspace. This space is chosen so that all of the errors move the code space to mutually orthogonal error subspaces. A measurement tells us in which of these subspaces the system resides, and hence we can infer

exactly what type of error occurred. The error can then be repaired by applying an appropriate unitary transformation. Any logical operation can be physically implemented as spin rotation and phase inversion or as a linear combination of such operations [Ref 91]. Although the quantum computer is more efficient than a classical machine in computations involving matrices of exponentially large size, decoherence errors remain one of the major obstacles to building a quantum computer. The effect of decoherence can be compensated for if one introduces redundancy into a quantum code. This entails that a quantum state is first encoded into a larger Hilbert space and the wavefunction is projected into a suitable subspace. Then by applying a unitary transform to the orthogonal complement of the subspace according to the measurement result, it is possible to correct errors due to decoherence. The obvious way to do this is to repeat the measurement, which might be difficult or even impossible in extremal decision making [Ref 76, 77]. Although by 1982 it became apparent that quantum codes can be expected to have far-reaching implications [Ref 82], until very recently the difficulties with quantum error correction seemed quite serious. But then in 1994 the nested quantum codes (codes within codes) were introduced by the author [Ref 91] and in 1996 Shor and Steane [Ref 96, 96] proposed 'good' quantum codes. These and other codes are still based on the redundant storage of bits with all the advantages and limitations which it entails. A new radical decision for error self-correction which greatly improves the prospects for quantum computing technology are autonomous self-correcting codes.

SELF-CORRECTING CODES

As we discussed earlier, in the existing classical and quantum codes one protects data through redundancy, and then checks for errors and revises them. In ideal circumstances there are syndromes for a code which enables one to determine the nature and location of errors. However, a different strategy for performing error correction can be considered. Instead of protecting a code through redundancy, what we want is to delegate the job of error correction to the code itself. Clearly we are seeking a self-correcting **intelligent** code, able to take care of itself, very much as the human brain does. There is no external supervisor which tells the brain when and where the error occurs and what has to be done to correct it. Encoding and decoding in the brain rely on the built-in rules and not on the addition of an ancilla of redundant bits. Introducing self-correcting codes, we depart from the principles of classical coding theory and provide codes with intrinsic built-in control over their own operations. Because we intend to construct codes which correct errors intrinsically, there is another feature which distinguishes our approach from conventional coding schemes. While in standard coding theory one deals with codewords represented by suitable **operands**, we seek suitable **operations**, which in themselves and by themselves can intelligently provide

an effective defence against errors. Because these optimal coding operations, as we will shortly find out, are quantum-mechanical in nature, we define our codes as matrix quantum codes where redundancy is of no fundamental significance. To define a matrix code rigorously we consider the space of all **n**-tuples of **0**'s, **1**'s and **-1**'s over the Galois field **GF(k)**, the finite field with **k** elements. The fundamental block codes we are concerned with are the vector code over **GF(3)** with **9** codewords (**2**-tuples), the Boolean code over **GF(2)** with **16** codewords (**4**-tuples), a matrix code over **GF(3)** with **81** codewords (**4**-tuples), and the **mod2** code over **GF(2)** with **16** codewords (**9**-tuples). These underlying codes can be nested with quantum operations into codes of various complexity; of particular importance among these are the duplication **8**-tuple quantum code and the triple commutator code.

CODEWORDS AND INWORDS

An **n**-ary block code with codewords of length **k** has $\mathbf{n^k}$ codewords in it, which in the standard set of matrix logic gives us the code **Ç(81, 4)** with $\mathbf{3^4 = 81}$ codewords:

0 0 0 0

0 0 0 1	0 0 0-1	0 0 1 0	0 0-1 0
0 1 0 0	0-1 0 0	1 0 0 0	-1 0 0 0
1 0 0 1	-1 0 0 1	1 0 0-1	-1 0 0-1
0 1 1 0	0-1 1 0	0 1-1 0	0-1-1 0
0 0 1 1	0 0-1 1	0 0 1-1	0 0-1-1
1 1 0 0	-1 1 0 0	1-1 0 0	-1-1 0 0
0 1 0 1	0-1 0 1	0 1 0-1	0-1 0-1
1 0 1 0	-1 0 1 0	1 0-1 0	-1 0-1 0
0 1 1 1	0 1 1-1	0 1-1 1	0-1 1 1
0 1-1-1	0-1 1-1	0-1-1 1	0-1-1-1
1 1 1 0	-1 1 1 0	1-1 1 0	1 1-1 0
-1-1 1 0	-1 1-1 0	1-1-1 0	-1-1-1 0
1 1 0 1	1-1 0 1	1 1 0-1	-1 1 0 1
-1-1 0 1	1-1 0-1	-1 1 0-1	-1-1 0-1
1 0 1 1	1 0-1 1	-1 0 1 1	1 0 1-1
1 0-1-1	-1 0-1 1	-1 0 1-1	-1 0-1-1
1 1 1 1	-1 1 1 1	1 1-1 1	1-1 1 1

1 1 1-1	-1 1 1-1	1-1-1 1	1 1-1-1
-1-1 1 1	1-1 1-1	-1 1-1 1	1-1-1-1
-1-1-1 1	-1 1-1-1	-1-1 1-1	-1-1-1-1

Excluding puncturing and insertion errors, both the proper codewords and the error codewords are taken from the same logical set:

$$X \in \Omega \qquad \text{and} \qquad \{\mathbb{X}\} \in \Omega.$$

Defining the weight or mass $w(X)$ of a codeword X as the algebraic sum of bits in a codeword:

$$w(X) = \sum_{i=1}^{n} x_i ,$$

one determines that in $\text{Ç}(81, 4)$ there are **31** codewords with positive mass, **31** codewords with the negative mass, and **19** massless codewords with zero weight. The matrix code is distinguished by nonzero codewords, whose mass nonetheless is zero. The total mass of the code $\text{Ç}(81, 4)$ is zero:

$$w(\sum_{i=1}^{81} X_i) = w(0) = 0.$$

The code $\text{Ç}(81, 4)$ contains within itself the lower level code $\text{Ç}(9, 2)$ comprising nine codewords:

0 0, 0 1, 1 0, 1 1, 0 -1, -1 0, -1 1, 1 -1, -1 -1,

three of which, 0 0, -1 1 and 1 -1, are massless $w(X) = 0$. Because in matrix logic we add or delete bits in pairs the code $\text{Ç}(9, 2)$ is important. Due to global phase symmetry it reduces to the fundamental $\text{Ç}(5, 2)$ code, comprising five fundamental codewords

0 0, 0 1, 1 0, 1 1, 1 -1.

Any matrix codeword can be expanded in the lower level $\text{Ç}(9, 2)$ code, with bits being treated as the spin-spin outer product interactions:

$$|0><0|, \quad |0><1|, \quad |1><0|, \quad |1><1|,$$

corresponding to the following array of spins, respectively:

$$\downarrow\downarrow, \quad \downarrow\uparrow, \quad \uparrow\downarrow, \quad \uparrow\uparrow.$$

Different arrays of spins give different matrix codewords. The codewords which emerge in square root polarization of logical vacuum,

1 1-1-1, 0 0 1 0, 0 1 0 0,

are special in that that they are created by the entanglement of the orthogonal states:

$|S_->< S_+| \Rightarrow$ 1 1-1-1, $|1><0| \Rightarrow$ 0 0 1 0, $|0><1| \Rightarrow$ 0 1 0 0.

Following the prescription of coding theory, we wrote the codewords as strings. This is a convenient approach, but there are no fundamental reasons for doing so. Moreover, because we are after a self-correcting code, we must distinguish two kinds of codes: codes without and with interaction. Classical theory is concerned with codes without interaction. The bits are sent from a

source to a receiver, and every effort is made to prevent bits interfering with each other, which could only result in errors. In a major reversal to this scheme, we are seeking **active** codes in which the interaction of bits is not avoided, but on the contrary is explored to the advantage of error-correction. To distinguish between the **passive** codewords of traditional codes and our **active** codewords we define the latter as **inwords**, where '**in**' stands for inner, interactive or intelligent, depending on taste. In the ground state the inwords are not simply an array of bits but an ordered entanglement, which occurs when we collect the array into a matrix,

$$X = x_1 x_2 x_3 x_4 \Rightarrow \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}$$

The content of codewords is now more visible, in contrast to the standard string presentation in coding theory. There is also considerably more structure in our codewords, which can be now naturally treated as matrix logical operators. A fundamental feature of the active code is the interaction between codewords where we study not only entanglements of bits but the interaction between these entanglements, the entanglement of entanglements. Any interaction whose outcome is the inword $X \in \mathcal{C}(81,4)$ is legal and is forbidden if $X \notin \mathcal{C}(81,4)$.

Since we have collected bits of codewords into matrices, the terminology commonly in use for matrices can be freely transferred to our interactive codes. It will be legitimate for us to speak about, for example, a transposed or conversely transposed codeword or the determinant of a codeword:

$$\tilde{X} = x_1 x_3 x_2 x_4, \quad \overleftarrow{X} = x_4 x_2 x_3 x_1, \quad \det X = x_1 x_4 - x_3 x_2,$$

keeping in mind the corresponding matrix operations

$$\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}^{\sim} = \begin{pmatrix} x_1 & x_3 \\ x_2 & x_4 \end{pmatrix}, \qquad \begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix}^{\leftarrow} = \begin{pmatrix} x_4 & x_2 \\ x_3 & x_1 \end{pmatrix}, \qquad \begin{vmatrix} x_1 & x_2 \\ x_3 & x_4 \end{vmatrix} = x_1 x_4 - x_3 x_2.$$

We shall use coding and matrix terms interchangeably, referring to a matrix as an inword and to an inword as a matrix, depending on context or simply for reasons of style. In the same spirit we distinguish singular and nonsingular inwords, the inner and outer products of the inwords, the trace and retrace, and so on. Likewise, we classify the inwords as belonging to different classes of symmetry, again deriving this idea from the matrix form of the inwords:

Symmetric:	$x_2 = x_3,$	$x_1 \neq x_4$
Asymmetric:	$x_2 \neq x_3,$	$x_1 = x_4$
Bisymmetric:	$x_2 = x_3,$	$x_1 = x_4$
(Bi)asymmetric:	$x_2 \neq x_3,$	$x_1 \neq x_4$
Antisymmetric:	$x_2 = - x_3,$	$x_1 = x_4 = 0$
(Bi)antisymmetric:	$x_2 = - x_3,$	$x_1 = - x_4 \neq 0$
Antipersymmetric:	$x_2 = x_3 = 0,$	$x_1 = - x_4$

Although from the string coding perspective it may seem that symmetric and asymmetric or antisymmetric and antipersymmetric codes are equivalent, they are not. In fact, we determine that the antipersymmetric code is most effective

of the self-correcting matrix codes. Matrix theory makes it possible to provide the code with a rich and powerful algebraic structure which in turn allows for the application of different coding techniques. We now have an algebra and can multiply, add, subtract, root and divide the inwords, so long as the result is also an inword in a chosen modulo system.

Dense encoding

In conventional coding theory errors are corrected with syndromes which detect the location and nature of errors. This scheme is fundamentally different from an active code, which has a built-in self-correction mechanism and requires no external intelligent agent to do the job. Basically we are emulating the properties of the intelligence code of the brain, in which the active interacting inwords themselves cancel the errors out. A distinctive feature of the intelligence code is that it is a self-correcting code with interaction, where the inwords interact with one another, generating new inwords. An interaction whose outcome is the inword $Z \in \text{Ç}(81,4)$ we consider legal and forbidden if $Z \notin \text{Ç}(81,4)$. Each inword is a logical sequence of interacting matrices. Although we do not use explicit markers, one must keep in mind that in an inword, after each 4 bits in a row, some matrix operation applies. The sequence

$$x_1\ x_2\ x_3\ x_4 x_5\ x_6\ x_7\ x_8$$

stands for

$$\begin{pmatrix} x_1 & x_2 \\ x_3 & x_4 \end{pmatrix} * \begin{pmatrix} x_5 & x_6 \\ x_7 & x_8 \end{pmatrix}$$

where "*" denotes the matrix operation.

The general operational condition for a matrix code is

$$X = Y ? Z$$

where L is a correct codeword $X \in \Omega$, A and B are the possible operator errors which the code can handle, and "?""*" is an unknown 'clever' operation, acting in $\{\mathbb{X}\}$, which we need to identify.

Let $Z = X * Y$ be an inword, and $\{\mathbb{X} * \mathbb{Y}\}$ a set of all errors which the inword may suffer. A code is self-correcting code if it has an error-correction operation "*" which intrinsically maps the error set or a subset of the error set back to Z:

$$\mathbb{X} * \mathbb{Y} = Z.$$

The optimal self-correcting code handles the highest number of errors. In coding theory one typically tries to have maximal Hamming coding distance, that is, using codewords separated as far as possible from each other. This 'as-far-as-possible' principle is not important for self-correcting codes. The

effectiveness of the code depends largely on the type of the operation "*" and not on how far apart codewords are from each other.

Whatever distaste we may have for a completely random universe, we cannot preclude the occurrence of quantum error to occur. We could never have a perfect world, nor an absolutely reliable code, whether quantum or classical. *Errare humanum est.* To err is in human nature, but also and no less, if not more, error is the distinctive feature of the indeterministic quantum world. What is essential for and what distinguishes human intelligence is the powerful ability to correct errors. Since errors are unavoidable, their control and intelligent management are of paramount importance for an effective code. Moreover, errors do not have absolute meaning: what may be a faulty word in one situation is a perfectly legitimate word in another. Since an error changes a logical matrix into another logical matrix, the error can be treated as an *unwanted* logical operation, but since it *is* a logical operation, we can associate it with a corresponding logical operator. Depending on the particular situation, these can be different operators. Consider, for example, the following errors, where the subscript indicates the damaged bits:

$$\begin{pmatrix}0\\1\end{pmatrix} \xrightarrow{A_1} \begin{pmatrix}1\\1\end{pmatrix}, \qquad \begin{pmatrix}1\\0\end{pmatrix} \xrightarrow{AP_1} \begin{pmatrix}-1\\1\end{pmatrix}.$$

In the first instance the Boolean state suffers the amplitude error in the upper position; in the second instance the amplitude error in the lower position and the phase error in the upper position. These errors we can induce with the following logical operators:

$$\mathbf{OR}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}, \qquad \mathbf{OR}^{-1}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}-1\\1\end{pmatrix}.$$

Applying the inverse of the operator, if such is available, one can correct the errors and restore the bits to the original value. Because the inverse operation is not universally available, this is generally not the case. Suppose we introduce the composite errors in the true state with the momentum operator:

$$\mathbf{M}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\-1\end{pmatrix}$$

Since **M** is singular, the recovery of the original state cannot be accomplished with an inverse. This brings into focus the importance of nonsingular codes where the errors are fully reversible. Particular importance attaches to the spin operator, under the impact of which the true and superposed logical states suffer a phase error:

$$\sigma|1\rangle = -|1\rangle, \qquad \sigma|s_-\rangle = |s_+\rangle$$

but the false state remains unaffected:

$$\sigma|0\rangle = |0\rangle.$$

Because σ is self-inverse, simple iteration will correct the error. This is an intelligent error control which fights errors with errors, cancelling themselves in superposition. An important method of elimination of error, not dependent

on the availability of the inverse, makes use of the knowledge operators. Suppose the errors had been induced on $|1\rangle$ by the operator **FALSE**:

$$\mathbf{FALSE}|1\rangle = |0\rangle.$$

FALSE does not have an inverse, but acting with **TRUE**, which is the knowledge operator, we eliminate the acquired errors, recovering the original state:

$$\mathbf{TRUE} \bullet \mathbf{FALSE}\ |1\rangle = |1\rangle.$$

Since inwords are matrices, when the two inwords **X** and **Y** are multiplied and the result $\mathbf{Z} = \mathbf{X} \bullet \mathbf{Y}$ is in $\mathcal{Q}(81, 4)$, the **Z** is an inword too, densely encoding the **X** and **Y**. There are two fundamentally different kinds of dense encoding: singular and nonsingular. In a nonsingular code, given **Z**, we can recover the **X** or **Y** or both, while in a singular code information on the input is irretrievably lost. Suppose that an error occurred in a bit or bits of **X** or/and **Y**. What may one expect to happen to **Z**? Let us study a specific example, taking $\mathbf{Z} = \mathbf{N}$, $\mathbf{X} = \mathbf{IF}^{-1}$ and $\mathbf{Y} = \mathbf{OR}$:

$$\mathbf{N} = \mathbf{IF}^{-1} \bullet \mathbf{OR}$$

which is explicitly

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$$

or in the string format

$$0\ 1\ 1\ 0 \quad \Leftrightarrow \quad 1\ 0\text{-}1\ 1\ 0\ 1\ 1\ 1.$$

In the event of errors in the interacting codewords the recovery procedure may fail. The interaction may fail to restore the correct inword if the bits flip and/or change phases. However, with correlated errors the recovery can be made successfully. Suppose we damage the three bits, in the 3rd, 4th and 7th positions with phase errors which induce the following change

$$1\ 0\text{-}1\ 1\ 0\ 1\ 1\ 1 \quad \Rightarrow \quad 1\ 0\ 1\text{-}1\ 0\ 1\text{-}1\ 1$$

In spite of the errors the product inword remains unaffected:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$$

Likewise we can damage the code with an array of amplitude and phase errors, again observing no change in the resulting inword:

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

In fact the product inword $\mathbf{Z} = 0110$ is stable against **40** different operator errors in **X** and **Y**. This example gives a clue about how one can go about finding self-correcting active matrix codes with interaction. The solution of the problem depends on the operation mediating the interaction of the codewords.

Correlated errors

We have classified the elementary coding errors, the amplitude and phase errors. These are local errors, concerning individual bits. Searching for effective methods for error correction and error prevention, it is important also to distinguish compound or global errors in which elementary errors occur jointly in many or even all bits of the inword. Here we must make a very important distinction between uncorrelated and correlated errors. Quantum codes are more powerful than classical one because bits are strongly quantum-mechanically correlated (entangled). Under this distinction, what causes problems for the classical recovery protocol can be turned to the advantage of the self-correcting code which can fight error with errors. Errors can be detrimental, but correlated errors can also be useful. In an active code, if an error occurs in one of the interacting codewords, it may spread to the other. A single error may duplicate, causing two errors, and these multiplied errors can propagate to the output data. To contain the propagation and to make our procedure fault-tolerant, we can make use of correlated errors. Given the two codewords **X** and **Y** we form a composition, the inword **Z = X*Y**, where the operation mediating the interaction must be explicitly identified. We consequently introduce the following **4** major classes of self-correcting codes.

MATRIX PRODUCT CODE X•Y

We say that the inword $Z = z_1z_2z_3z_4$ is a matrix product code of the inwords $X = x_1x_2x_3x_4$ and $Y = y_1y_2y_3y_4$,

$$Z = X \bullet Y,$$

if the bits of **Z** are the scalar products of the rows of **X** and the columns of **Y**:

$$Z = x_1y_1+x_2y_3 \quad x_1y_2+x_2y_4 \quad x_3y_1+x_4y_3 \quad x_3y_2+x_4y_4$$

Unless we work in the **mod2** system, the **(2x2)** matrix product code, as well as other codes which we consider, must take account of the forbidden interactions reducing the size of the code. Of the **6561** possible product inwords **1982** are inadmissible.

TRANSMULTIPLICATION CODE X♦Y

We say that the inword $Z = z_1z_2z_3z_4$ is a transmultiplication code of the inwords $X = x_1x_2x_3x_4$ and $Y = y_1y_2y_3y_4$,

$$Z = X \blacklozenge Y,$$

if the bits of **Z** are the products of the corresponding bits of **X** and **Y**:

$$Z = x_1y_1 \quad x_2y_2 \quad x_3y_3 \quad x_4y_4.$$

ANTICOMMUTATION CODE {X,Y}

We say that the inword $Z = z_1z_2z_3z_4$ is a anticommutation code of the inwords $X = x_1x_2x_3x_4$ and $Y = y_1y_2y_3y_4$, when **Z** is the composition:

$$Z = \{X,Y\} = X \bullet Y + Y \bullet X.$$

COMMUTATION CODE [X, Y]

We say that the inword $Z = z_1z_2z_3z_4$ is a commutation code of the inwords $X = x_1x_2x_3x_4$ and $Y = y_1y_2y_3y_4$,

$$Z = [\,X, Y\,],$$

if **Z** is the Lie product of **X** and **Y**:

$$Z = X \bullet Y - Y \bullet X.$$

In existing codes the syndrome measurements may reveal information about the errors without revealing anything about the data. In self-correcting codes this recovery procedure is turned upside-down: a quantum operation recovers the data without revealing anything about the errors. Consider the commutator [X, Y]. It acts nontrivially, if remains unchanged in spite of the errors in the codewords **X** and/or **Y**, in that the commutator retains the data bits, but provides no information about errors. To read information reliably in quantum codes based on redundancy, we need to know how it was stored, otherwise the measurements are bound to damage the coherence which is so crucial to quantum error correction. But a self-correcting code repairs itself intrinsically: we don't know and we don't need to know how the bits were stored.

The active codes with interaction which we have introduced remain fault-tolerant in the presence of errors. Consider as an example the commutator inword for the **TIME** operator:

0 0 0 1 0 1 1 1

which in a more familiar notation is $[\wedge, \vee]$, and suppose that the amplitude errors occurred in the **5**th and **8**th positions, so that the inword becomes

0 0 0 1 1 1 1 0

Apparently the inword $[\wedge, \vee]$ has been changed into the inword $[\wedge, \theta\vee]$. However, in spite of the local disturbance of the code, the information carried by the inword remains intact:

$$[\wedge, \vee] = [\wedge, \theta\vee].$$

With our method we can introduce many other errors into the inword, which will be intelligently self-corrected by the commutator quantum operation, for example:

$$[\wedge, \vee] = [\downarrow, \uparrow]$$

$$= [\,\mathbf{H}, \wedge\,] = [\,\mathbf{H}, \vee\,] = [\uparrow, \mathbf{H}\,] = [\downarrow, \mathbf{H}]$$

$$= [\,\wedge, \dot{H}\,] = [\,\dot{H}, \vee\,] = [\,H., \uparrow\,] = [\,\downarrow, H.\,] = [\,\dot{H}, \uparrow\,] = [\,\wedge, H.], \text{ etc.}$$

Altogether there are **136** commutator inwords for the **TIME** operator, half of which is a global phase change of the other half:

$$[X, Y] = [-X, -Y]$$

Theorem

If $[X, Y]$ is an inword for time $[X, Y] = \mathbf{TIME}$, then the errors due to simultaneous local unitary rotations do not disturb the code:

$$[X, Y] = [\theta Y, \theta X]$$

Proof

The unitary transform is linear in the commutator

$$\theta\,[X, Y] = [\theta X, \theta Y].$$

But time is also antiunitary

$$\theta\,[X, Y] = -\,[X, Y] = [Y, X]$$

Hence,

$$[X, Y] = [\theta Y, \theta X].$$

□

We are naturally interested which of the active codes provide for maximal fault-tolerance, and which inword, if any, is most robust with regard to global operator errors? An inword can be encoded in different matrix codes, which deliver different fault-tolerances. For example, the inword 0 1 1 0 will resist:

40 operator errors in the matrix product code $X \bullet Y$
100 operator errors in the transmultiplication code $X \blacklozenge Y$
40 operator errors in the anticommutation code $\{X, Y\}$
136 operator errors in the commutation code $[X, Y]$

We see that the commutation code provides the highest error self-correction capability. However, not every codeword of the matrix code $\mathcal{C}(81, 4)$ can be expanded in the commutation and anticommutation codes. While 0 1 1 0, for example, has both $\mathbf{NOT} = [X,Y]$ and $\mathbf{NOT} = \{X,Y\}$, there is no commutator code for its complement 1 0 0 1, which exists only as the anticommutator, $\mathbf{YES}=\{X, Y\}$. The fundamental transmultiplication code for **YES** involves the two fundamental self-potent operators:

1 1 0 1 1 0 1 1.

The code is stable to **100** operator errors, as any transmultiplication code with $w(X) = 2$ will be, while the corresponding matrix product code controls only **40** errors. On the intersection of the product and transproduct codes we find **4** inwords, shared by both codes:

1 0 0 1 1 0 0 1, 1 0 0-1 1 0 0-1, -1 0 0 1-1 0 0 1, -1 0 0 1-1 0 0 1

Together with the empty inword 0 0 0 0, the commutator code contains **27** inwords whose fault-tolerance varies, as is indicated by subscript in the **13** commutator inwords and **13** anti-inwords with global phase reversal:

1 0 0-1,	1 1-1-1,	0 1 1 0,	-1 1 1 1,	1 1 1-1,
180	80	136	80	80
1-1 1-1,	0 0 1 0,	0 0 1 0,	-1 1 0 1,	1 1 0-1,
80	136	136	104	104
	-1 0 1 1,	1 0 1-1,	0-1 1 0.	
	104	104	136	

Of the active matrix logical codes the commutation code is optimal and in the commutation code the spin code is the most effective for autonomous error correction. The spin σ is the only antipersymmetric inword; it is the complement of the adder + which is the sum of the canonical pair of inwords:

$$\sigma = \overline{\wedge + \vee}$$

Equivalently, it is the difference of the respective unitary transforms of the canonical inwords:

$$\sigma = \theta\wedge - \wedge \qquad \text{and} \qquad \sigma = \theta\vee - \vee.$$

We can derive σ by subtracting two outer product codes

$$\sigma = |0\rangle\langle 0| - |1\rangle\langle 1|.$$

If prior to that the first and the third states are exchanged, we get instead of σ, the **TIME** operator

$$\blacktriangledown = |1\rangle\langle 0| - |0\rangle\langle 1|.$$

$\blacktriangledown$ induces the phase error, simultaneously swapping the lower and upper components of a logical state; σ reverses the phase of the lower component, leaving the upper component intact. The antisymmetric $\blacktriangledown$ is the operator difference of the asymmetric operators **IF** and **IMPLY** or their duals a^* and a:

$$\blacktriangledown = \mathbf{IF} - \mathbf{IMPLY} = a^* - a$$

This in contrast to the antipersymmetric spin inword, found as the commutator

$$\sigma = [\mathbf{IMPLY}, \mathbf{IF}].$$

QUANTUM CODES AND THE INTELLIGENCE CODE

For ordinary codes without interaction, two codes are equivalent if their generating matrices are equivalent. But active codes can produce equivalent codewords with different matrices. This provides a defence against errors and reasons for preferring one code over other. The self-correcting codes are robust and can remain effective in spite of the extended presence of errors. The optimal active code is the spin code, intrinsically stable against **180** global operator errors, which is about **500** local quantum errors. Because there are

still errors which the spin code would not correct, we need to introduce additional constraints to improve the code. There are two distinct directions such an improvement can take: heuristic rules and natural laws. For example, for the commutator code decoding can be improved somewhat by counting the number of zero entries in the generating codewords. For σ this number is among 1, 2, 3, 4, 5, 6, for N - 1, 3, 4, 5, M - 2, 4, for a* - 3, 4, 5, 6, - all different, which allows for additional discrimination of quantum errors. Coding theorists have proposed ingenious coding schemes, most prominently the parity syndrome due to Hamming. But the codes created by nature are different from those created by man. We can introduce some clever heuristic rules to manage spin errors or we can place spins in a particular physical set-up and let the nature take its course. One nontrivial idea is to make use of the stable atoms as natural 'codewords' [Ref 92, 94]. All of the quantum spins are stored in a single atom, sharing the same interaction bus, similar to phonons in an optical resonator. We store data and carry out a search on individual atoms. Whatever happens to a stable atomic inword, it still will be the same atom. Whatever one does to a stable atom, short of a nuclear reaction, it will remain the same atom. There is no need to take any precautions: the code would fail only if nuclear errors occurred, which is highly unlikely for a stable atomic codeword. When we have assembled all the elements of an atomic code, one can enact quantum computation without fear of decoherence.

The errors are controlled naturally by the laws of spin interaction. In the duplication code each pair of bits is a pair of fermion spins, where each 4-bit logical nibble is represented by a 8-bit byte. The lower-level code is inscribed into the higher-level code, which in turn can be inscribed into still higher-level code. An atom is a quantum inword, and different atoms provide for different reliabilities of a code. When we choose the isospin invariant atom of oxygen from the various possible atomic inwords only those which comply with the Pauli exclusion principle will be actually physically realized. This feature suggests a natural way of constraining the number of errors of a code. For example if the bits are bosons, then the codeword **01** may suffer 3 (uncorrelated) errors

00, 10, 11,

but if the bits are (strongly correlated) fermions, there is only one possible error **10**. When one bit flips, the Pauli exclusion rule will force the other bit coherently to flip too. These and other less intuitive features at the heart of quantum mechanics are important for improving the fault-tolerance of the quantum codes. One requires stable atoms not susceptible to decay to provide for effective storage and transmission of quantum information. As the cipher and the key of a code become intertwined, the error set gets smaller. Such are the self-dual codes in which the generator matrix is also the parity check matrix. Quantum codes explore rules that are fundamental in origin, and the fact that the inwords are atoms allows one not only to draw parallels between

the theory of codes and quantum physics, but to relate them in a fundamental sense.

Error correction in the quantum computer relies on the built-in mechanism. Here we depart from the redundancy principle, widely explored in coding theory. Quantum coding schemes which rely on redundancy would not handle a quantum computer. For a complex system of quantum gates, say, several millions of atoms, which is nothing extraordinary as the basis for a computer, the redundant ancilla may require a number of bits larger than the number of particles in the visible universe. But with natural physical constraints, where we engage the laws of physics themselves to control the 'errors', we can dramatically reduce the phase space of the problem. Likewise, the brain achieves effective computations through a built-in mechanism which is fundamentally different from the efficiency arising from redundancy. What distinguishes intelligence coding is that it corrects errors without revealing anything about the state of the brain. We are unaware of both the location of an error, and of how it is corrected. As in *blindsight* we make the right choices, but obtain no information about how we actually do it. Intelligence code operations restore an inword, corrupted with errors, to the intended value without revealing any information about the nature or location of errors. We cannot instruct a particular neuron in the brain to inspect what errors occur and where and then to change the logical state properly. This puts the theory of quantum codes in direct contact with the theory of the intelligence code. The quantum codes correct decoherence errors in a state intrinsically without knowledge of the state itself. One repairs an unknown universe, without actually knowing how it is constructed.

We have constructed self-correcting codes where inwords were represented by pairs of interacting nibbles. This is the extension of the idea of the duplication codes studied earlier, in which a pair of bits represents a single bit of the higher-order code [Ref 91]. Combining different active codes we can increase the fault-tolerance of the code. For example, the set inword 1111, encoded as the product of two commutation codes $\mathbf{1}=[\sigma]\cdot[\mathbf{M}]$ will resist $(180\times80)+84=14484$ operator errors, which includes **84** corrections handled by the matrix product code $\mathbf{1} = \mathbf{X}\cdot\mathbf{Y}$.

Another important connection between the intelligence code and quantum codes is nesting. Consciousness begins to operate only after a high level of hierarchy is achieved, and in fact the idea of nested codes first emerges in the context of cognitive studies, only later extended to quantum codes [Ref 82, 94]. In the nested code each bit of an inword, when inspected at higher resolution, is itself an encoded inword. And when we examine the encoded inword with still better resolution, we discover that it too is really an array of interacting inwords. And so on. The reason that nesting is useful is that it enables the brain to recover from errors efficiently. With enough levels of nesting we can improve the performance of the code and diminish the error rate. Quantum error recovery is not flawless because it is itself a quantum

computation that is prone to error. If the recovery procedure is designed carelessly, then it might even introduce more errors than it corrects.Then we have derived no benefit from using a nested code; in fact, the total number of errors can be even higher than without any nesting coding. So we are obliged to consider all the possible ways that recovery might fail. Only then is nested coding guarantied to pay off. With the aid of computer simulations we have determined an active spinor code $\sigma_Z = [X, Y]$ code which self-corrects **180** global errors. When the code is nested to the triple commutator

$$\sigma_Z = [X, [Y, Z]],$$

there are **8160** global errors which spin code can effectively control. Further nesting increases the self-correcting capability of the code:

$$[A,B] \Rightarrow [A, [C,D]] \Rightarrow [A, [C, [E,F]]] \Rightarrow [A, [C, [E, [K,L]]]] \Rightarrow \ldots$$

The number of correctable errors increases as the length of the inwords increases and with natural confinement of spins, the probability of errors can be made arbitrarily small, satisfying the accuracy threshold required for effective quantum computation. Self-correcting codes correct arbitrarily many errors, and make possible arbitrarily long computations. This is in marked contrast with quantum codes based on the principle of redundancy. Although it may seem that by choosing codewords as large as we please we can make the probability of error smaller, this turns out not to be the case. As the complexity of the code increases, the complexity of the recovery procedure also increases. Eventually we reach the point where it takes so long to perform recovery that it is likely that more errors will accumulate before the recovery is completed, and the ability of the code to correct errors is thus compromised. A self-correcting code, of course, cannot suffer from similar drawbacks since fault-tolerance is welded into the code itself and no error recovery protocol is needed, while the accuracy can be improved indefinitely.

The self-correcting intelligent code of the brain has been a major source of inspiration for quantum codes, and will continue to be so in the future. The fundamental coding units of the intelligence code are the gauge invariant codons of tautology and contradiction. Because the inference rules are universal tautologies, the intelligence code can be described as a covariant inference rule of inference rules, a global matrix tautology. In scalar format we have the universal constant

$$\bar{\alpha} + \alpha \equiv 1,$$

where the term α can be anything from a single logical variable to an atomic bracket or a fundamental product and its various reductions, which we discussed earlier. The simplicity of the constant is misleading. To assure nontrivial stability of the codons, it is instructive to introduce entanglement, confining the logical terms under the complement:

$$\bar{\alpha} + \alpha = 1 - \alpha + \alpha = \overline{\alpha - \alpha} = \bar{0} \equiv 1.$$

This is only the beginning of the complexification of a codon, which must remain unchanged under arbitrary local gauge transformation. There are

many ways to obtain intelligence code cognitive constants. The canonical matrix form of the intelligence code's codons is

$$\overline{<x|L|y><x|\theta L^*|y>} \equiv 1$$

where **L** and L^* are the dual logical operators. The unitary transform of the dual of an operator is the operator's complement:

$$L^* = \overline{\theta L} = \theta \bar{L} \quad \Rightarrow \quad \theta L^* = \theta\theta\bar{L} = \bar{L}$$

which for implication gives the co-dual, required for the **Modus Ponens** inference rule:

$$\overline{<x|\rightarrow|y><x|a^*|y>} \equiv 1$$

The codons are the 'reaction centres', which free logical elements join in an absolutely predictable fashion. One builds up concatenated **Modus Ponens**, for example, with the octet of the doubly-even self-dual oxygen $_8O^{16}$ spins:

$$\overline{\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow}$$

where in conditions of chemical stability the three-level couplings deter the transfer of spins, preventing quantum errors.
Since

$$L + \bar{L} \equiv 1 \qquad \text{and} \qquad L \bullet \bar{L} \equiv 0,$$

we can express the fundamental cognitive constants of contradiction and tautology respectively:

$$\overline{<x|L + \theta L^*|y>} \equiv 0 \qquad \text{and} \qquad \overline{<p|L \bullet \theta L^*|q>} \equiv 1.$$

In matrix logic the tautology constant extends in **84** matrix products and in **24** anticommutators, but not a single commutator for **ONE** exists in the intelligence code. The absence of the antisymmetric Lie products for the tautology is significant. The total number of cognizable codons, including the fundamental syllogism inference, is **26**, which equals the number of bosonic dimensions in string theory. Bosons are cognizable, as opposed to fermions. At higher levels of deductive thinking consciousness is asymmetrically shifted towards the bosonic limit of Matrix Statistics, a conclusion relevant for experimental verification.

In addition to the **Modus Ponens** family of **4** canonical inferences, the intelligence code allows for (a) the triplet of codons with masses of the codeword **w(L) = 2**:

$$\overline{<\perp><\top>} \equiv 1, \quad \overline{<\dashv><\vdash>} \equiv 1, \quad \overline{<\mathbf{I}><\mathbf{N}>} \equiv 1,$$

(b) the **6**-tet of codons with masses of the codeword **w(L) = 1**:

$$\overline{<\backslash><|>} \equiv 1, \quad \overline{<\backslash><\wedge>} \equiv 1, \quad \overline{<\backslash><\uparrow>} \equiv 1,$$
$$\overline{<|><\wedge>} \equiv 1, \quad \overline{<|><\uparrow>} \equiv 1, \quad \overline{<\wedge><\uparrow>} \equiv 1,$$

(c) the **12**-tet of codons with masses of the codeword **w(L)= 1** and **w(L)= 2**:

$$\overline{<\wedge><\top>} \equiv 1, \quad \overline{<\backslash><\top>} \equiv 1, \quad \overline{<\perp><|>} \equiv 1, \quad \overline{<\perp><\uparrow>} \equiv 1,$$

$\overline{<\wedge><N>} \equiv 1, \quad \overline{<\uparrow><N>} \equiv 1, \quad \overline{<I><\backslash>} \equiv 1, \quad \overline{<I></>} \equiv 1,$

$\overline{<\wedge><\vdash>} \equiv 1, \quad \overline{</><\vdash>} \equiv 1, \quad \overline{<\dashv><\backslash>} \equiv 1, \quad \overline{<\dashv><\uparrow>} \equiv 1.$

The intelligence code is a complex structure.The logical operators can conspire anticommutatively only in **21** and commutatively only in **13** dimensions. A striking feature is the loss of symmetry with respect to negation and identity. **NOT** can be generated both as an anticommutator and a commutator, for example:

$$\{\mathbf{AND}, \overline{\mathbf{AND}}\} = [\mathbf{M}, \mathbf{AND}] = \mathbf{NOT}.$$

YES, however, does not exist as a commutator, but **140** anticommutators for identity are found in the intelligence code, among them

$$\{a^*, M\} = \mathbf{TRUE} + W(\mathbf{TRUE}) = \mathbf{YES}$$

Since technically there no distinction between **YES** and **NOT** and the spin operators σ_0 and σ_x, we can choose the computational basis states in the code subspace to be the simultaneous eigenstates of the inwords, with the +1 eigenvalue corresponding to false, and the -1 eigenvalue to true. This brings the intelligence code into relation with the stabilizer formalism of quantum codes, where a quantum code is characterized as the space of simultaneous eigenstates [Ref 24]. Spin operators square to identity and has equal numbers of eigenvectors with eigenvalue +1 and -1, so that specifying an eigenvalue reduces the dimension of the space by half. Quantum error changes the eigenvalues of inwords, and ideally, two independent errors always change the eigenvalues in distinct ways.

The intelligence code adds, subtracts, multiplies and divides matrix inwords. The burden of explaining why and how these operations deliver an intelligent reasoning capability is eased significantly by exploring the gauge symmetry. Altogether there are **25** transmultiplicative cognizable codons used by the code, a single supercommutator codon, **337** anticommutator codons, and **817** commutator codons. Once again we observe that the quantum operation of commutation provides for the most powerful matrix code. The Lie commutator algebra is of fundamental significance both for quantum computer codes and for the intelligence code. The non-Abelian operator operations are the operations found at the core of the mechanism of logical thought:

$$[\mathbf{X}, \mathbf{Y}] = -[\mathbf{Y}, \mathbf{X}]$$
$$[\mathbf{X}, \textbf{scalar matrix}] \equiv \mathbf{0}$$
$$[\mathbf{X}, \mathbf{Y} \pm \mathbf{X}] = [\mathbf{X}, \mathbf{Y}]$$
$$[\mathbf{X}, \mathbf{Y}+\mathbf{Z}] = [\mathbf{X}, \mathbf{Y}] + [\mathbf{X}, \mathbf{Z}]$$
$$[\mathbf{X}+\mathbf{Y}, \mathbf{Z}] = [\mathbf{X}, \mathbf{Z}] + [\mathbf{Y}, \mathbf{Z}]$$
$$[\mathbf{X}, \mathbf{YZ}] = \mathbf{Y}[\mathbf{X}, \mathbf{Z}] + [\mathbf{X}, \mathbf{Y}]\mathbf{Z}$$
$$[\mathbf{XY}, \mathbf{Z}] = \mathbf{X}[\mathbf{Y}, \mathbf{Z}] + [\mathbf{X}, \mathbf{Z}]\mathbf{Y}$$
$$[\mathbf{X}, [\mathbf{Y}, \mathbf{Z}]] + [\mathbf{Y}, [\mathbf{Z}, \mathbf{X}]] + [\mathbf{Z}, [\mathbf{X}, \mathbf{Y}]] = \mathbf{0}$$

The algebra of thoughts is effectively a Lie algebra with an antisymmetric product satisfying Jacobi identity. The latter, when realized by logical operators, gives a cognitive constant: any contradiction can be expressed as Jacobi identity, where the three triple commutators with cyclic shifts sum up to the empty operator. For the major basis set {**NOT, AND, OR**} we get

$$[\wedge, [\vee, \mathbf{N}\,]\,] + [\vee, [\mathbf{N}, \wedge\,]\,] + [\mathbf{N}, [\wedge, \vee\,]\,] = \mathbf{0}$$

where the inscribed commutators are the **TIME** operators:

$$\begin{aligned}[\wedge, \blacktriangledown\,] + [\vee, \blacktriangle\,] + [\mathbf{N}, \blacktriangledown\,] &= \wedge\blacktriangledown - \blacktriangledown\wedge + \vee\blacktriangle - \blacktriangle\vee + \mathbf{N}\blacktriangledown - \blacktriangledown\mathbf{N} \\ &= (\wedge+\mathbf{N})\blacktriangledown - \blacktriangledown(\wedge+\mathbf{N}) + \vee\blacktriangle - \blacktriangle\vee \\ &= \vee\blacktriangledown - \blacktriangledown\vee + \vee\blacktriangle - \blacktriangle\vee \\ &= \vee(\blacktriangledown + \blacktriangle) - (\blacktriangledown + \blacktriangle)\vee = \mathbf{0}\end{aligned}$$

The **TIME** are congruent for the minor basis set {**NOT, AND**} and reversed for **OR**. If we choose a different ordering the **TIME** will be negated:

$$[\wedge, \blacktriangle\,] + [\vee, \blacktriangledown\,] + [\mathbf{N}, \blacktriangle\,] = \mathbf{0},$$

however, once again it is congruent in the first and the third terms, and reversed in the second term.

Matrix algebra has provided new important insights into the internal structure of the intelligence code, for example, revealing unexpectedly that **TRUE** is the product of the canonical operators

$$\mathbf{TRUE} = \mathbf{AND} \bullet \mathbf{OR}.$$

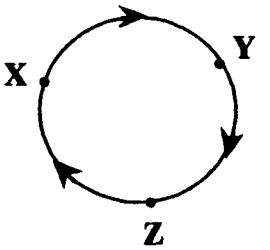
However, matrix algebra cannot give the universal constants which we need and which Lie algebra with an antisymmetric product can provide. With the aid of Lie algebra we determine that the momentum operator can be obtained from the operators of the major basis set:

$$\mathbf{M} = [\mathbf{N}\,, \wedge\vee\,] = [\mathbf{N}\,, \wedge\,]\vee + \wedge\,[\mathbf{N}\,, \vee\,] = \blacktriangle\vee + \wedge\blacktriangle.$$

Any triplet of logical operators, maximally entangled by Jacobi identity, when confined under the complement, can be employed as the fundamental cognitive constant of tautology:

$$\overline{<[\mathbf{X}, [\mathbf{Y}, \mathbf{Z}]+[\mathbf{Y}, [\mathbf{Z}, \mathbf{X}]+[\mathbf{Z}, [\mathbf{X}, \mathbf{Y}]>} \equiv \mathbf{1}.$$

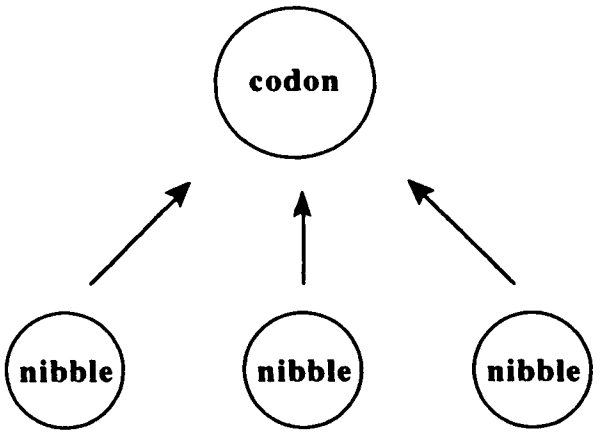
Given two operators, it is possible to construct the third one, which jointly under cyclic shift would universally guarantee an invariant inference. We have a system with a closed topology in which each operator interacts with the commutator of the other two:



Any matrix-logical Jacobi identity is the empty operator of a contradiction. Although the proof of the inverse is still lacking, the non-Abelian quantum rules of the Lie algebra emerge as fundamental rules of the intelligence code with self-reference. We reach an even deeper level of the code when we consider the logical charges of triplets in Jacobi identity. While the global charge is always an integer: 0 for contradiction and 1 for tautology, the constituent charges are fractional. This is similar to how quark charges add up to an integer charge of a nucleon. If we treat the complement operator as a nibble,

$$\overline{<L><\bar{L}>} = <1> - <L><\bar{L}> \equiv 1,$$

a cognitive inference codon is the covariant entangelment of three nibbles:



The determination of the fundamental physical structure of the intelligence code may require knowledge of correlations beyond quantum electrodynamics, the simplest gauge theory. In Schrödinger theory the physics of these correlations is incorporated in the wavefunction. The effect of Pauli correlations is explicitly manifested by the requirement that the wavefunction be antisymmetric in an interchange of the coordinates of any two particles. However, if the cognitive brain makes use of the deeper-level matrix processes, we see only the tip of the iceberg.

EVERYTHING IS LOGIC

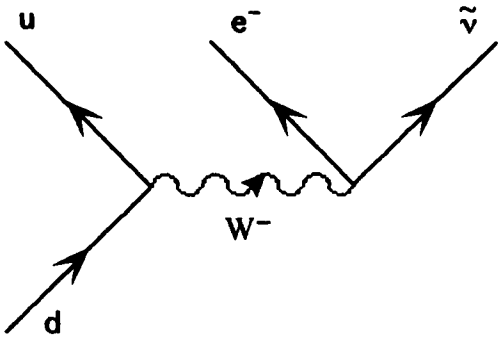
This is a crazy theory. The question is whether it is crazy enough.

Niels Bohr

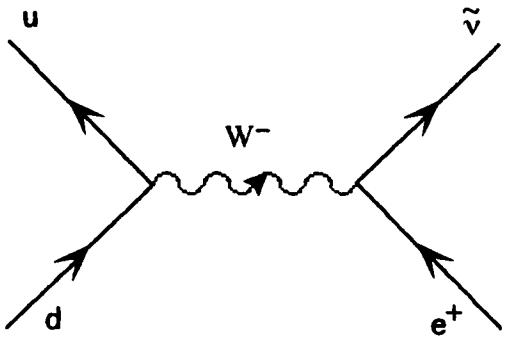
QUARK LOGIC

Matrix theory reveals that logic is intrinsically woven into the fundamental laws of physics. The formalism of logic, which is generally thought to apply only to the description of our thoughts, brains or computers is relevant at the fundamental level of particles and fields. For all practical purposes the algebra of the fundamental interactions of leptons and quarks in QED and QCD is the logical algebra, which is particular unexpected for quarks. With the support of matrix logic the regularities at the fundamental level can be represented and analysed in an unconventional way, opening new avenues for resolving the difficulties present in the quark models.

The interaction of quarks has been clarified with the discovery of W^+, W^- and Z^0 particles, the carriers of the weak force. A down quark **d** decays into an up quark **u** by emitting a W^-, which later breaks into an electron e^- and an antineutrino $\tilde{\nu}_e$:



This diagram of β-decay at the fundamental level is equivalent to the interaction of a down quark and a positron: the outgoing electron e^- is simply replaced by the incoming positron e^+:



In this way the down quark **d** turns into an up quark **u**, giving off an antineutrino. The electric charge of **d** is -1/3 and that of **u** is 2/3 and the emission of the W^- particle provides the required charge conservation: the electric charge entering a vertex must be equal to the charge going out.

The treatment of quark interactions in the formalism of matrix theory has shown that the decay of a quark, described above, can be pictured as the DeMorgan duality transformation: the logical **AND** can be invariantly replaced by the dual **OR**, provided the logical states are locally and globally negated:

$$<p|\wedge|q> \;=\; \overline{<\bar{p}|\vee|\bar{q}>}$$

Considering the averages

$$<q|\wedge|q> \;=\; 1-<\bar{q}|\vee|\bar{q}>$$

and integrating we have

$$\int_0^1 <q|\wedge|q>\,dq \;=\; \int_0^1 (1-<\bar{q}|\vee|\bar{q}>)dq$$

$$\int_0^1 q^2 dq \;=\; \int_0^1 dq - \int_0^1 (\bar{q}^2 + 2q\bar{q})dq$$

$$\frac{1}{3} \;=\; 1-\frac{2}{3}$$

which swapping the sides gives

$$-\frac{1}{3} \;=\; \frac{2}{3}-1$$

This agrees with the composition of the electric charges in the vertex of the quark diagram. An admissible logical transformation corresponds to the admissible particle transformation:

charge(d) = charge(u) - charge(W^-).

The neutrino connects a continuum to a quantized spectrum and in this respect it has been suggested that logic may rely on a neutrino-producing process and in principle is measurable [Ref 81]. To advance the argument, we conjecture that a catalogue of fundamental particle interactions is dictated by the

corresponding **legal** logical catalogue. Logic goes with its roots to the basic level of matter, and it is possible to connect every admissible particle interaction and corresponding logical duality transformation. In this framework physics is matrix logic in a fundamental sense, which, if true, opens up an opportunity to predict the outcome of particle interaction simply on the basis of logical principles. We have considered ternary quark interaction, but matrix logic admits another duality transformation, involving a half-quark charge:

$$-\frac{1}{6} = \frac{5}{6} - 1$$

due to the conversion principle, and a 1/2 charge:

$$-\frac{1}{2} = \frac{1}{2} - 1$$

due to the dualities

$<p|\mathbf{TRUE}|q> = \overline{<\bar{p}|\mathbf{TRUE}|\bar{q}>}$ or $<p|\mathbf{FALSE}|q> = \overline{<\bar{p}|\mathbf{FALSE}|\bar{q}>}$.

which are in effect the identity transformations.

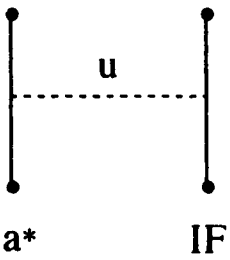
The quark 1/3 joins with the half-quark charge to give 1/2 charge of **TRUE** and **FALSE**:

$$\frac{1}{6} + \frac{1}{3} = \frac{1}{2}$$

while 2/3 mediates the conversion of the creation operator into implication:

$$\frac{1}{6} + \frac{2}{3} = \frac{5}{6}$$

as shown in the diagram below:



Considering the group of irreducible Kronecker representations of matrix logical strings, we observe that its basic units carry the elementary quark charges:

$$\mu(<q|q>) = \frac{2}{3}, \qquad \mu(\overline{<q|q>}) = \frac{1}{3},$$

$$\mu(\mathbf{YES}) = \frac{2}{3}, \qquad \mu(\mathbf{NOT}) = \frac{1}{3},$$

and so are the canonical pair of logical operators:

$$\mu(\vee) = \frac{2}{3}, \qquad \mu(\wedge) = \frac{1}{3}.$$

Generally the charge of a fundamental product, representing a logical thought, is between 0 and 1:

$$0 \le \mu(\alpha) \le 1.$$

The logical charges of contradiction and tautology are universally zero and one:

$$\mu(\mathbf{NONE}) = 0, \qquad \mu(\mathbf{ONE}) = 1.$$

The logical charges of the (bi)(anti)(per)symmetric operators, momentum, time and spin, are universally zero:

$$\mu(\mathbf{M}) = 0, \qquad \mu(\blacktriangledown) = 0, \qquad \mu(\sigma) = 0.$$

The charges of the logical energy are added in quark fractions, serially or in parallel:

$$\mu(\mathbf{H}) = \int dq < \mathbf{H} > \ = \frac{1}{3} \qquad \mu(\mathbf{H}^2) = \int dq < \mathbf{H}^2 > \ = \frac{2}{3}$$

This is similar to the logical algebra

$$\mathbf{NOT} \bullet \mathbf{NOT} = \mathbf{YES},$$

where the charge of **YES** is twice of that of **NOT**

$$\mu(\mathbf{NOT}) = 1/3, \qquad \mu(\mathbf{YES}) = 2/3.$$

The unbroken topological phases also carry the quark logical charges

$$\mu(\mathrm{H.}) = 1/3, \qquad \mu(\mathrm{.H}) = 2/3.$$

Inspired by the strange correspondence between quark charges and logical charges, already established in our early studies [Ref 94], we now build this idea into a more elaborate scheme for quark logic which in a spectacular way turns upside-down many standard logical relations we are familiar with. As we emphasized earlier, classical logic is a scalar and in the deliberately simplified Boolean code makes use of true 1 and false 0 bits. If an error occurs in a bit, changing 1 to 0 or vice versa, then the information is irreversibly lost and is not recoverable. In a matrix code a logical state is determined by two bits, and when we change from Boolean to matrix logic we change from a one-dimensional code to a two-dimensional code:

$$1 \Rightarrow \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \qquad 0 \Rightarrow \begin{pmatrix} 1 \\ 0 \end{pmatrix}.$$

Although we have added an extra bit to the code, its reliability does not improve by much. There are errors we can't repair. For example, if we measure the codeword (0 0), it could very well occurred due to a flip in the second bit of $|1>$ or in the first bit of $|0>$. With the aim of improving reliability we will now take a further step and add the parity check bits to the basic matrix code, which increases the number of degrees of freedom from two to three:

$$\begin{pmatrix} 0 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}, \quad \begin{pmatrix} 1 \\ 1 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 0 \\ 0 \end{pmatrix} \Rightarrow \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}.$$

The undecided states (1 1) and (0 0) already have parity and the check bit of course is 0. Just as the two-dimensional true and false states |1> and |0> the three-dimensional true and false

$$|\underline{1}> = \begin{pmatrix}0\\1\\1\end{pmatrix}, \qquad |\underline{0}> = \begin{pmatrix}1\\0\\1\end{pmatrix}$$

form a basis set in which any state is expanded as the linear combination:

$$\alpha\begin{pmatrix}0\\1\\1\end{pmatrix} + \beta\begin{pmatrix}1\\0\\1\end{pmatrix}$$

where $\alpha, \beta = 0, 1$. We get the state (011) for $\alpha = 1$, $\beta = 0$, the state (101) for $\alpha = 0, \beta = 1$, (110) for $\alpha = 1, \beta = 1$, and (000) for $\alpha = 0, \beta = 0$.

Each logical state is now determined by three elements, which is one of the reason for defining our new formulation as **quark logic**. Because the parity check added an extra dimension to the logical states, we are naturally required to redesign the (2×2) logical operators of the standard model, as (3×3) matrix operators with parity.

The undecided states in the standard model can be obtained by acting with an operator on the normalized state, for example

$$\textbf{OR}\ |1> = \begin{pmatrix}0&1\\1&1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}1\\1\end{pmatrix}, \qquad \textbf{AND}\ |0> = \begin{pmatrix}0&0\\0&1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}0\\0\end{pmatrix}.$$

Likewise, in quark logic the operators can reform the true and false states

$$\begin{pmatrix}0\\1\\1\end{pmatrix} \quad \text{and} \quad \begin{pmatrix}1\\0\\1\end{pmatrix}$$

into the undecided states, for example:

$$\underline{\textbf{OR}}\ |\underline{0}> = \begin{pmatrix}0&1&1\\1&1&0\\1&0&1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix}, \qquad \underline{\textbf{AND}}\ |\underline{1}> = \begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$$

DOUBLE PARITY CHECK

It is only natural that the parity check which we have imposed on the logical state is also imposed on the quark logical operators. Now, however, it is not simply a parity but a double parity check, which leads to the matrix linear **Ç(16,9)** code [Ref 94]. For completeness we add a parity check for the parity checks themselves, resulting in square matrices needed to introduce the operations of matrix algebra. In the **mod2** system the product and sum of any two codeword matrices is again a codeword matrix. The code **Ç(16,9)** is thus closed under matrix addition and multiplication, hence a ring. The code is described by a system of linear equations with binary coefficients:

$$x_{11} + x_{12} + x_{13} = 0, \qquad x_{11} + x_{21} + x_{31} = 0$$
$$x_{21} + x_{22} + x_{23} = 0, \qquad x_{12} + x_{22} + x_{32} = 0$$
$$x_{31} + x_{32} + x_{33} = 0, \qquad x_{13} + x_{23} + x_{33} = 0$$

This reduces to a system of **5** equations, after one redundant equations is omitted: the four parity checks uniquely determine the fifth.

$$\underline{\mathbf{N}} = \begin{pmatrix} 0\,1\,1 \\ 1\,0\,1 \\ 1\,1\,0 \end{pmatrix} \quad \underline{\mathbf{I}} = \begin{pmatrix} 1\,0\,1 \\ 0\,1\,1 \\ 1\,1\,0 \end{pmatrix} \quad \underline{\wedge} = \begin{pmatrix} 0\,0\,0 \\ 0\,1\,1 \\ 0\,1\,1 \end{pmatrix} \quad \underline{\vee} = \begin{pmatrix} 0\,1\,1 \\ 1\,1\,0 \\ 1\,0\,1 \end{pmatrix}$$

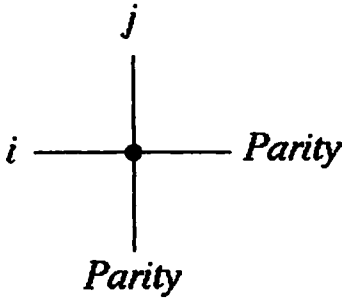
$$\underline{\rightarrow} = \begin{pmatrix} 1\,1\,0 \\ 0\,1\,1 \\ 1\,0\,1 \end{pmatrix} \quad \underline{\leftarrow} = \begin{pmatrix} 1\,0\,1 \\ 1\,1\,0 \\ 0\,1\,1 \end{pmatrix} \quad \underline{\setminus} = \begin{pmatrix} 0\,0\,0 \\ 1\,0\,1 \\ 1\,0\,1 \end{pmatrix} \quad \underline{/} = \begin{pmatrix} 0\,1\,1 \\ 0\,0\,0 \\ 0\,1\,1 \end{pmatrix}$$

$$\underline{\uparrow} = \begin{pmatrix} 1\,0\,1 \\ 0\,0\,0 \\ 1\,0\,1 \end{pmatrix} \quad \underline{\downarrow} = \begin{pmatrix} 1\,1\,0 \\ 1\,0\,1 \\ 0\,1\,1 \end{pmatrix} \quad \underline{\perp} = \begin{pmatrix} 0\,0\,0 \\ 1\,1\,0 \\ 1\,1\,0 \end{pmatrix} \quad \underline{\top} = \begin{pmatrix} 1\,1\,0 \\ 0\,0\,0 \\ 1\,1\,0 \end{pmatrix}$$

$$\underline{\dashv} = \begin{pmatrix} 0\,1\,1 \\ 0\,1\,1 \\ 0\,0\,0 \end{pmatrix} \quad \underline{\vdash} = \begin{pmatrix} 1\,0\,1 \\ 1\,0\,1 \\ 0\,0\,0 \end{pmatrix} \quad \underline{\mathbf{1}} = \begin{pmatrix} 1\,1\,0 \\ 1\,1\,0 \\ 0\,0\,0 \end{pmatrix} \quad \underline{\mathbf{0}} = \begin{pmatrix} 0\,0\,0 \\ 0\,0\,0 \\ 0\,0\,0 \end{pmatrix}$$

Table 5
Matrix codewords of the double parity check code **Ç(16,9)**.

We know that after the parity check bits are appended to the codewords, there is no longer any difference between them and the data or information bits. One simply 'forgets' which bits are checks and which are data, and deals with the three-dimensional codewords, acted upon by (3×3) quark operators. Clearly the double parity check supports a more robust code with greater intrinsic structure. Since an error simultaneously distorts the parity of the i-th row and the j-th column in an operator, when these are specified, the incorrect entry will be found on the intersection a_{ij}. The error is cross-checked in two dimensions:



Clearly, the global overall parity check applies to all objects of quark logic:

$$\sum_{i,j} \underline{x_{ij}} = 0$$

Another way of stating the same fact is to consider the **mod2** mass of an operator which universally vanishes:

$$w(\mathbf{L}) \equiv 0.$$

For the convenience of matrix representation we have added a parity check to the parity check itself. When parity checks are added only to data bits and the fifth extra bit is omitted, we get an octal block code, comprising the nibble of data bits and the nibble of parity bits:

$$\underbrace{x_1\ x_2\ x_3\ x_4}_{\text{data bits}}\ x_5\ \underbrace{x_6\ x_7\ x_8}_{\text{parity bits}}$$

The octal representation code is in a direct relation to the oxygen duplication code in which **8** spins determine a codeword [Ref 89, 91]. Expressing parity bits via data bits, the octal codewords can be written as

$$\mathbf{X} = x_1\ x_2\ x_3\ x_4\ (x_1+x_2)\ (x_3+x_4)\ (x_1+x_3)\ (x_2+x_4).$$

The data nibbles are **16** logical **(2×2)** operators **L**, and nothing prevents us from identifying the four parity bits as a **(2×2)** logical operator too. We denote such an 'operator' by ⌟.Then a particle of the octal codeword takes the form of the interaction

$$\mathbf{X} = \mathbf{L}^{*}\lrcorner.$$

We have an active code in which each inword is an interaction of the two standard matrix operators.

We now develop and investigate a logic algebra for the double parity code. The catalogue of fundamental multiplicative interactions in the **Ç(16,9)** code is provided by the product table given below:

•	I	N	△	⩡	⇉	⇇	⇓	⇑	⟍	⟋	⊥	⫪	⫤	⫣	1
I	N	I	⟋	⇉	⩡	⇓	⇇	⟍	⇑	△	⫪	⊥	⫤	⫣	1
N	I	N	△	⩡	⇉	⇇	⇓	⇑	⟍	⟋	⊥	⫪	⫤	⫣	1
△	⟍	△	0	△	⊥	⟍	⊥	⟍	0	△	0	⊥	△	⟍	⊥
⩡	⇇	⩡	△	N	⇓	I	⇉	⫣	⟍	⫤	⊥	1	⟋	⇑	⫪
⇉	⇓	⇉	⟋	I	⇇	N	⩡	⫣	⇑	⫤	⫪	1	△	⟍	⊥
⇇	⩡	⇇	⫤	⇓	N	⇉	I	⟍	⫣	△	1	⊥	⟋	⇑	⫪
⇓	⇉	⇓	⫤	⇇	I	⩡	N	⇑	⫣	⟍	1	⫪	△	⟍	⊥
⇑	⟍	⇑	⟋	⫪	⟋	⫪	⇑	0	⇑	0	⫪	0	⟋	⇑	⫪
⟍	△	⟍	△	⊥	△	⊥	⟍	0	⟍	0	⊥	0	△	⟍	⊥
⟋	⇑	⟋	0	⟋	⫪	⇑	⫪	⇑	0	⟋	0	⫪	⟋	⇑	⫪
⊥	⊥	⊥	△	⟍	⟍	△	△	⟍	⟍	△	⊥	⊥	0	0	0
⫪	⫪	⫪	⟋	⇑	⇑	⟋	⟋	⇑	⇑	⟋	⫪	⫪	0	0	0
⫤	⫣	⫤	0	⫤	1	⫣	1	⫣	0	⫤	0	1	⫤	⫣	1
⫣	⫤	⫣	△	1	⫤	1	⫣	0	⫣	0	1	0	⫤	⫣	1
1	1	1	1	⫣	⫣	⫤	⫤	⫣	⫣	⫤	1	1	0	0	0

Comparing the product table for quarks with the corresponding standard (2×2) logic code, given in Table 6, we observe that these differ from each other in several important respects. The first key difference is that in quark logic the roles of the NOT and the YES are reversed: the NOT acts as identity, and the YES as negation,

$$\underline{N}\,\underline{L} = \underline{I}\,\underline{L}, \qquad \underline{I}\,\underline{L} = \underline{N}\underline{L},$$

which has fundamental consequences.

•	I	N	∧	∨	→	←	↓	↑	\	/	⊥	⊤	⊣	⊢	1	0
I	I	N	∧	∨	→	←	↓	↑	\	/	⊥	⊤	⊣	⊢	1	0
N	N	I	/	→	∨	↓	←	\	↑	∧	⊤	⊥	⊣	⊢	1	0
∧	∧	\	∧	⊥	∧	⊥	\	0	\	0	⊥	0	∧	\	⊥	0
∨	∨	←	⊣	↓	N	→	I	\	⊢	∧	1	⊥	/	↑	⊤	0
→	→	↓	⊣	←	I	∨	N	↑	⊢	/	1	⊤	∧	\	⊥	0
←	←	∨	∧	N	↓	I	→	⊢	\	⊣	⊥	1	/	↑	⊤	0
↓	↓	→	/	I	←	N	∨	⊢	↑	⊣	⊤	1	∧	\	⊥	0
↑	↑	/	0	/	⊤	↑	⊤	↑	0	/	0	⊤	/	↑	⊤	0
\	\	∧	0	∧	⊥	\	⊥	\	0	∧	0	⊥	∧	\	⊥	0
/	/	↑	/	⊤	/	⊤	↑	0	↑	0	⊤	0	/	↑	⊤	0
⊥	⊥	⊥	∧	\	\	∧	∧	\	\	∧	⊥	⊥	0	0	0	0
⊤	⊤	⊤	/	↑	↑	/	/	↑	↑	/	⊤	⊤	0	0	0	0
⊣	⊣	⊢	⊣	1	⊣	1	⊢	0	⊢	0	1	0	⊣	⊢	1	0
⊢	⊢	⊣	0	⊣	1	⊢	1	⊢	0	⊣	0	1	⊣	⊢	1	0
1	1	1	⊣	⊢	⊢	⊣	⊣	⊢	⊢	⊣	1	1	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table 6
The **mod2** product table of logical operators in the standard model.

From the product table we read directly that quark logic also exchanges the projective and self-dual vacuum operators: the projectors of the standard model are converted into the self-duals:

$$\underline{AND}^2 = \underline{0}, \qquad \underline{NOR}^2 = \underline{0}.$$

and the self-duals become projectors:

$$\underline{ON}^2 = \underline{ON}, \qquad \underline{OFF}^2 = \underline{OFF}.$$

However, the third fundamental self-dual, the momentum operator, remains self-dual due to the irrelevance of the ± sign in the **mod2** system:

$$\begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}, \qquad \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}^2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

Vacuum roots

In the standard model the polarizations of the logical vacua gives the octet, an **even** number, of the square roots, but in quark logic there is just the triplet, an **odd** number, of the fundamental square roots:

$$\sqrt{\underline{\mathbf{0}}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

The sum total of the vacuum roots is the quark affirmation:

$$\underline{\wedge} + \underline{\uparrow} + \underline{\mathbf{1}} = \underline{\mathbf{N}}.$$

UNITARIZATION OF IMPLICATION AND SUPERCOMMUTATORS

The fundamental operator of implication in (3×3) logic obtains special, previously lacking, properties which involve the unitarity so crucial to quantum information.

Theorem

In quark logic the implications operators are **4**-potent :

$$\underline{\rightarrow}^4 = \underline{\rightarrow}, \qquad \underline{\leftarrow}^4 = \underline{\leftarrow}$$

Proof

$$\underline{\rightarrow}^2 = \underline{\leftarrow}, \quad \underline{\rightarrow}^3 = \underline{\mathbf{N}}, \quad \underline{\rightarrow}^4 = \underline{\rightarrow}$$

$$\underline{\leftarrow}^2 = \underline{\rightarrow}, \quad \underline{\leftarrow}^3 = \underline{\mathbf{N}}, \quad \underline{\leftarrow}^4 = \underline{\leftarrow}$$

Arbitrary transformations in the **Ç(16,9)** code are given by (3×3) matrices. To have a realizable quantum theory we need unitary matrices. In the **Ç(16,9)** code the condition of unitarity has to be modified because the role of identity is realized by **NOT**, and not by **YES**. Hence the operator is unitary if $\mathbf{L} \cdot \mathbf{L}^* = \mathbf{NOT}$. There are the quadruplet of such operators, all of which are symmetric:

$$\mathbf{YES} \cdot \mathbf{YES} = \mathbf{NOT} \cdot \mathbf{NOT} = \mathbf{OR} \cdot \mathbf{OR} = \mathbf{NAND} \cdot \mathbf{NAND} = \mathbf{NOT}.$$

The asymmetric implications **IMPLY** and **IF** are distinguished as the only unitary pair:

$$\underline{\rightarrow} \cdot \underline{\leftarrow} = \underline{\mathbf{N}}, \qquad \underline{\leftarrow} \cdot \underline{\rightarrow} = \underline{\mathbf{N}}.$$

The unitary (3×3) matrices form a three-dimensional representation of the **SU(3)** group, which is well known in nuclear physics but new to logic.

According to the properties of the **SU(3)** group, an arbitrary transformation of the states is given by **8** real parameters. The octal oxygen code, performing a search in atoms, is the physically suitable quantum system.

The double parity **mod2** code provides for the **supersymmetry**

$$[A, B] = \{A, B\}.$$

which in the standard model is possible only for orthogonal matrices yielding zero products. Many other logical relations turn upside-down in the double parity code, very much as a visual image passing through eye turns upside-down on the retina. Noncommuting $\wedge$ and $\vee$: **[AND, OR] ≠ 0**, now commute:

$$[\underline{\wedge}, \underline{\vee}] = \underline{\mathbf{0}},$$

and of course supercommute:

$$\{\underline{\wedge}, \underline{\vee}\} = \mathbf{0},$$

as well as a number of other, previously noncommuting, products:

$$\{\underline{\rightarrow}, \underline{\leftarrow}\} = \{\underline{\uparrow}, \underline{\downarrow}\} = \{\underline{\backslash}, \underline{/}\} = \{\underline{\perp}, \underline{\dashv}\} = \{\underline{\mathrm{I}}, \underline{\vdash}\} = \mathbf{0}$$

The operator **YES** emerges with three important commutations:

$$\{\underline{\mathbf{YES}}, \underline{\mathbf{NOT}}\} = \{\underline{\mathbf{YES}}, \underline{\mathbf{NOR}}\} = \{\underline{\mathbf{YES}}, \underline{\mathbf{ONE}}\} = \mathbf{0},$$

which is a dual concomitant to commutators in standard model:

$$[\mathbf{NOT}, \mathbf{YES}] = [\mathbf{NOT}, \mathbf{ONE}] = [\mathbf{NOT}, \mathbf{H}] = \mathbf{0}$$

Clearly, the operator **NOT** supercommutes with any codeword:

$$\{\underline{\mathbf{NOT}}, \underline{\mathbf{L}}\} = \mathbf{0}.$$

{,}	I	N	$\wedge$	$\vee$	$\rightarrow$	$\leftarrow$	$\downarrow$	$\uparrow$	\	/	$\perp$	$\dashv$	I	$\vdash$	1	0
I	0	0						0							0	0
N	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
$\wedge$		0	0	0												0
$\vee$		0	0	0												0
$\rightarrow$		0			0	0										0
$\leftarrow$		0			0	0										0
$\downarrow$		0					0	0								0
$\uparrow$	0	0					0	0								0
\		0							0	0						0
/		0							0	0						0
$\perp$		0									0	0				0
$\dashv$		0									0		0			0
I		0										0		0		0
$\vdash$		0											0	0		0
1	0	0													0	0
0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Table. 7
Supercommuting products in quark logic

MATRIX BASE

The additive matrix base of matrix logic {**AND, ON, OFF, NOR**} is a set of linear independent matrices from which any logical matrix is obtained as a linear combination

$$\mathbf{L} = \alpha\begin{pmatrix}0&0\\0&1\end{pmatrix}+\beta\begin{pmatrix}0&0\\1&0\end{pmatrix}+\gamma\begin{pmatrix}0&1\\0&0\end{pmatrix}+\omega\begin{pmatrix}1&0\\0&0\end{pmatrix}$$

with the binary factors $\alpha,\beta,\gamma,\omega = 0,1$. The same property obtains for the matrix base {**AND**, **ON**, **OFF**, **NOR**} in quark logic, where any (3×3) operator is a linear combination

$$\underline{\mathbf{L}} = \alpha\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}+\beta\begin{pmatrix}0&0&0\\1&0&1\\1&0&1\end{pmatrix}+\gamma\begin{pmatrix}0&1&1\\0&0&0\\0&1&1\end{pmatrix}+\omega\begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix}$$

We know that the elements of the matrix base {**AND, ON, OFF, NOR**} are the tensor products of the true $|1\rangle$ and false $|0\rangle$ logical states:

$$|1\rangle\langle 1| = \begin{pmatrix}0&0\\0&1\end{pmatrix}, \qquad |1\rangle\langle 0| = \begin{pmatrix}0&0\\1&0\end{pmatrix},$$

$$|0\rangle\langle 1| = \begin{pmatrix}0&1\\0&0\end{pmatrix}, \qquad |0\rangle\langle 0| = \begin{pmatrix}1&0\\0&0\end{pmatrix}.$$

Taking a clue from the standard model, we can derive corresponding tensor products in quark logic:

$$\begin{pmatrix}0\\1\\1\end{pmatrix}(0\,1\,1) = \begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} = \underline{\wedge}, \qquad \begin{pmatrix}0\\1\\1\end{pmatrix}(1\,0\,1) = \begin{pmatrix}0&0&0\\1&0&1\\1&0&1\end{pmatrix} = \underline{\backslash},$$

$$\begin{pmatrix}1\\0\\1\end{pmatrix}(0\,1\,1) = \begin{pmatrix}0&1&1\\0&0&0\\0&1&1\end{pmatrix} = \underline{/}, \qquad \begin{pmatrix}1\\0\\1\end{pmatrix}(1\,0\,1) = \begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix} = \underline{\uparrow}.$$

where the true and false logical states are represented by the vectors:

$$|\underline{1}\rangle = \begin{pmatrix}0\\1\\1\end{pmatrix}, \qquad |\underline{0}\rangle = \begin{pmatrix}1\\0\\1\end{pmatrix}$$

Having assembled the key elements of quark logic, we now can proceed to design an operational logical calculus. This entails the computation of the atomic products $\langle\underline{x}|\underline{\mathbf{L}}|\underline{y}\rangle$ and fundamental strings composed of such products.

An unexpected result of this analysis is that the 3-dimensional quark logic and the standard model are linked as logical complements. The complement of the (3×3) atomic product equals the atomic product of the corresponding (2×2) logical dual

$$\overline{\langle\underline{x}|\underline{\mathbf{L}}|\underline{y}\rangle} = \langle x|\mathbf{L}^*|y\rangle.$$

Parity rotates unitarily: quark (3×3) logic is a dual unitary image of (2×2) matrix logic:

$$<\underline{x}|\underline{L}|\underline{y}> = <x|\theta\mathbf{L}|y>.$$

As opposed to the contraction of the complement in standard model:

$$\overline{<x|\mathbf{L}|y>} = <x|\bar{\mathbf{L}}|y>,$$

the complement operator in quark logic is a scalar operator and is not contractable

$$\overline{<\underline{x}|\underline{\mathbf{L}}|\underline{y}>} \neq <\underline{x}|\underline{\bar{\mathbf{L}}}|\underline{y}>.$$

This is a direct consequence of the invariance of the parity bits, for an operator **L**, denoted by $\hookleftarrow(\mathbf{L})$, under complementation:

$$\hookleftarrow(\mathbf{L}) \equiv \hookleftarrow(\bar{\mathbf{L}})$$

while

$$\underline{\mathbf{L}} \neq \underline{\bar{\mathbf{L}}}$$

The reduction of the connective operator set to the basis set takes into account the reversal of the roles **NOT** and **YES** in quark logic. Whereas

$$<x|\rightarrow|y> = <x|\mathbf{NOT}\cdot\mathbf{OR}|y>$$

in quark logic

$$<\underline{x}|\underline{\rightarrow}|\underline{y}> = <\underline{x}|\underline{\mathbf{YES}}\cdot\underline{\mathbf{OR}}|\underline{y}>$$

DeMorgan's dualities in quark logic also reflect the changeover of the roles of the negation and affirmation operators:

$$<\underline{x}|\underline{\wedge}|\underline{y}> = \overline{<\underline{x}|\theta\underline{\vee}|\underline{y}>}, \qquad <\underline{x}|\underline{\vee}|\underline{y}> = \overline{<\underline{x}|\theta\underline{\wedge}|\underline{y}>}$$

where

$$\theta\underline{\mathbf{L}} = \underline{\mathbf{YES}}\cdot\underline{\mathbf{L}}\cdot\underline{\mathbf{YES}}$$

corresponds to the canonical unitary transformation in matrix logic

$$\theta\mathbf{L} = \mathbf{NOT}\cdot\mathbf{L}\cdot\mathbf{NOT}$$

It is important to keep in mind that the dual nature of the complement operator in standard model does not apply in quark logic. Whereas in the standard model

$$\overline{<x|\mathbf{L}|y>} = <x|\bar{\mathbf{L}}|y>$$

and the scalar and operator complements cancel out:

$$\overline{<x|\bar{\mathbf{L}}|y>} = <x|\mathbf{L}|y>$$

in quark logic one must carefully distinguish between the scalar and operator complements (modes?) which are different:

$$\overline{<\underline{x}|\underline{\mathbf{L}}|\underline{y}>} \neq <\underline{x}|\underline{\bar{\mathbf{L}}}|\underline{y}>.$$

Moreover, since

$$<\underline{x}|\underline{\bar{\mathbf{L}}}|\underline{y}> = <\underline{x}|\underline{\mathbf{L}}|\underline{y}>$$

it follows that

$$\overline{<\underline{x}|\underline{\mathbf{L}}|\underline{y}>} = \overline{<\underline{x}|\underline{\bar{\mathbf{L}}}|\underline{y}>}$$

where the complements **do not cancel out!** This has a bearing on the DeMorgan equalities, since both

$$<\underline{x}|\underline{\wedge}|\underline{y}> = \overline{<\underline{x}|\theta\underline{\vee}|\underline{y}>}, \qquad <\underline{x}|\underline{\vee}|\underline{y}> = \overline{<\underline{x}|\theta\underline{\wedge}|\underline{y}>}$$

and

$$\langle \underline{x}|\underline{\wedge}|\underline{y}\rangle = \overline{\langle \underline{x}|\overline{\theta}\underline{\vee}|\underline{y}\rangle}, \qquad \langle \underline{x}|\underline{\vee}|\underline{y}\rangle = \overline{\langle \underline{x}|\overline{\theta}\underline{\wedge}|\underline{y}\rangle}$$

are legitimate. We illustrate this with the following specific computation:

$$\langle \underline{x}|\underline{\wedge}|\underline{y}\rangle = \overline{\langle \underline{x}|\theta\underline{\vee}|\underline{y}\rangle} = \overline{\langle \underline{x}|\underline{\downarrow}|\underline{y}\rangle} = \langle \underline{x}|\overline{\underline{\downarrow}}|\underline{y}\rangle$$

(1)

$$\langle \underline{0}|\underline{\wedge}|\underline{0}\rangle = (101)\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = (101)\begin{pmatrix}0\\1\\1\end{pmatrix} = 1$$

$$\langle \underline{0}|\underline{\wedge}|\underline{1}\rangle = (101)\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix} = (101)\begin{pmatrix}0\\0\\0\end{pmatrix} = 0$$

$$\langle \underline{1}|\underline{\wedge}|\underline{0}\rangle = (011)\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = (011)\begin{pmatrix}0\\1\\1\end{pmatrix} = 0$$

$$\langle \underline{1}|\underline{\wedge}|\underline{1}\rangle = (011)\begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix} = (011)\begin{pmatrix}0\\0\\0\end{pmatrix} = 0$$

(2)

$$\overline{\langle \underline{0}|\underline{\downarrow}|\underline{0}\rangle} = \overline{(101)\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}} = \overline{(101)\begin{pmatrix}1\\0\\1\end{pmatrix}} = \overline{0} = 1$$

$$\overline{\langle \underline{0}|\underline{\downarrow}|\underline{1}\rangle} = \overline{(101)\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}} = \overline{(101)\begin{pmatrix}1\\1\\0\end{pmatrix}} = \overline{1} = 0$$

$$\overline{\langle \underline{1}|\underline{\downarrow}|\underline{0}\rangle} = \overline{(011)\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}} = \overline{(011)\begin{pmatrix}1\\0\\1\end{pmatrix}} = \overline{1} = 0$$

$$\overline{\langle \underline{1}|\underline{\downarrow}|\underline{1}\rangle} = \overline{(011)\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}} = \overline{(011)\begin{pmatrix}1\\1\\0\end{pmatrix}} = \overline{1} = 0$$

The results of the computation are the same. Complementating the operator produces no observable effect:

(3)

$$\overline{\langle \underline{0}|\underline{\bar{\downarrow}}|\underline{0}\rangle} = \overline{(101)\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}} = \overline{(101)\begin{pmatrix}1\\0\\1\end{pmatrix}} = \bar{0} = 1$$

$$\overline{\langle \underline{0}|\underline{\bar{\downarrow}}|\underline{1}\rangle} = \overline{(101)\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}\begin{pmatrix}1\\1\\0\end{pmatrix}} = \overline{(101)\begin{pmatrix}1\\1\\0\end{pmatrix}} = \bar{1} = 0$$

$$\overline{\langle \underline{1}|\underline{\bar{\downarrow}}|\underline{0}\rangle} = \overline{(011)\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix}} = \overline{(011)\begin{pmatrix}1\\0\\1\end{pmatrix}} = \bar{1} = 0$$

$$\overline{\langle \underline{1}|\underline{\bar{\downarrow}}|\underline{1}\rangle} = \overline{(011)\begin{pmatrix}0&0&1\\0&1&0\\1&0&0\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix}} = \overline{(011)\begin{pmatrix}1\\1\\0\end{pmatrix}} = \bar{1} = 0$$

Recall that the roles of **NOT** and **YES** have changed over now: **NOT** affirms, and **YES** negates, in total reversal to standard matrix logic. This helps us to understand a complex behaviour of quark logical vectors. Whereas in the standard model

$$|x\rangle = \mathbf{YES}|x\rangle, \qquad |\bar{x}\rangle = \mathbf{NOT}|x\rangle$$

in quark logic

$$|\underline{x}\rangle = \underline{\mathbf{NOT}}|\underline{x}\rangle$$

whereas

$$|\underline{0}\rangle = \underline{\mathbf{YES}}|\underline{1}\rangle, \qquad |\underline{1}\rangle = \underline{\mathbf{YES}}\,|\underline{0}\rangle.$$

Explicitly:

$$\begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}\begin{pmatrix}0\\1\\1\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}, \qquad \begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}\begin{pmatrix}1\\0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix}.$$

Whereas in the standard model the true and false are orthonormal and complementary

$$\langle x|x\rangle = 1, \qquad \langle x|\bar{x}\rangle = 0.$$

in quark logic they are not complementary

$$|\bar{\underline{0}}\rangle \neq |\underline{1}\rangle$$

and congruently with the **TIME**'s eigenstates, they are not orthogonal,

$$\langle \underline{1}\,|\underline{0}\rangle \neq 0$$

but **mod2** self-dual

$$\langle \underline{0}\,|\underline{0}\rangle = 0, \qquad \langle \underline{1}\,|\underline{1}\rangle = 0$$

and mutually normalized

$$\langle \underline{1}\,|\underline{0}\rangle = 1.$$

Complementation of quark logical states $|\underline{0}>$ and $|\underline{1}>$ deactivates the parity bits:

$$|\bar{\underline{0}}> = \begin{pmatrix}0\\1\\0\end{pmatrix}, \qquad |\bar{\underline{1}}> = \begin{pmatrix}1\\0\\0\end{pmatrix}.$$

This restores the orthogonality

$$<\bar{\underline{0}}|\bar{\underline{1}}> = 0,$$

and entails the correspondence

$$<\bar{\underline{x}}|\underline{L}|\bar{\underline{y}}> = <\bar{x}|L|\bar{y}>$$

where one simply throws away all parity bits which give the equivalent standard (2×2) model with unitarily rotated operators:

$$<\bar{\underline{x}}|\underline{L}|\bar{\underline{y}}> = <x|\theta L|x>$$

Likewise

$$<\underline{x}|\underline{L}|\underline{y}> = <x|\theta L|y>$$

since quark logic with parity checks is indistinguishable from complementary logic with orthonormal states without parity:

$$<\underline{x}|\underline{L}|\underline{y}> = <\bar{\underline{x}}|\underline{L}|\bar{\underline{y}}>$$

Puncturing errors convert quark logical states to canonical states, but **asymmetrically** for the true $|\underline{1}>$ and false $|\underline{0}>$. Indicating by subscript the position of a puncturing error, one can convert $|\underline{0}>$ into a true, false or superposed state,

$$\Lambda_1(|\underline{0}>) = \begin{pmatrix}0\\1\end{pmatrix}, \qquad \Lambda_2(|\underline{0}>) = \begin{pmatrix}1\\1\end{pmatrix}, \qquad \Lambda_3(|\underline{0}>) = \begin{pmatrix}1\\0\end{pmatrix},$$

but the true state $|\underline{1}>$ cannot be punctured into a false state:

$$\Lambda_1(|\underline{1}>) = \begin{pmatrix}1\\1\end{pmatrix}, \qquad \Lambda_2(|\underline{1}>) = \begin{pmatrix}0\\1\end{pmatrix}, \qquad \Lambda_3(|\underline{1}>) = \begin{pmatrix}0\\1\end{pmatrix}.$$

Conversion to decimals

Complementation produces a less secure code without parity but with remarkable singularities where one is at liberty to invariantly change over from an operator to the complement and vice versa. In the standard model the scalar complement operator acts as the matrix complement. In particular

$$\overline{<x|y>} = <x|\overline{\mathbf{YES}}|y> = <x|\mathbf{NOT}|y>$$

Quark logic violates the operator complement rule of the standard model. Whereas

$$<x|\mathbf{YES}|y> \neq <x|\overline{\mathbf{YES}}|y>$$

$$<x|\mathbf{NOT}|y> \neq <x|\overline{\mathbf{NOT}}|y>$$

in the double parity code the equivalence and inequivalence functions remain invariant under complementation:

$$<\underline{x}|\underline{\mathbf{NOT}}|\underline{y}> = <\underline{x}|\overline{\underline{\mathbf{NOT}}}|\underline{y}>$$

$$<\underline{x}|\underline{\mathbf{YES}}|\underline{y}> = <\underline{x}|\overline{\underline{\mathbf{YES}}}|\underline{y}>$$

We know that in quark logic the operator **NOT** is affirmation, and therefore

$$<\underline{x}|\underline{\mathbf{NOT}}|\underline{y}> = <\underline{x}|\underline{y}>$$

or explicitly

$$<\underline{x}|\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}|\underline{y}> = <\underline{x}|\underline{y}>$$

On the other hand it is true that

$$<\underline{x}|\underline{y}> = <\underline{x}|\mathbf{I}|\underline{y}>$$

or explicitly

$$<\underline{x}|\underline{y}> = <\underline{x}|\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}|\underline{y}>,$$

where

$$\mathbf{I} = \begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}$$

is the (3×3) unit matrix. Combining the two results we get the equality

$$<\underline{x}|\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}|\underline{y}> = <\underline{x}|\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}|\underline{y}>,$$

where the two matrices are complements of each other. Thus we have proven the invariance:

$$<\underline{x}|\underline{\mathbf{NOT}}|\underline{y}> = <\underline{x}|\overline{\underline{\mathbf{NOT}}}|\underline{y}>$$

We now prove the similar invariance for **YES**, which in quark logic acts as the negation operator. In the atomic product

$$<\underline{x}|\underline{\mathbf{YES}}|\underline{y}> = <\underline{x}|\begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}|\underline{y}>$$

the operator **YES**, applied to true or false states

$$\begin{pmatrix}0\\1\\1\end{pmatrix} \quad \text{or} \quad \begin{pmatrix}1\\0\\1\end{pmatrix},$$

changes over bits in the first and second position while leaving the third position intact. Consequently the action of the operator is identical to the action of the perturbation matrix

$$\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}$$

Consequently, it is legitimate to write the equality

$$\langle \underline{x}| \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{pmatrix} |\underline{y}\rangle = \langle \underline{x}| \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} |\underline{y}\rangle,$$

in which one determines that the two operators in both sides are complements of each other. We thus proven the invariance

$$\langle \underline{x}|\underline{\mathbf{YES}}\,|\underline{y}\rangle = \langle \underline{x}|\overline{\underline{\mathbf{YES}}}\,|\underline{y}\rangle$$

The two proofs, which we have obtained by making use of special properties of **NOT** and **YES**, are the particular cases of the invariance of the atomic bracket under complementation in quark logic. We now prove the general rule.

Theorem

$$\langle \underline{x}|\overline{\underline{L}}\,|\underline{y}\rangle = \langle \underline{x}|\underline{L}\,|\underline{y}\rangle$$

Proof

$$\begin{aligned} \langle \underline{x}|\overline{\underline{L}}\,|\underline{y}\rangle &= \langle \underline{x}| \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} - \underline{L}\,|\underline{y}\rangle \\ &= \langle \underline{x}| \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} |\underline{y}\rangle - \langle \underline{x}|\underline{L}\,|\underline{y}\rangle \\ &= 0 - \langle \underline{x}|\underline{L}\,|\underline{y}\rangle \\ &= \langle \underline{x}|\underline{L}\,|\underline{y}\rangle \end{aligned}$$

In the standard model the world gets inverted in the mirror; in quark logic it stays the same because it does not have sides (is unilateral) very much like in René Margritte's painting. Clearly for any quark codeword

$$\underline{L} \in \text{Ç}(16,9) \qquad \text{but} \qquad \overline{\underline{L}} \notin \text{Ç}(16,9)$$

and of course the complementation destroys parity. Significantly, the complement of the quark identity **NOT** is actual matrix identity, while the complement of **NAND** is actual matrix negation:

$$\overline{\underline{\mathbf{NOT}}} = \overline{\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \overline{\underline{\mathbf{NAND}}} = \overline{\begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

Naturally

$$\left(\overline{\underline{\mathbf{NAND}}}\right)^2 = \overline{\underline{\mathbf{NOT}}}$$

but also

$$\left(\underline{\mathbf{NAND}}\right)^2 = \underline{\mathbf{NOT}}$$

which is one of the four fundamental squarings to the quark identity:

$$\left(\underline{\mathbf{OR}}\right)^2 = \underline{\mathbf{NOT}}, \qquad \left(\underline{\mathbf{NOT}}\right)^2 = \underline{\mathbf{NOT}}, \qquad \left(\underline{\mathbf{YES}}\right)^2 = \underline{\mathbf{NOT}}.$$

TRANSMULTIPLICATION

In **standard** model matrix multiplication is restricted by the set of forbidden interactions while transmultiplication applies universally:

$$\langle x|A|y\rangle\langle x|B|y\rangle = x|A \blacklozenge B|y\rangle.$$

In quark logic it is the other way round: the matrix product is unrestricted, but transmultiplication generally does not apply because it destroys the required parity code. There is, however, a notable exception to this general rule, where the code loses parity, yet continues to compute properly.

First we recall the two fundamental transmultiplications in the standard model:

$$\langle x|{\rightarrow}|y\rangle\langle x|{\leftarrow}|y\rangle = \langle x|{\rightarrow}\blacklozenge{\leftarrow}|y\rangle = \langle x|y\rangle$$

due to

$${\rightarrow}\blacklozenge{\leftarrow} = \mathbf{YES}$$

and

$$\langle x|\vee|y\rangle\langle x|\downarrow|y\rangle = x|\vee\blacklozenge\downarrow|y\rangle = x|\mathbf{NOT}|y\rangle$$

due to

$$\vee\blacklozenge\downarrow = \mathrm{NOT}$$

Transmultiplication theorem

(a)

$$\langle \underline{x}|\underline{\rightarrow}|\underline{y}\rangle\langle \underline{x}|\underline{\leftarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{NOT}}|\underline{y}\rangle$$

(b)

$$\langle \underline{x}|\underline{\vee}|\underline{y}\rangle\langle \underline{x}|\underline{\downarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{YES}}|\underline{y}\rangle$$

Proof

The proofs for **(a)** and **(b)** are similar.

(a)

$$\langle \underline{x}|\underline{\rightarrow}|\underline{y}\rangle\langle \underline{x}|\underline{\leftarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\rightarrow}\blacklozenge\underline{\leftarrow}|\underline{y}\rangle$$

$$= \langle \underline{x}|\begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}\blacklozenge\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}|\underline{y}\rangle$$

$$= \langle \underline{x}|\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}|\underline{y}\rangle$$

$$= \langle \underline{x}|\begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{NOT}}|\underline{y}\rangle$$

The last step in the proof is due to the theorem: $\langle \underline{x}|\overline{\underline{\mathbf{L}}}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{L}}|\underline{y}\rangle$.

(b)

$$\langle \underline{x}|\underline{\vee}|\underline{y}\rangle\langle \underline{x}|\underline{\downarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{YES}}|\underline{y}\rangle$$

Proof

$$\langle \underline{x}|\underline{\vee}|\underline{y}\rangle\langle \underline{x}|\underline{\downarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\vee}\blacklozenge\underline{\downarrow}|\underline{y}\rangle$$

$$= \langle \underline{x}|\begin{pmatrix}0&1&1\\1&1&0\\1&0&1\end{pmatrix}\blacklozenge\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}|\underline{y}\rangle$$

$$= \langle \underline{x}|\begin{pmatrix}0&1&0\\1&0&0\\0&0&1\end{pmatrix}|\underline{y}\rangle$$

$$= \langle \underline{x}|\begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{YES}}|\underline{y}\rangle$$

Once again the last step in the proof follows from the theorem:

$$\langle \underline{x}|\overline{\underline{\mathbf{L}}}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{L}}|\underline{y}\rangle$$

Theorem

In the atomic product quark logic does not distinguish between transmultiplication and matrix multiplication of implications:

$$\langle \underline{x}|\underline{\rightarrow}\blacklozenge\underline{\leftarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\rightarrow}\bullet\underline{\leftarrow}|\underline{y}\rangle$$

Proof

In the preceding theorem we have determined that

$$\langle \underline{x}|\underline{\rightarrow}\blacklozenge\underline{\leftarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{NOT}}|\underline{y}\rangle$$

But the pair of operators $\underline{\rightarrow}$ and $\underline{\leftarrow}$ is the only pair of two different logical operators whose matrix product is **NOT**:

$$\underline{\rightarrow}\bullet\underline{\leftarrow} = \underline{\mathbf{NOT}}.$$

This, combined with the property

$$\langle \underline{x}|\overline{\underline{\mathbf{L}}}|\underline{y}\rangle = \langle \underline{x}|\underline{\mathbf{L}}|\underline{y}\rangle$$

proves the theorem.

Implications in quark logic are also characterised by suprasymmetry, a remarkable and unique property of invariance vis-à-vis different matrix logical operations:

$$\underline{\rightarrow}\bullet\underline{\leftarrow} = \underline{\rightarrow}+\underline{\leftarrow} = \underline{\mathbf{N}}$$

Since it is also true that

$$\underline{\rightarrow}\blacklozenge\underline{\leftarrow} = \overline{\underline{\mathbf{N}}},$$

the interaction of implications, be it transmultiplicative, additive or multiplicative, will retain the invariance of the atomic product

$$\langle \underline{x}|\underline{\rightarrow}\blacklozenge\underline{\leftarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\rightarrow}+\underline{\leftarrow}|\underline{y}\rangle = \langle \underline{x}|\underline{\rightarrow}\bullet\underline{\leftarrow}|\underline{y}\rangle$$

Whatever the nature of a matrix operation introduced deliberately or by a coding error between the implication operators, the result of the evaluation is unchanged.

BASIS SETS

The irreducible fundamental basis sets in the standard model and in quark logic are both triplets of logical operators, with quark logic taking into account the reverse roles for negation and affirmation:

STANDARD MODEL {NOT, AND, OR}	QUARK LOGIC {YES, AND, OR}
N	I
∧	△
∨	⩒
I = N N	N = I I
\ = ∧ N	⟍ = △ I
/ = N ∧	⟋ = I △
↑ = N ∧ N	↥ = I △ I
→ = N ∨	⇉ = I ⩒
← = ∨ N	⇇ = ⩒ I
↓ = N ∨ N	↧ = I ⩒ I
⊥ = ∧ ∨	⊥ = △ I ⩒
⊣ = ∨ ∧	⫤ = ⩒ I △
I = N ∧ ∨	I = I △ I ⩒
⊢ = ∨ ∧ N	⫣ = ⩒ I △ I
1 = ∨ ∧ ∨	1 = ⩒ I △ I ⩒
0 = ∧ N ∧	0 = △ △

The coding theory approach leads to a new way of defining a logical basis. A fundamental combinatorial minor basis of quark logic, a counterpart of {NOT, AND} in the standard model, is the basis {YES , AND}, since any (3×3) logical operator, excluding the vacuum, is obtained from the two matrices by recombining the rows and columns in YES and AND. In this manner the AND gives rise to the octet of operators

$$\text{AND} \Rightarrow \{ \diagdown, \diagup, \uparrow, \mathbf{1}, \perp, \mathrm{I}, \dashv, \vdash \}$$

and YES to the following five operators

$$\text{YES} \Rightarrow \{ \mathrm{N}, \veebar, \rightrightarrows, \leftleftarrows, \downarrow \}$$

In a peculiar way this reflects the fact that the **16** Boolean operators are divided into a subset of **6** nonsingular and subset **10** singular operators, which correspond, respectively, to the sextet

$$\{ \mathrm{I}, \mathrm{N}, \veebar, \rightrightarrows, \leftleftarrows, \downarrow \}$$

and the decuplet

$$\{\underline{\Delta}, \underline{\wedge}, \underline{\angle}, \underline{\uparrow}, \underline{\perp}, \underline{\mathrm{I}}, \underline{\dashv}, \underline{\vdash}, \underline{\mathbf{1}}, \underline{\mathbf{0}}\}$$

The corresponding sextet and decuplet are the same in the standard model and in quark logic. However, in contrast to the standard model, in quark logic these subsets are the coding derivatives of the basis set, revealing a new fundamental feature of logical theory, comprising singular and nonsingular operations.

In classical logic one distinguishes between logical variables and logical connective operators. Matrix theory has contributed to a further understanding of logic, showing that logical operators can be derived as tensor products of logical states, a fundamental innovation which extends to quark logic. With a single outer product we generate the decuplet of operators:

$$\{\underline{\Delta}, \underline{\wedge}, \underline{\angle}, \underline{\uparrow}, \underline{\perp}, \underline{\mathrm{I}}, \underline{\perp}, \underline{\dashv}, \underline{\vdash}, \underline{\mathbf{1}}, \underline{\mathbf{0}}\}$$

Explicitly:

$$\underline{\Delta} = |\underline{1}\rangle\langle\underline{1}|, \quad \underline{\wedge} = |\underline{1}\rangle\langle\underline{0}|, \quad \underline{\angle} = |\underline{0}\rangle\langle\underline{1}|, \quad \underline{\uparrow} = |\underline{0}\rangle\langle\underline{0}|,$$
$$\underline{\perp} = |\underline{1}\rangle\langle\underline{1}|, \quad \underline{\mathrm{I}} = |\underline{0}\rangle\langle\underline{1}|, \quad \underline{\dashv} = |\underline{1}\rangle\langle\underline{1}|, \quad \underline{\vdash} = |\underline{1}\rangle\langle\underline{0}|,$$
$$\underline{\mathbf{1}} = |\underline{1}\rangle\langle\underline{1}|, \quad \underline{\mathbf{0}} = |\underline{0}\rangle\langle\underline{0}|,$$

This corresponds to the decuplet of singular logical operators generated as the outer product in the standard model. The sextet of nonsingular operators

$$\underline{\mathbf{YES}}, \underline{\mathbf{NOT}}, \underline{\mathbf{OR}}, \underline{\mathbf{IMPLY}}, \underline{\mathbf{IF}}, \underline{\mathbf{NAND}}$$

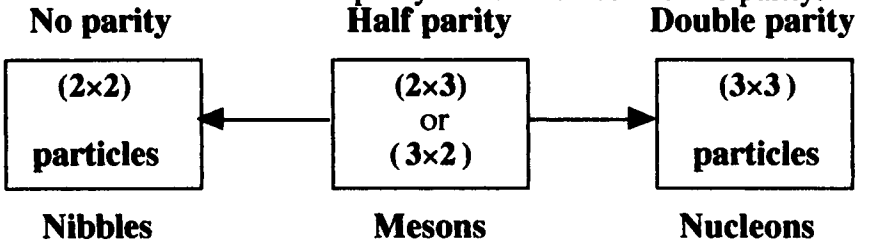
with $\det \underline{\mathbf{L}} = \pm 2$ are linear combinations of these product, just as

$$\mathbf{YES}, \mathbf{NOT}, \mathbf{OR}, \mathbf{IMPLY}, \mathbf{IF}, \mathbf{NAND}$$

with $\det \mathbf{L} = \pm 1$ are linear combinations in the standard model.

LOGICAL MESONS

An important contribution of quark logic to logical inquiry lies in introducing entirely new class of logical objects which are neither logical states nor logical operators but occupy a middle ground between these two key elements. These are the rectangular (2×3) and (3×2) logical matrices. If one defines the 3-component logical states as quarks, and 9-component logical operators, comprising 3 logical quarks, as nucleons, then (2x3) and (3x2) matrices, containing 2 quarks, are new fundamental logical objects, which one naturally defines as mesons. Mesons are distinguished by half-parity, in contrast to nucleons with double parity and to nibbles with no parity:



Logical mesons, denoted by π, can realise several important functions. Acting on matrix logical states $|x\rangle$, an appropriate meson, such as $\pi(\underline{0}\ \underline{1})$, adds a required parity check, transforming $|x\rangle$ into the corresponding logical quark

$$\pi|x\rangle = |\underline{x}\rangle$$

Explicitly

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}|1\rangle = \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\1\\1\end{pmatrix} = |\underline{1}\rangle,$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}|0\rangle = \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix} = |\underline{0}\rangle,$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}|\mathbf{1}\rangle = \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}1\\1\\0\end{pmatrix} = |\underline{\mathbf{1}}\rangle,$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}|\mathbf{0}\rangle = \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix} = |\underline{\mathbf{0}}\rangle.$$

Mesons are the generators of quark logic

Any (3×3) logical operator is the product interaction $\underline{\mathbf{L}} = \pi \bullet \tilde{\pi}$ where π is a (3×2) meson, and $\tilde{\pi}$ is a (2×3) meson, for example:

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&1&1\end{pmatrix} = \begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix} = \underline{\mathbf{I}}, \qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&1&1\\1&0&1\end{pmatrix} = \begin{pmatrix}0&1&1\\1&0&1\\1&1&0\end{pmatrix} = \underline{\mathbf{N}},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&1&1\\1&1&0\end{pmatrix} = \begin{pmatrix}0&1&1\\1&1&0\\1&0&1\end{pmatrix} = \underline{\vee}, \qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&1&0\\1&0&1\end{pmatrix} = \begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix} = \underline{\downarrow},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&1&0\\0&1&1\end{pmatrix} = \begin{pmatrix}1&1&0\\0&1&1\\1&0&1\end{pmatrix} = \underline{\rightarrow}, \qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&0&1\\1&1&0\end{pmatrix} = \begin{pmatrix}1&0&1\\1&1&0\\0&1&1\end{pmatrix} = \underline{\leftarrow},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&0&0\\0&1&1\end{pmatrix} = \begin{pmatrix}0&0&0\\0&1&1\\0&1&1\end{pmatrix} = \underline{\wedge}, \qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&0&0\end{pmatrix} = \begin{pmatrix}1&0&1\\0&0&0\\1&0&1\end{pmatrix} = \underline{\uparrow},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&0&0\\1&0&1\end{pmatrix}=\begin{pmatrix}0&0&0\\1&0&1\\1&0&1\end{pmatrix}=\underline{\setminus},\qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&1&1\\0&0&0\end{pmatrix}=\begin{pmatrix}0&1&1\\0&0&0\\0&1&1\end{pmatrix}=\underline{/},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&0&0\\1&1&0\end{pmatrix}=\begin{pmatrix}0&0&0\\1&1&0\\1&1&0\end{pmatrix}=\underline{\perp},\qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&1&1\\0&1&1\end{pmatrix}=\begin{pmatrix}0&1&1\\0&1&1\\0&0&0\end{pmatrix}=\underline{\dashv},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&1&0\\0&0&0\end{pmatrix}=\begin{pmatrix}1&1&0\\0&0&0\\1&1&0\end{pmatrix}=\underline{\top},\qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&0&1\\1&0&1\end{pmatrix}=\begin{pmatrix}1&0&1\\1&0&1\\0&0&0\end{pmatrix}=\underline{\vdash},$$

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&1&0\\1&1&0\end{pmatrix}=\begin{pmatrix}1&1&0\\1&1&0\\0&0&0\end{pmatrix}=\underline{1},\qquad \begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}0&0&0\\0&0&0\end{pmatrix}=\begin{pmatrix}0&0&0\\0&0&0\\0&0&0\end{pmatrix}=\underline{0}.$$

Mesons are the generators of the standard model

Any matrix logical operator from the standard set is the product interaction of a pair of mesons:

$$\mathbf{L} = \tilde{\pi}\bullet\pi$$

where π is a (3×2) meson, and $\tilde{\pi}$ is a (2×3) meson, which is the transposition of

$$\underline{\mathbf{L}} = \pi\bullet\tilde{\pi}$$

where π is a (3×2) meson, and $\tilde{\pi}$ is a (2×3) meson. Take for example

$$\begin{pmatrix}1&0\\0&1\\1&1\end{pmatrix}\begin{pmatrix}1&0&1\\0&1&1\end{pmatrix}=\begin{pmatrix}1&0&1\\0&1&1\\1&1&0\end{pmatrix}=\underline{\mathbf{YES}},$$

If we transpose the mesons, which effectively leaves (2×3) meson unchanged and swaps columns in the (3×2) meson, we get the standard operator:

$$\begin{pmatrix}1&0&1\\0&1&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\\1&1\end{pmatrix}=\begin{pmatrix}1&0\\0&1\end{pmatrix}=\mathbf{YES}.$$

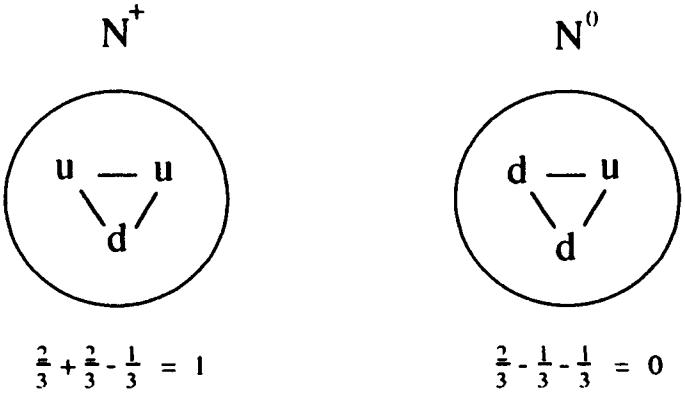
Similarly for all others. We see that quark logic is the 'outer' product and the standard model is the 'inner' product of mesons. Meson logic exchanges quark logic and standard matrix logic. It is possible to link both by duality: what is true at the quark level is false at the (2×2) level. The logical particle $\underline{\mathbf{L}}$ corresponds to the particle L, which is unitarily rotated, or equivalently to the negated dual particle:

$$\langle \underline{x}|\underline{\mathbf{L}}|\underline{y}\rangle = \langle x|\theta\mathbf{L}|y\rangle = \langle y|\overline{\mathbf{L}^{*}}|y\rangle.$$

Reversing the order of interacting mesons one exchanges (2×2) and (3×3) logics, making it natural to treat the logical operators as particles and meson logic as a field mediating the interaction.

MONOLOGIC

Quarks are the elementary building blocks of the heavy particles, the baryons. According to quantum chromodynamics mesons are formed of pairs of quark and antiquark, and nucleons of the triplets of **up** and **down** quarks with fractional charges 2/3 and -1/3 respectively. The integer electric charges, 0 and +1, of neutron N^0 and proton N^+ are the sum total of the charges of the constituent quarks:



In the modern theory, quark confinement is induced by a condensation of magnetic monopoles that is analogous to the condensation of Cooper pairs in a superconductor. The condensation of particles with electrical charges causes the expulsion of the magnetic field from the superconductor. Conversely, the condensation of particles with magnetic charges would cause the exclusion of the electric field. When the 'electromagnetism' is that of quarks and **SU(3)**, this is precisely the confinement of quarks.

If the topological and Noether charges in the brain are related in the same way as magnetism is related to electric charge, quark logic provides the framework for deriving the quark symmetries from logical considerations alone. When a three-dimensional logical state is populated with the quark charges, with the aid of the nonsingular logical operators of quark logic we can collect them into the integer charges of a proton or of a neutron:

$$(-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\underline{\mathbf{YES}}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = -\tfrac{1}{9}+\tfrac{8}{9}+\tfrac{8}{9} = 1$$

and

$$(-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\underline{\mathbf{NOT}}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = -\tfrac{4}{9}+\tfrac{2}{9}+\tfrac{2}{9} = 0$$

Likewise

$$(-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\underline{\mathbf{IMPLY}}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = (-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\underline{\mathbf{IF}}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = 1$$

and

$$(-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\underline{\mathbf{OR}}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = 0$$

None of the operators we employed to derive the integer charges is functionally complete. With the aid of the functionally complete monoconnective **NAND** we can isolate the quark charge:

$$(-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\underline{\mathbf{NAND}}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = (-\tfrac{1}{3}\ \tfrac{2}{3}\ \tfrac{2}{3})\begin{pmatrix}1&1&0\\1&0&1\\0&1&1\end{pmatrix}\begin{pmatrix}-\frac{1}{3}\\ \frac{2}{3}\\ \frac{2}{3}\end{pmatrix} = -\tfrac{1}{9}+\tfrac{2}{9}+\tfrac{2}{9} = \tfrac{1}{3}.$$

Monologic can be compared to monopole. Experimentally we never observe isolated monopoles because they are not elementary particles but collective excitations. In logic it does not necessarily 'take two to tango'. The minor functionally complete basis sets, like { **NOT**, **AND**} or { **NOT**, **OR**}, involve two operators. But monologics {**NAND**} or {**NOR**}, with a single functionally complete operator, also exist, changing a dialogue into a monologue. Logical 'monopoles' can be 'isolated' and given to consciousness directly.

CONSCIOUSNESS AS THE MACROANALOGUE OF QUARKS

Through a joint effort of matrix logic, topology and quantum theory, infophysics has made a considerable advance in accounting for thinking processes in the brain. Are we in a position to answer the question what thought is from the point of view of physics. We began with the formulation of the matrix principle and the conversion theorem, and determined that

cognitive logic requires a new statistics, Matrix Statistics, comprising Fermi-Dirac and Bose-Einstein as two eigenstate limits. Paradoxically, if thoughts are fermions they may occupy the same phase space, and if they are bosons, only a finite number of them can accumulate. In their unsuccessful search for the time operator, physicists always thought that it must carry a Hermitian eigenvalue, just as any quantum operator does. We introduce the time operator which carries a complex value and is observable only cognitively. Treating consciousness as a topological effect, in the context of gauge symmetry, we established dualities which connect the continuum of consciousness to the discretum of the quantum field and formulated explicitly the underlying law of the logical mind-brain:

$$IF^{IF} = e^{a^*}$$

This is the operator topological wave which encompasses the physics of consciousness. We have shown that logic can be formulated finitely in the language of the creation and annihilation operators, and that logical evaluation can be achieved as an ordering of operators reminiscent of the ordering in the second-quantized field theory. Consciousness is superquantized. According to the conversion theorem a quantum field theory can be converted into a logical calculus and, conversely, a logical theory based on matrix implication can be converted into a quantum theory with annihilation and creation. The nilpotent annihilation and creation operators appear to be as fundamental for logic as they are for fundamental physics. Found at the very foundation of logic, they enable us to derive logical theory in all its complexity from first principles. Finally we descended to the fundamental level of quarks, the frontier of modern physics.

Physics is married to facts and evidence, often deducing general principles by reverse reasoning. Logic is in love with axioms, and if these are guessed correctly, the facts must follow. However, the conceptual and computational reform accomplished in matrix logic reveals that logic, too, should not be invented but has to be studied in the same way as we study real-world physics. To capture the underlying physics of the conscious mind we must unify the concept of logic with the fundamental language of physics. While logic is mostly of concern to theorists and until recently was developing in a postulatory fashion, our objective was to transform it into an exact and eventually into an experimental science. After all, logic is the only window on the mind we have so far, and therefore, the more we know about the laws of logic, the more we know about the thinking brain. To understand how strange the brain is, we have to understand how simple it is. Consciousness is tied up to and inevitably utilizes physical degrees of freedom. The quarks and leptons, the building blocks of matter, are also the elementary building blocks of the brain. In the end it is an ordered system of atoms, and ultimately the system of leptons and quarks, which gives rise to consciousness. But the gap is wide. There is a great disparity between the energy scales at which the (bio)logical brain operates and those of quarks. At the same time we observe a perfect

numerical match between the logical charges of the nonnegative symmetric operators and the electric charges of the fundamental quarks:

$$\begin{pmatrix}\textbf{up}\\ \textbf{down}\end{pmatrix}, \quad \begin{pmatrix}\textbf{charm}\\ \textbf{strange}\end{pmatrix}, \quad \begin{pmatrix}\textbf{truth}\\ \textbf{beauty}\end{pmatrix}.$$

There are six fundamental quarks and six nontautological symmetric logical operators, whose logical charges exhibit the same regularities [Ref 91]:

$$\mu(\mathbf{N}) = \int_0^1 (2q - 2q^2)dq = \frac{1}{3}, \qquad \mu(\mathbf{I}) = \int_0^1 (1 - 2q + 2q^2)dq = \frac{2}{3}$$

$$\mu(\wedge) = \int_0^1 q^2 dq = \frac{1}{3}, \qquad \mu(\wedge) = \int_0^1 (2q - 2q^2)dq = \frac{2}{3}$$

$$\mu(\uparrow) = \int_0^1 (1 - 2q + q^2)dq = \frac{1}{3}, \qquad \mu(\downarrow) = \int_0^1 (1 - q^2)dq = \frac{1}{3}$$

Is it merely a striking coincidence or do we lack a some fundamental insight into quark physics and some unknown relation between quarks and consciousness? Could the laws of logic, the same laws that govern intelligent thought, be important at the fundamental level? The intriguing correspondence between matrix logic and quarks inspires us to regard logic not simply as an abstract construct but as a fundamental structure underlying real physical interactions. In this framework logic obtains the status of a fundamental science, capable not only of explaining physics but predicting physics. Conversely, quantum physics could be viewed and studied as a logic in a fundamental sense. Just read the logic off from Hilbert space, and vice versa, complexify logic to obtain the Hilbert space of quantum mechanics.

According to a commonly held view, the brain 'communicates' with the outside world through the electrons of its constituent atoms. It is a general consensus that only the electrons of the outer shells are relevant to life, including the brain. Could it be that the 'seat of consciousness' is in fact 'inside' the atoms, and more weird still, inside the nucleons? Although the energy scales of quarks and the biological brain are quite incompatible, there are no obstacles to considering that the basic code of thoughts is a fundamental macroanalogue of quarks.

The ideas of physics have often been used in logic and brain science. But never the reverse. Can matrix logic predict physical relations? In this study we have been interested in the fundamental laws of the intelligent brain and not in physics. It turns out, however, that the results we have obtained go far beyond such a goal: logic is becoming relevant to physics *per se*. Quite unexpectedly, the concept of quarks derives support from matrix logic, fields generally considered to have nothing in common. In spite of the

malfunctioning of Boolean logic in the theory of quarks, in our quest to unravel the structure of nuclear matter we continue to think in terms of classical logic, which effectively reduces all logical states to either **true** or **false**. Matrix logic, naturally accommodating the ternary triangle of the **true**, **false** and **undecided**, could very well be what is required to fully resolve the quark puzzle instead of trying to force it into the procrustean bed of binary logic. Most importantly, matrix logic predicts the existence of a deeper-level fundamental half-quark or iqon charge. We have considered the charges of **symmetric** operators which are in striking correspondence to the quark charges 1/3 and 2/3. But we also have **nonsymmetric** logical operators which are consistent with charge 1/6. Such an elementary charge is carried by the creation and annihilation operators:

$$\int_0^1 < a^* > dq = \int_0^1 < a > dq = \frac{1}{6}.$$

Apparently the different charges are all multiples of the elementary or iqon charge ï. A doublet of iqons is quark 1/3; a quadruplet is quark 2/3. **TRUE** and **FALSE**, like 'nucleons' of a deeper level, are the triplets of iqons or equivalently a quark plus an iqon:

$$3\,ï = 1/3 + ï.$$

Finally, the implications **IF** and **IMPLY** are the **5**-lets or a quark plus a corresponding fraction:

$$5\,ï = 2/3 + ï = 1/3 + 2\,ï.$$

Likewise

$$5\,ï = 1 - ï.$$

Never before have the laws of logic been used in physics in a fundamental way. It would be a true achievement of matrix logic if its prediction of the existence of a half-quark could indeed be substantiated by experiment.

The conversion theorem puts the logical implications and the annihilation and creation operators into the duality relation:

$$a^* \Leftrightarrow \leftarrow \qquad \text{and} \qquad a \Leftrightarrow \rightarrow.$$

Because such a transformation must exchange the iqon charge 1/6 of a and a^* and the implication charge 5/6, one would need the quark charge to mediate the interaction between the logical and quantum field operators.

Matrix theory mixes logic and physics, leading to a unification of information and physical categories. It not only enhances the computational power of logic and solves problems inaccessible to other forms of logic, it also paves a way to the fusion of physics and logical theories. We are embarking on a challenging project whose goal is to express particle physics in logical terms and, with the aid of matrix logic, to rewrite the fundamental interactions as

logical interactions. One naturally asks in reverse: whether elementary particles are actually elementary primitives of logic, thought and consciousness? Just as we never observe an isolated quark, we cannot isolate and extract a thought from the brain for observation. Is there more to this than just a metaphor, a formal analogy, or this is a way to explore the deepest elements of human thought? Could the quark be given directly to consciousness? As we mentioned, the energy scales of quarks and of the brain are far apart, but T-duality may be a new way of conceptually connecting the physics of the large and physics of the small. This idea directs our efforts to the still poorly understood issue of the macro-micro duality, which conceivably may provide the symmetry exchange between the strong interactions of quarks and the 'weak' logical interactions. Although the numerical argument is persuasive, it alone is not enough and one could be sceptical of the conclusion in view of energy considerations. The energy constraints would prevent the brain from working on quarks literally, but a higher-level duality of dualities may explain the correspondence between the fundamental level and logic. The logical mesons which we introduced earlier are asymmetric (quasi)particles. When two conformable mesons interact in direct or reverse order, the resulting particle is symmetric, and will be compactified or boosted to a space of different dimensions, nibble or quark, suggesting that m(atrix) logic could be a new unexpected way to link QED and QCD. The quark negation acquires the third eigenvalue of energy

$$\lambda(\underline{\mathbf{YES}}) = 1, -1, 2,$$

in comparison with the pair of quantum logical numbers in the standard model:

$$\lambda(\mathbf{NOT}) = 1, -1.$$

All the facts point in the direction of an important synthesis. Quantum quantities such as energy, momentum and spin have macroscopic analogues. According to the results of this study even the 'impossible' coherent quantum superposition has found a macroanalogue in nonorientable Möbius topology. Except for quarks we have found macroanalogues for all quantum quantities, but a macroanalogue for the quarks stays a mystery. Do quarks have a macroanalogue? In a bold move we conclude that consciousness is the macroanalogue of quark and the brain behaves as it were a 'macroscopic nucleus'. The idea that the brain 'works' on quarks, if taken literally, immediately runs into conflict with the energy scales involved in quarks. Quantum thinking, however, is detached from and not bound to a substrate; it is an abstract cognitive method which provides information about actual objects but without actually describing them directly. One of the reasons we have difficulty in understanding consciousness and have not been able to put a finger on thoughts, because they are entanglements. In cognitive thoughts (nibbles) the constituent bits are in correlation, their state cannot be specified individually, only the combined state can be known, which is similar to the

nonlocality in the Einstein-Podolsky-Rosen paradox. The unexpected links between logic, particle physics and topology lead to an important synthesis. The thinking brain requires new fundamental concepts which go beyond the scope of both classical and quantum physics. The future experimental study of consciousness will depend on the identification the iqon particle, the fundamental charge 1/6 inside the quark predicted by matrix logic. In the long run we may not fully understand consciousness until we fully understand and isolate quarks. Future neuroscientists may look upon our present idea that we think with currents of electrical energy running through circuits of cellular wires of neurons in the same way as we look at Aristotle's naïve idea that the heart is the seat of the soul.

POSTSCRIPT

As a student in the late 1960s I was struck by the profundity of Leon Brillouin's conjecture that in the biosciences, as in physics, some unknown wave could be of fundamental significance. It took a major effort to find the superquantized wave and many years of study and research to obtain the operator matrix wave equation of the logical mind-brain, which now appears so beautifully simple:

$$\mathbf{IF}^{\mathbf{IF}} = \cos a^* + \sin a^*.$$

Mathematics is the key to human insight. I am happy to see the finite final result, as any physicist would be. Even though the original hope that the 'cognitive waves' could be accommodated by a first-quantized Schrödinger wavefunction did not materialize, the research effort was not in vain. Having understood the fundamental infophysics of thought, we can begin to understand consciousness.

APPENDIX

24+2 ROOTS OF IS

THE SQUARE ROOTS

$$\begin{pmatrix}1&0\\0&1\end{pmatrix},\ \begin{pmatrix}-1&0\\0&-1\end{pmatrix},\ \begin{pmatrix}0&1\\1&0\end{pmatrix},\ \begin{pmatrix}0&-1\\-1&0\end{pmatrix},\ \begin{pmatrix}1&0\\0&-1\end{pmatrix},\ \begin{pmatrix}-1&0\\0&1\end{pmatrix},\ \begin{pmatrix}-1&1\\0&1\end{pmatrix},$$
$$\begin{pmatrix}1&-1\\0&-1\end{pmatrix},\ \begin{pmatrix}1&1\\0&-1\end{pmatrix},\ \begin{pmatrix}-1&-1\\0&1\end{pmatrix},\ \begin{pmatrix}-1&0\\1&1\end{pmatrix},\ \begin{pmatrix}1&0\\-1&-1\end{pmatrix},\ \begin{pmatrix}1&0\\1&-1\end{pmatrix},\ \begin{pmatrix}-1&0\\-1&1\end{pmatrix}.$$

THE THIRD ROOTS

$$\begin{pmatrix}0&1\\-1&-1\end{pmatrix},\ \begin{pmatrix}0&-1\\1&-1\end{pmatrix},\ \begin{pmatrix}-1&-1\\1&0\end{pmatrix},\ \begin{pmatrix}-1&1\\-1&0\end{pmatrix}.$$

THE FORTH ROOTS

$$\begin{pmatrix}0&1\\-1&0\end{pmatrix},\ \begin{pmatrix}0&-1\\1&0\end{pmatrix}.$$

THE SIXTH ROOTS

$$\begin{pmatrix}0&-1\\1&1\end{pmatrix},\ \begin{pmatrix}0&1\\-1&1\end{pmatrix},\ \begin{pmatrix}1&1\\-1&0\end{pmatrix},\ \begin{pmatrix}1&-1\\1&0\end{pmatrix}.$$

THE MATRIX ROOTS

$$\sqrt[\begin{pmatrix}0&1\\0&0\end{pmatrix}]{\begin{pmatrix}1&0\\0&1\end{pmatrix}} = \begin{pmatrix}1&1\\0&1\end{pmatrix},\qquad \sqrt[\begin{pmatrix}0&0\\1&0\end{pmatrix}]{\begin{pmatrix}1&0\\0&1\end{pmatrix}} = \begin{pmatrix}1&0\\1&1\end{pmatrix}.$$

40 FACTORIZATIONS OF NOT

$\begin{pmatrix}0&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $\begin{pmatrix}1&0\\0&1\end{pmatrix}\begin{pmatrix}0&1\\1&0\end{pmatrix}$, $\begin{pmatrix}0&1\\-1&0\end{pmatrix}\begin{pmatrix}-1&0\\0&1\end{pmatrix}$, $\begin{pmatrix}-1&0\\0&1\end{pmatrix}\begin{pmatrix}0&-1\\1&0\end{pmatrix}$,

$\begin{pmatrix}0&-1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\0&-1\end{pmatrix}$, $\begin{pmatrix}1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&1\\-1&0\end{pmatrix}$, $\begin{pmatrix}0&-1\\-1&0\end{pmatrix}\begin{pmatrix}-1&0\\0&-1\end{pmatrix}$, $\begin{pmatrix}-1&0\\0&-1\end{pmatrix}\begin{pmatrix}0&-1\\-1&0\end{pmatrix}$,

$\begin{pmatrix}0&1\\1&1\end{pmatrix}\begin{pmatrix}1&-1\\0&1\end{pmatrix}$, $\begin{pmatrix}0&1\\-1&1\end{pmatrix}\begin{pmatrix}-1&1\\0&1\end{pmatrix}$, $\begin{pmatrix}0&-1\\1&1\end{pmatrix}\begin{pmatrix}1&1\\0&-1\end{pmatrix}$, $\begin{pmatrix}0&1\\-1&-1\end{pmatrix}\begin{pmatrix}-1&-1\\0&1\end{pmatrix}$,

$\begin{pmatrix}0&-1\\1&-1\end{pmatrix}\begin{pmatrix}1&-1\\0&-1\end{pmatrix}$, $\begin{pmatrix}0&-1\\-1&1\end{pmatrix}\begin{pmatrix}-1&-1\\0&-1\end{pmatrix}$, $\begin{pmatrix}0&-1\\-1&-1\end{pmatrix}\begin{pmatrix}-1&1\\0&-1\end{pmatrix}$, $\begin{pmatrix}0&1\\1&-1\end{pmatrix}\begin{pmatrix}1&1\\0&1\end{pmatrix}$,

$\begin{pmatrix}1&1\\0&1\end{pmatrix}\begin{pmatrix}-1&1\\1&0\end{pmatrix}$, $\begin{pmatrix}1&1\\0&-1\end{pmatrix}\begin{pmatrix}1&1\\-1&0\end{pmatrix}$, $\begin{pmatrix}-1&1\\0&1\end{pmatrix}\begin{pmatrix}1&-1\\1&0\end{pmatrix}$, $\begin{pmatrix}1&-1\\0&-1\end{pmatrix}\begin{pmatrix}-1&1\\-1&0\end{pmatrix}$,

$\begin{pmatrix}-1&-1\\0&1\end{pmatrix}\begin{pmatrix}-1&-1\\1&0\end{pmatrix}$, $\begin{pmatrix}-1&-1\\0&-1\end{pmatrix}\begin{pmatrix}1&-1\\-1&0\end{pmatrix}$, $\begin{pmatrix}-1&1\\0&-1\end{pmatrix}\begin{pmatrix}-1&-1\\-1&0\end{pmatrix}$, $\begin{pmatrix}1&-1\\0&1\end{pmatrix}\begin{pmatrix}1&1\\1&0\end{pmatrix}$,

$\begin{pmatrix}1&0\\1&1\end{pmatrix}\begin{pmatrix}0&1\\1&-1\end{pmatrix}$, $\begin{pmatrix}-1&0\\1&1\end{pmatrix}\begin{pmatrix}0&-1\\1&1\end{pmatrix}$, $\begin{pmatrix}1&0\\1&-1\end{pmatrix}\begin{pmatrix}0&1\\-1&1\end{pmatrix}$, $\begin{pmatrix}-1&0\\-1&1\end{pmatrix}\begin{pmatrix}0&-1\\1&-1\end{pmatrix}$,

$\begin{pmatrix}1&0\\-1&-1\end{pmatrix}\begin{pmatrix}0&1\\-1&-1\end{pmatrix}$, $\begin{pmatrix}-1&0\\-1&-1\end{pmatrix}\begin{pmatrix}0&-1\\-1&1\end{pmatrix}$, $\begin{pmatrix}-1&0\\1&-1\end{pmatrix}\begin{pmatrix}0&-1\\-1&-1\end{pmatrix}$, $\begin{pmatrix}1&0\\-1&1\end{pmatrix}\begin{pmatrix}0&1\\1&1\end{pmatrix}$,

$\begin{pmatrix}1&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\-1&1\end{pmatrix}$, $\begin{pmatrix}1&-1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\1&-1\end{pmatrix}$, $\begin{pmatrix}1&1\\-1&0\end{pmatrix}\begin{pmatrix}-1&0\\1&1\end{pmatrix}$, $\begin{pmatrix}-1&-1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\-1&-1\end{pmatrix}$,

$\begin{pmatrix}-1&1\\-1&0\end{pmatrix}\begin{pmatrix}-1&0\\-1&1\end{pmatrix}$, $\begin{pmatrix}1&-1\\-1&0\end{pmatrix}\begin{pmatrix}-1&0\\-1&-1\end{pmatrix}$, $\begin{pmatrix}-1&-1\\-1&0\end{pmatrix}\begin{pmatrix}-1&0\\1&-1\end{pmatrix}$, $\begin{pmatrix}-1&1\\1&0\end{pmatrix}\begin{pmatrix}1&0\\1&1\end{pmatrix}$.

136 COMMUTATORS OF NOT

1	-1	-1	1	-1	0	-1	1	-1
2	-1	-1	1	-1	0	0	0	-1
3	-1	-1	1	-1	0	1	-1	-1
4	-1	-1	1	-1	1	-1	1	0
5	-1	-1	1	-1	1	0	0	0
6	-1	-1	1	-1	1	1	-1	0
7	-1	-1	1	0	-1	-1	1	-1
8	-1	-1	1	0	0	-1	1	0
9	-1	-1	1	0	0	0	0	-1
10	-1	-1	1	0	1	-1	1	1
11	-1	-1	1	0	1	0	0	0
12	-1	-1	1	0	1	1	-1	-1
13	-1	-1	1	1	-1	-1	1	0
14	-1	-1	1	1	0	-1	1	1
15	-1	-1	1	1	0	0	0	-1
16	-1	-1	1	1	1	0	0	0
17	-1	0	0	0	-1	-1	1	-1
18	-1	0	0	0	-1	-1	1	0
19	-1	0	0	0	-1	-1	1	1
20	-1	0	0	0	0	-1	1	-1
21	-1	0	0	0	0	-1	1	0
22	-1	0	0	0	0	-1	1	1
23	-1	0	0	0	1	-1	1	-1
24	-1	0	0	0	1	-1	1	0
25	-1	0	0	0	1	-1	1	1
26	-1	1	-1	-1	-1	-1	1	0
27	-1	1	-1	-1	-1	0	0	0
28	-1	1	-1	-1	-1	1	-1	0
29	-1	1	-1	-1	0	-1	1	1
30	-1	1	-1	-1	0	0	0	1
31	-1	1	-1	-1	0	1	-1	1
32	-1	1	-1	0	-1	-1	1	-1
33	-1	1	-1	0	-1	0	0	0
34	-1	1	-1	0	-1	1	-1	1
35	-1	1	-1	0	0	-1	1	0
36	-1	1	-1	0	0	0	0	1
37	-1	1	-1	0	1	-1	1	1
38	-1	1	-1	1	-1	0	0	0
39	-1	1	-1	1	0	-1	1	-1
40	-1	1	-1	1	0	0	0	1
41	-1	1	-1	1	1	-1	1	0
42	0	-1	1	-1	-1	1	-1	-1
43	0	-1	1	-1	0	0	0	-1
44	0	-1	1	-1	0	1	-1	0
45	0	-1	1	-1	1	-1	1	-1
46	0	-1	1	-1	1	0	0	0
47	0	-1	1	-1	1	1	-1	1
48	0	-1	1	0	0	-1	1	-1
49	0	-1	1	0	0	0	0	-1
50	0	-1	1	0	0	1	-1	-1
51	0	-1	1	0	1	-1	1	0
52	0	-1	1	0	1	0	0	0
53	0	-1	1	0	1	1	-1	0
54	0	-1	1	1	-1	-1	1	-1
55	0	-1	1	1	0	-1	1	0
56	0	-1	1	1	0	0	0	-1
57	0	-1	1	1	1	-1	1	1
58	0	-1	1	1	1	0	0	0
59	0	-1	1	1	1	1	-1	-1
60	0	0	0	-1	-1	1	-1	-1
61	0	0	0	-1	-1	1	-1	0
62	0	0	0	-1	-1	1	-1	1
63	0	0	0	-1	0	1	-1	-1
64	0	0	0	-1	0	1	-1	0
65	0	0	0	-1	0	1	-1	1
66	0	0	0	-1	1	1	-1	-1
67	0	0	0	-1	1	1	-1	0
68	0	0	0	-1	1	1	-1	1
69	0	0	0	1	-1	-1	1	-1
70	0	0	0	1	-1	-1	1	0
71	0	0	0	1	-1	-1	1	1
72	0	0	0	1	0	-1	1	-1
73	0	0	0	1	0	-1	1	0
74	0	0	0	1	0	-1	1	1
75	0	0	0	1	1	-1	1	-1
76	0	0	0	1	1	-1	1	0
77	0	0	0	1	1	-1	1	1
78	0	1	-1	-1	-1	-1	1	1
79	0	1	-1	-1	-1	0	0	0
80	0	1	-1	-1	-1	1	-1	-1
81	0	1	-1	-1	0	0	0	1
82	0	1	-1	-1	0	1	-1	0
83	0	1	-1	-1	1	1	-1	1
84	0	1	-1	0	-1	-1	1	0
85	0	1	-1	0	-1	0	0	0
86	0	1	-1	0	-1	1	-1	0
87	0	1	-1	0	0	-1	1	1
88	0	1	-1	0	0	0	0	1
89	0	1	-1	0	0	1	-1	1
90	0	1	-1	1	-1	-1	1	-1
91	0	1	-1	1	-1	0	0	0
92	0	1	-1	1	-1	1	-1	1

93	0	1	-1	1	0	-1	1	0
94	0	1	-1	1	0	0	0	1
95	0	1	-1	1	1	-1	1	1
96	1	-1	1	-1	-1	1	-1	0
97	1	-1	1	-1	0	0	0	-1
98	1	-1	1	-1	0	1	-1	1
99	1	-1	1	-1	1	0	0	0
100	1	-1	1	0	-1	1	-1	-1
101	1	-1	1	0	0	0	0	-1
102	1	-1	1	0	0	1	-1	0
103	1	-1	1	0	1	-1	1	-1
104	1	-1	1	0	1	0	0	0
105	1	-1	1	0	1	1	-1	1
106	1	-1	1	1	0	-1	1	-1
107	1	-1	1	1	0	0	0	-1
108	1	-1	1	1	0	1	-1	-1
109	1	-1	1	1	1	-1	1	0
110	1	-1	1	1	1	0	0	0
111	1	-1	1	1	1	1	-1	0
112	1	0	0	0	-1	1	-1	-1
113	1	0	0	0	-1	1	-1	0
114	1	0	0	0	-1	1	-1	1

115	1	0	0	0	0	1	-1	-1
116	1	0	0	0	0	1	-1	0
117	1	0	0	0	0	1	-1	1
118	1	0	0	0	1	1	-1	-1
119	1	0	0	0	1	1	-1	0
120	1	0	0	0	1	1	-1	1
121	1	1	-1	-1	-1	0	0	0
122	1	1	-1	-1	0	0	0	1
123	1	1	-1	-1	0	1	-1	-1
124	1	1	-1	-1	1	1	-1	0
125	1	1	-1	0	-1	-1	1	1
126	1	1	-1	0	-1	0	0	0
127	1	1	-1	0	-1	1	-1	-1
128	1	1	-1	0	0	0	0	1
129	1	1	-1	0	0	1	-1	0
130	1	1	-1	0	1	1	-1	1
131	1	1	-1	1	-1	-1	1	0
132	1	1	-1	1	-1	0	0	0
133	1	1	-1	1	-1	1	-1	0
134	1	1	-1	1	0	-1	1	1
135	1	1	-1	1	0	0	0	1
136	1	1	-1	1	0	1	-1	1

40 ANTICOMMUTATORS OF NOT

1	-1	-1	-1	0	0	0	0	-1
2	-1	-1	1	0	0	-1	-1	0
3	-1	0	0	0	0	-1	-1	-1
4	-1	0	0	0	0	-1	-1	0
5	-1	0	0	0	0	-1	-1	1
6	-1	1	-1	0	0	-1	-1	0
7	-1	1	1	0	0	0	0	1
8	0	-1	-1	-1	-1	0	0	0
9	0	-1	-1	0	-1	-1	1	0
10	0	-1	-1	0	-1	0	0	0
11	0	-1	-1	0	-1	1	-1	0
12	0	-1	-1	0	0	-1	1	-1
13	0	-1	-1	0	0	0	0	-1
14	0	-1	-1	0	0	1	-1	-1
15	0	-1	-1	1	-1	0	0	0
16	0	-1	1	-1	0	-1	-1	0
17	0	-1	1	1	0	1	1	0
18	0	0	0	-1	-1	-1	-1	0
19	0	0	0	-1	0	-1	-1	0
20	0	0	0	-1	1	-1	-1	0

21	0	0	0	1	-1	1	1	0
22	0	0	0	1	0	1	1	0
23	0	0	0	1	1	1	1	0
24	0	1	-1	-1	0	-1	-1	0
25	0	1	-1	1	0	1	1	0
26	0	1	1	-1	1	0	0	0
27	0	1	1	0	0	-1	1	1
28	0	1	1	0	0	0	0	1
29	0	1	1	0	0	1	-1	1
30	0	1	1	0	1	-1	1	0
31	0	1	1	0	1	0	0	0
32	0	1	1	0	1	1	-1	0
33	0	1	1	1	1	0	0	0
34	1	-1	-1	0	0	0	0	-1
35	1	-1	1	0	0	1	1	0
36	1	0	0	0	0	1	1	-1
37	1	0	0	0	0	1	1	0
38	1	0	0	0	0	1	1	1
39	1	1	-1	0	0	1	1	0
40	1	1	1	0	0	0	0	1

180 COMMUTATORS OF THE DIAGONALIZED NOT

1	-1	-1	-1	-1	-1	0	-1	-1
2	-1	-1	-1	-1	-1	1	0	-1
3	-1	-1	-1	-1	0	0	-1	0
4	-1	-1	-1	-1	0	1	0	0
5	-1	-1	-1	-1	1	0	-1	1
6	-1	-1	-1	-1	1	1	0	1
7	-1	-1	0	-1	-1	-1	-1	-1
8	-1	-1	0	-1	-1	0	-1	-1
9	-1	-1	0	-1	-1	1	-1	-1
10	-1	-1	0	-1	0	-1	-1	0
11	-1	-1	0	-1	0	0	-1	0
12	-1	-1	0	-1	0	1	-1	0
13	-1	-1	0	-1	1	-1	-1	1
14	-1	-1	0	-1	1	0	-1	1
15	-1	-1	0	-1	1	1	-1	1
16	-1	-1	1	-1	-1	-1	0	-1
17	-1	-1	1	-1	-1	0	-1	-1
18	-1	-1	1	-1	0	-1	0	0
19	-1	-1	1	-1	0	0	-1	0
20	-1	-1	1	-1	1	-1	0	1
21	-1	-1	1	-1	1	0	-1	1
22	-1	0	-1	-1	-1	1	-1	-1
23	-1	0	-1	-1	-1	1	0	-1
24	-1	0	-1	-1	-1	1	1	-1
25	-1	0	-1	-1	0	1	-1	0
26	-1	0	-1	-1	0	1	0	0
27	-1	0	-1	-1	0	1	1	0
28	-1	0	-1	-1	1	1	-1	1
29	-1	0	-1	-1	1	1	0	1
30	-1	0	-1	-1	1	1	1	1
31	-1	0	1	-1	-1	-1	-1	-1
32	-1	0	1	-1	-1	-1	0	-1
33	-1	0	1	-1	-1	-1	1	-1
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GLOSSARY

COGNITIVE DIAGRAM is a topology corresponding to the mathematical structure of m(atrix) logic.

CONSCIOUSNESS is a nonorientable singularity of a self-observing system whose output is fed back to the input.

FACTOID is an intuitive and theoretically consistent but experimentally untested fact.

GRAY HOLE is a topological analog of a black hole, characterizing a vacuum information system, almost isolated from the outside universe.

INFOPHYSICS is a general theory of physics which incorporates information into physical description.

IMPROBABILITY is a quantized truth-value, defined outside the unit interval of classical convex probability.

INTELLIGENCE CODE is a universal renormalized logical calculus with intrinsic self-correction.

INWORD (inner, intelligent, interacting) is a codeword, formed through interaction of codewords.

IQON (half-quark charge) is an elementary charge 1/6, predicted in matrix logic.

KNOWL(EDGE) OPERATOR is an operator, which acting on arbitrary logical state, universally computes ("knows") its own eigenstate.

L-BRANE (logical brane, membrane) is a cognitively extended object with matrix degrees of freedom on which a thought can end.

LOGICAL GAUGE (tautology) is a constant of motion, unaffected under arbitrary local change of interpretation of logical variables.

LOGICAL MOMENTUM is a matrix divisor of zero (vacuum), equal by average to the average of the differentiation operator.

LOGICAL QUANTUM NUMBER is an eigenvalue(s) of a matrix-logical operator.

MATRIX STATISTICS is a supersymmetric logical ordering, containing bosonic and fermionic subsymmetries as the two eigenstate limits.

NIBBLE is a free logical operator, formed as an entanglement of four elementary bits.

QUANTUM COMPUTER is a computational system which makes use of quantum interference and entanglement of information bits.

SECRET METRIC is a pseudo-Euclidian cognitive separation invariably equal to a linear logical distance (relative truth-value).

SEPTONIC LOGIC (aesthetic consciousness) is a fundamental (sub)octonionic limit of human intelligence.

SUPERBIT (undecided bit, quantum bit, qubit) is a coherent superposition of orthogonal Boolean bits, first identified in matrix logic.

SUPRASYMMETRY is an invariance of a logical operation with respect to addition and product.

TIME OPERATOR is a (bio)logical antisymmetric comparison operator with imaginary eigenvalues.

TOPOLOGICAL CHARGE is a defect (knot) on a field line which cannot be smoothed out.

TRUTH INTEGRAL is a logical sum of all possible truth histories, independent of the variable of integration.

VIRTUAL LOGIC is a system of not cognizable operations of the thought process that only have logical meaning in their ultimate output.